Vor(P), DT(P), and F-rep – towards Optimal Modeling

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Agenda

- Motivation
- Computational Geometry
  - Methodology, Algorithmic Paradigms
  - Voronoi diagram
  - Delaunay triangulation
- Functional Representation
  - Definitions
  - HyperFun
  - Examples

Graphics & Visual Computing

ACM Computing Curriculum
at http://www.computer.org/education/cc2001/final/gv.htm:

The area encompassed by Graphics and Visual Computing (GV) is divided into four interrelated fields:

- *Computer graphics.*
- *Visualization.*
- *Virtual reality.*
- *Computer vision.*

Computer Graphics

Computer graphics is the art and science of communicating information using images that are generated and presented through computation. This requires:

(a) the design and construction of models that represent information in ways that support the creation and viewing of images,

(b) the design of devices and techniques through which the person may interact with the model or the view,

(c) the creation of techniques for rendering the model, and

(d) the design of ways the images may be preserved. The goal of comput graphics is to engage the person's visual centers alongside other cognitive centers in understanding.

„Computer Graphics...

- … can be formulated as a radiometrically „weighted“ counterpart of computational geometry…
- … rendering is done through the application of a simulation process to quantitative models of light and materials to predict/synthesize appearance“
-
- D. Dobkin & S. Teller, 1999

Computer Graphics...

- ... must account geometry
- material properties: reflectance/color, refractive index, opacity, and (for light sources) emmisivity
- radiometry
- output for viewing: explicitly or implicitly psychophysics

- by D. Dobkin & S. Teller, 1999

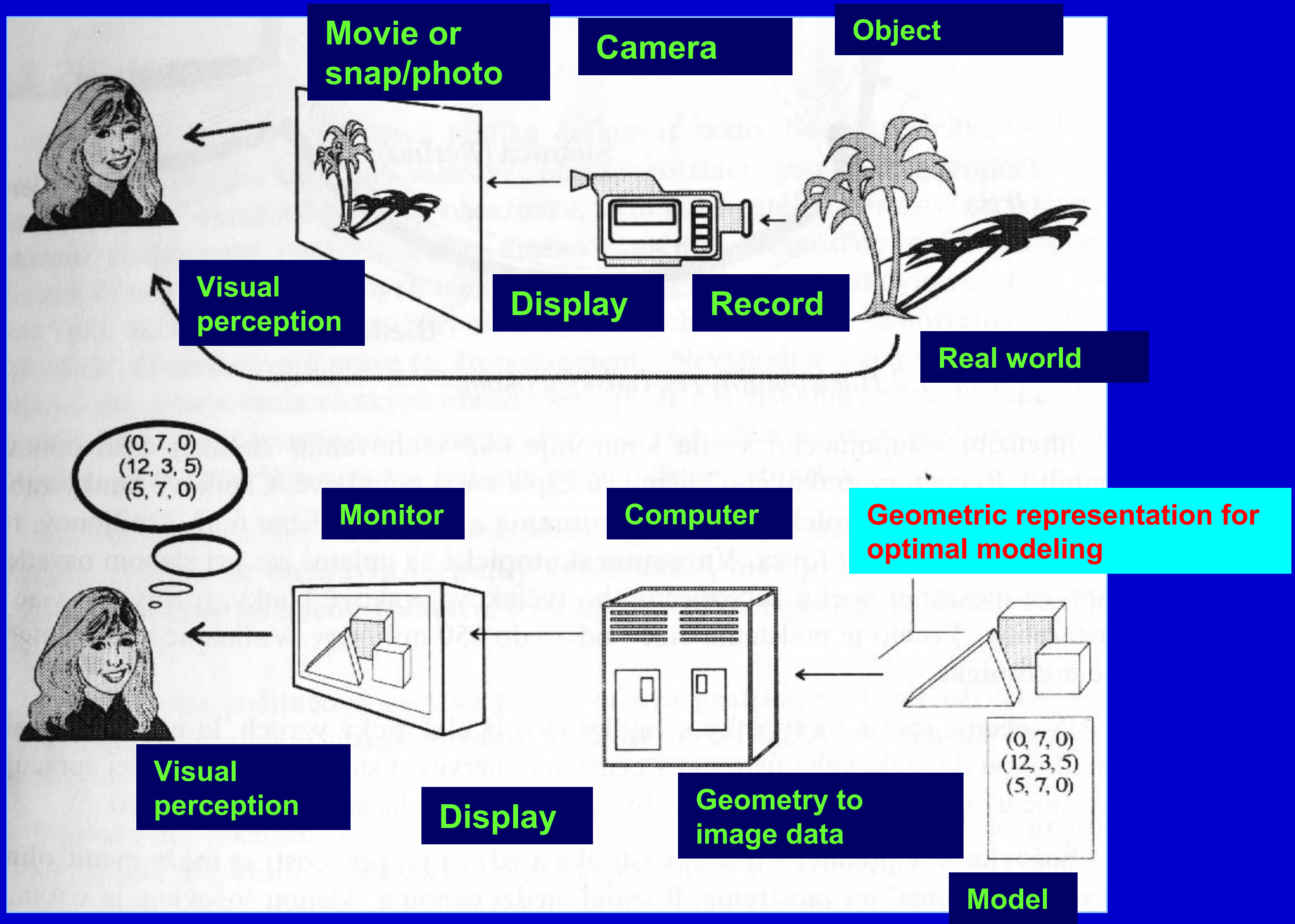
Material Description

Geometric Description

Light Sources Description

Image Synthesis (Rendering Algorithm)

- # Analogy: photography & computer graphics



- ***ISO: Computer graphics:*** *methods & techniques for construction, manipulation, storage and displaying pictures using computer.*

Object Representations

- Point-based Graphics
- Curves and Surfaces
- Solid Modeling
  - Boundary Representation (mesh, MR)
  - Spatial Enumeration Models
    - Spatial-Occupancy Enumeration (Voxel)
    - Binary Space Partitioning (BSP) Trees
    - Octrees
  - Constructive Solid Geometry (CSG)
  - Function Representation (F-rep)

Object Representations

- **Elementary Objects**
  - Primitives, regular polyhedra, ...
  - Sweeps
  - Free-form patches
  - (Super-)Quadrics
  - Terrain (DTM, DEM)
  - Fractal Mountains
  - Soft Objects
  - Particle Systems
  - Natural Phenomena…
- **Transformations**
  - linear ones
  - twist, blending ... (Verbiegeoperationen)
  - local operations
- **Combining methods**
  - Boolean Operations with Elementary Objects (CSG)
  - F-rep
  - (Solid Modeler UI)

Criteria

Modeling:

- Representation Power
- Transformation / Combination
- Interactivity Support
- Multiple Use, Generality

Rendering:

- Representation Precision
- Memory Requirements

Modeling

Task:

Create the Object Description for later Processing within the Rendering- and Output-Modul.

Generated via:

- User Interaction
- Automatically (eg „Object-Scanner“, range images, ...)

B-rep ->Volume Graphics

Gallery at :

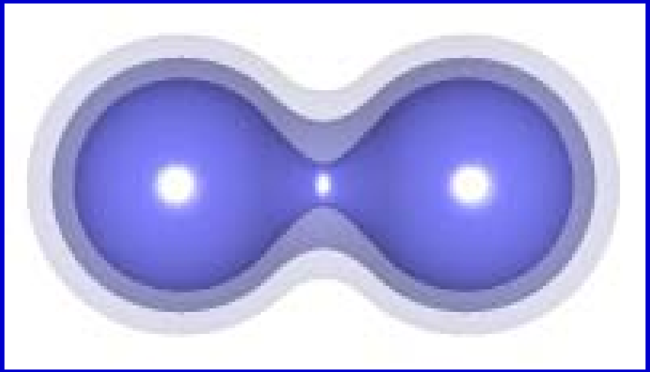
- *http://vg.swan.ac.uk/gallery/index.html*

Book:

- – ***Chen et al. 2000. Volume Graphics.***

vlib - GNU library at :

- *http://vg.swan.ac.uk/*



B-rep -> Point-based Graphics

Gallery at ETH:

- http://graphics.ethz.ch/pointshop3d/gallery.html
- **Book:**
  - No book up to now.

PointShop3D at ETH:

- http://graphics.ethz.ch/pointshop3d/

B-rep -> Quadrics

- **Idea:**
  - Quadrics are all objects, which is possible to describe using quadratic polynomials.

- **Definition:**
  - explicitly :

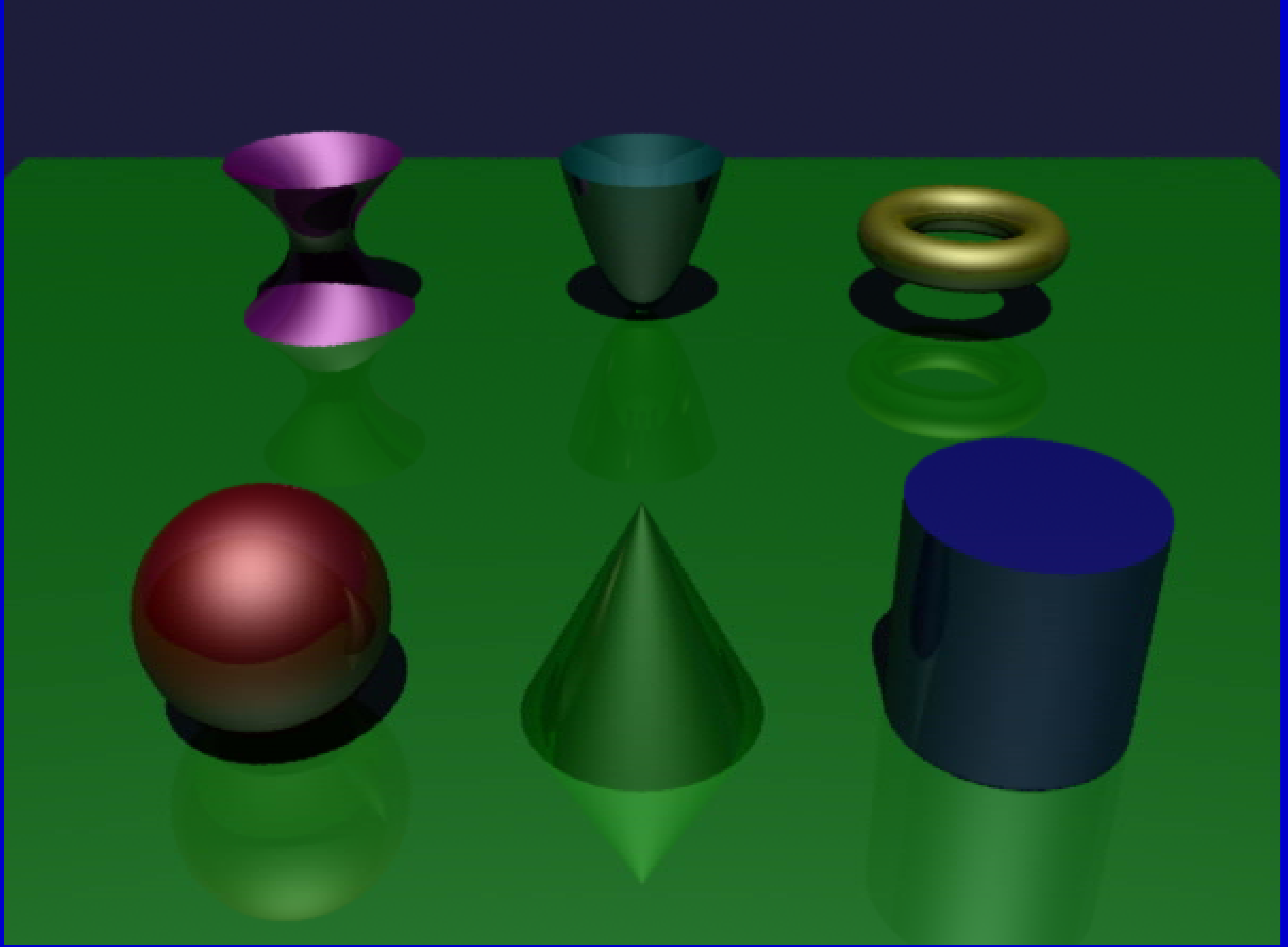
$$x^2 + y^2 + z^2 = r^2$$

  - parametric:

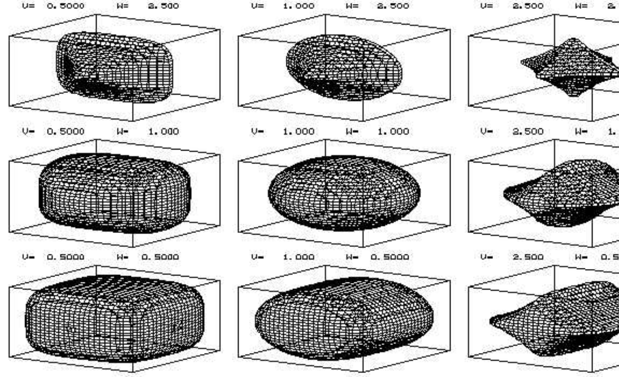
$$x = r\cos\alpha\cos\beta$$

$$y = r\cos\alpha\sin\beta$$

$$z = r\sin\alpha$$



B-rep -> Super-Quadrics



Superellipsoids by A. Pasko

Extending the idea:

Quadric is just the special case of Super-Quadric, when the exponents are general (not all equal to 2).

Examples

– Hyperellipsoid:

$$|x|^{n1} + |y|^{n2} + |z|^{n3} = 1$$

– Hypertorus:

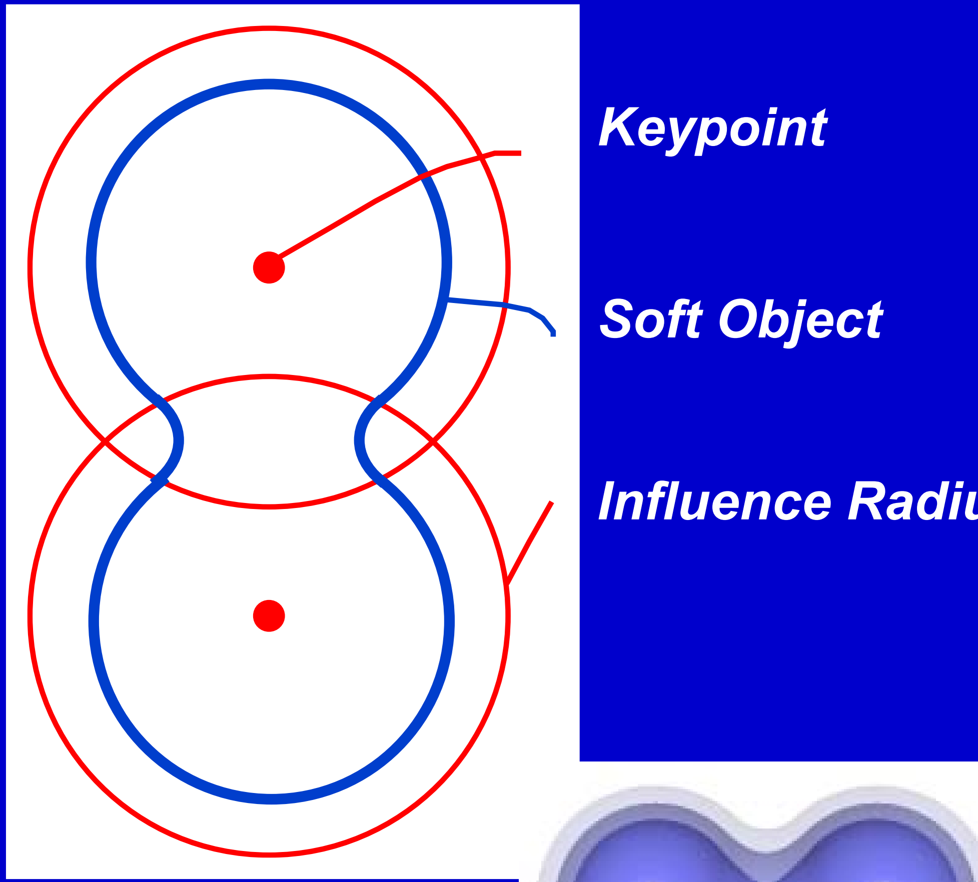
$$\left(\left(|x|^{n} + |y|^{n} \right)^{1/n} - 1 \right)^{m} + z^{m} = r^{m}$$

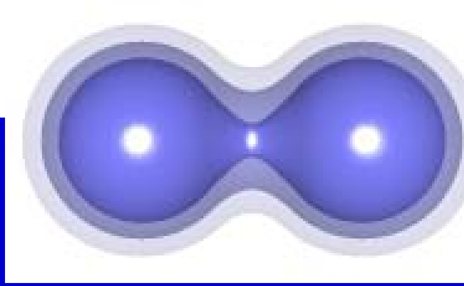
B-rep -> Soft Objects

Definition:

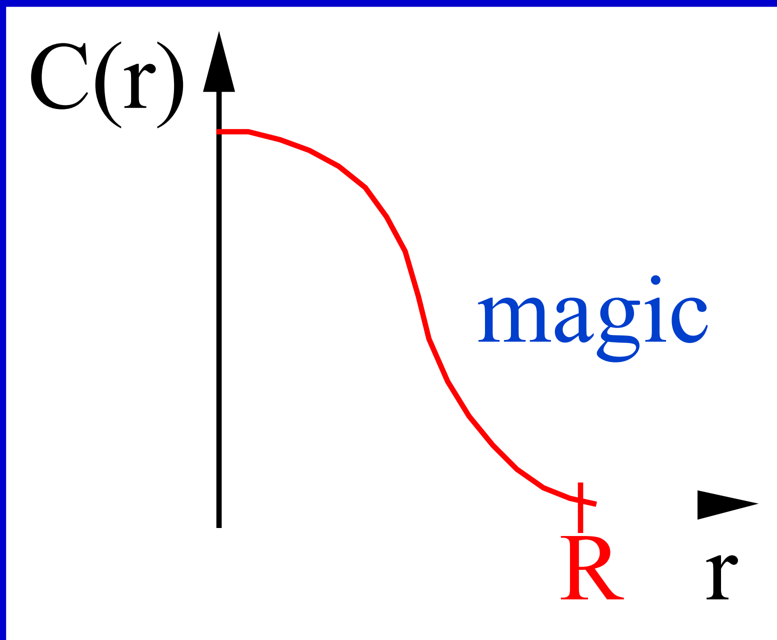
Potential field surrounds the keypoints.

The soft object is defined by isosurfaces in the field.





Influence ß-function (Kraftfeld)



$$C(r) = 2\frac{r^3}{R^3} - 3\frac{r^2}{R^2} + 1$$

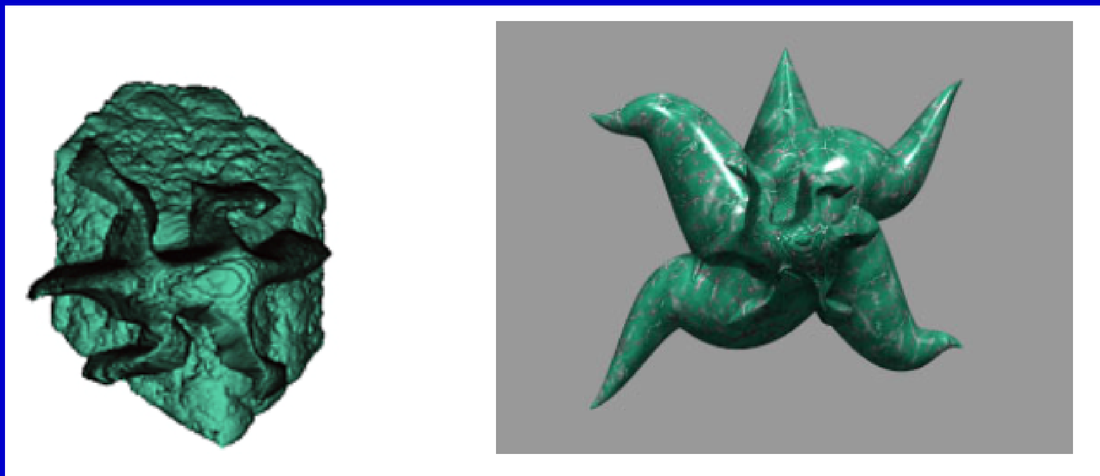
oder

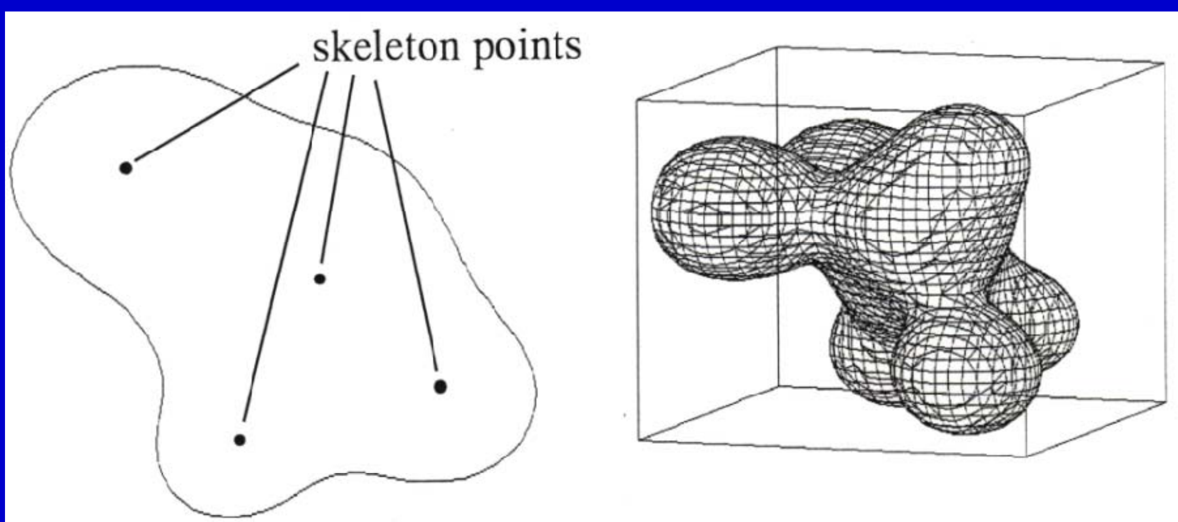
$$C(r) = a\frac{r^6}{R^6} + b\frac{r^4}{R^4} + c\frac{r^2}{R^2} +$$

$$a = -0.4 \quad b = 1.8 \quad c = -2.4$$

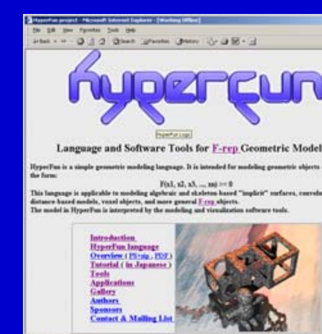
There are other forms by other authors, motivated by various requirements, see the survey by (Pasko, 1995).

E.g. – blobs, skeletal surfaces, …

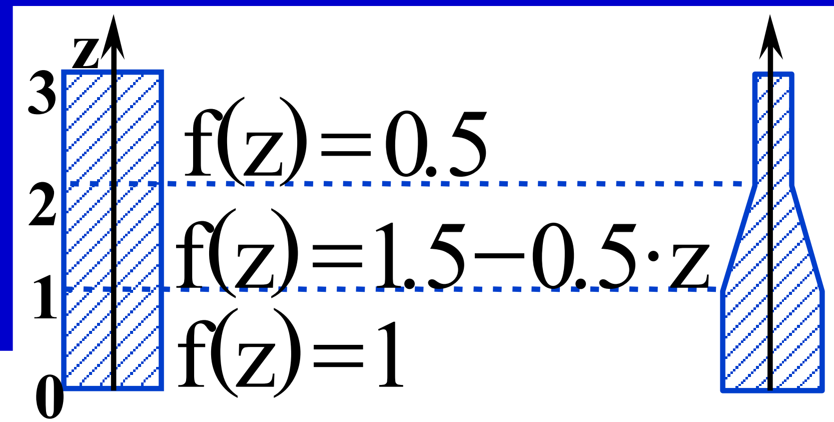


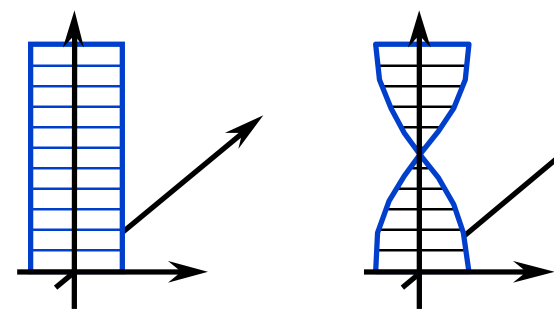


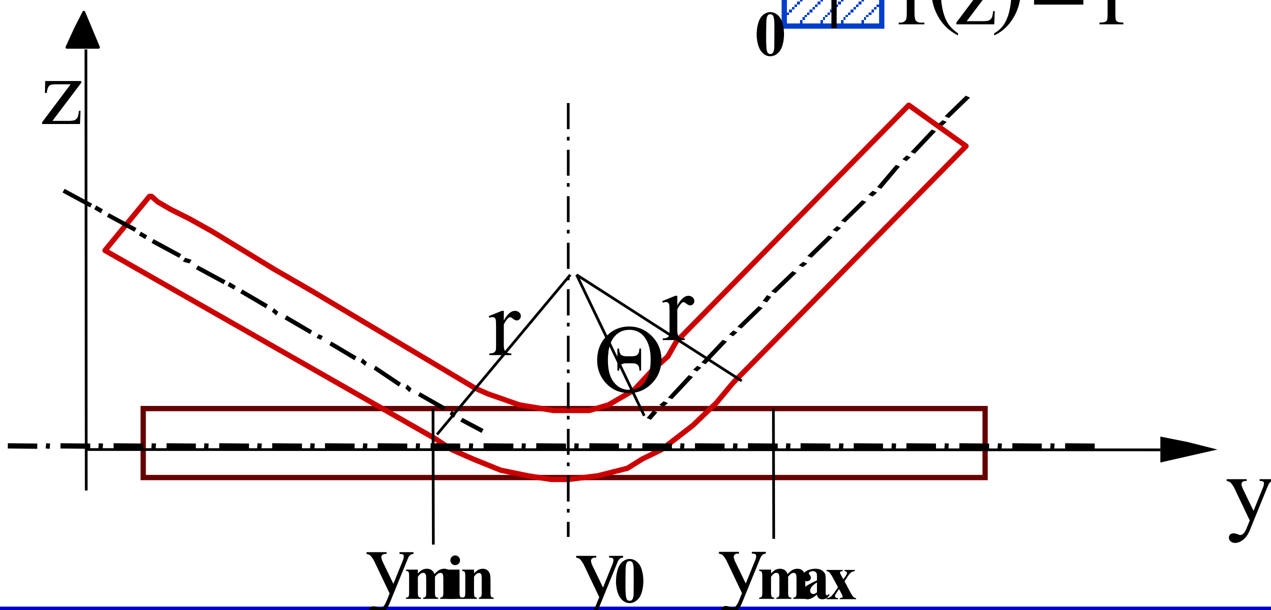
Images by A. Sherstyuk, A. Pasko, HyperFun page…



What about Unary Operations? Tapering, Twist, Bend...







What about Binary Operations? e.g. Parametric Blending

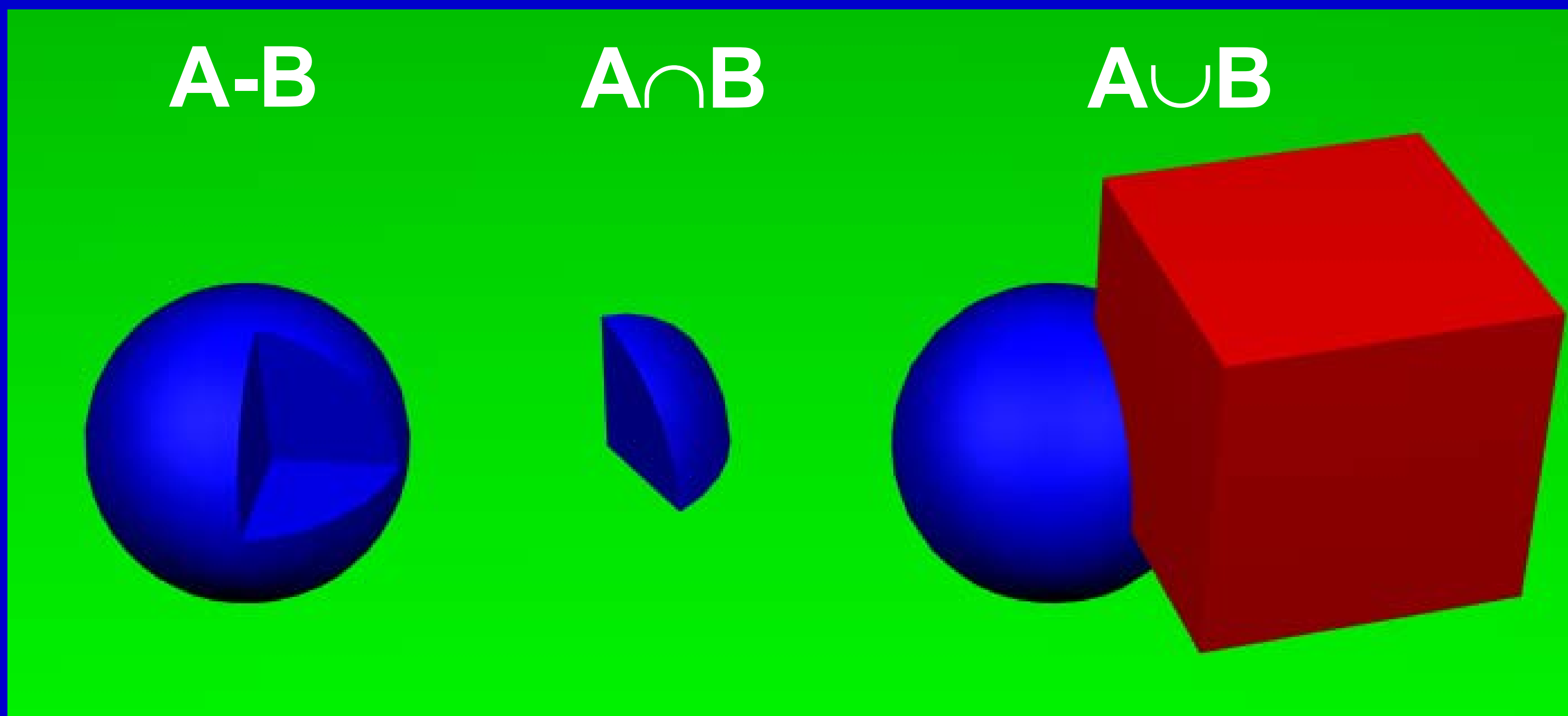
- Numbers, a, b, 0.5*(a + b)
- Points A, B, 0.3*A + 0.7*B
- Rotations, 4-tuples, quaternions
- Curve construction as weighted sum of points
- Surfaces
- Images… brightness, contrast, saturation, sharpening… image analogies, SIGGRAPH 2001
- … Warping, Morphing, Caricatures

Hm, Constructive Solid Geometry

- Composition of primitives
- Primitives: sphere, cone, cube, cylinder, ...
- Operations: $\cap$, -, $\cup$, ...
- Primitives in the leafs and operations build the rest nodes of the CSG-tree

Boolean Operations

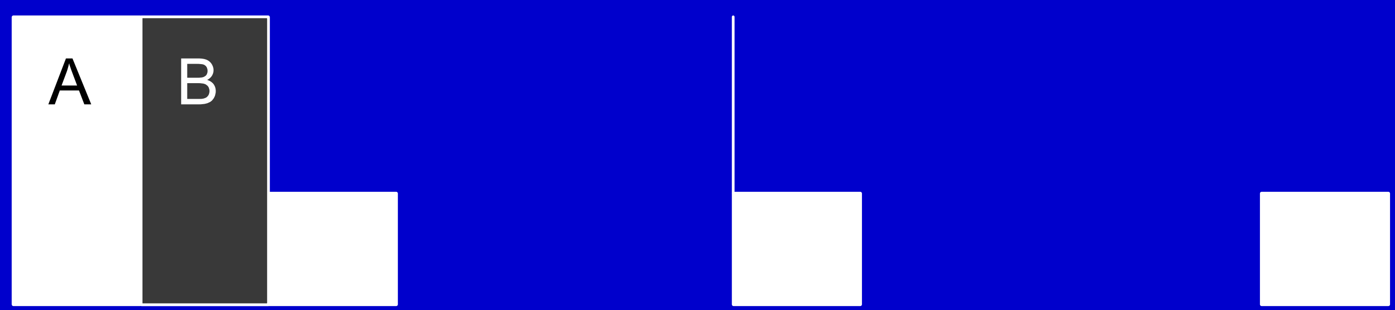
Using 3 operators enables for all possible combinations



Regular Boolean Operations

Basic Operations:

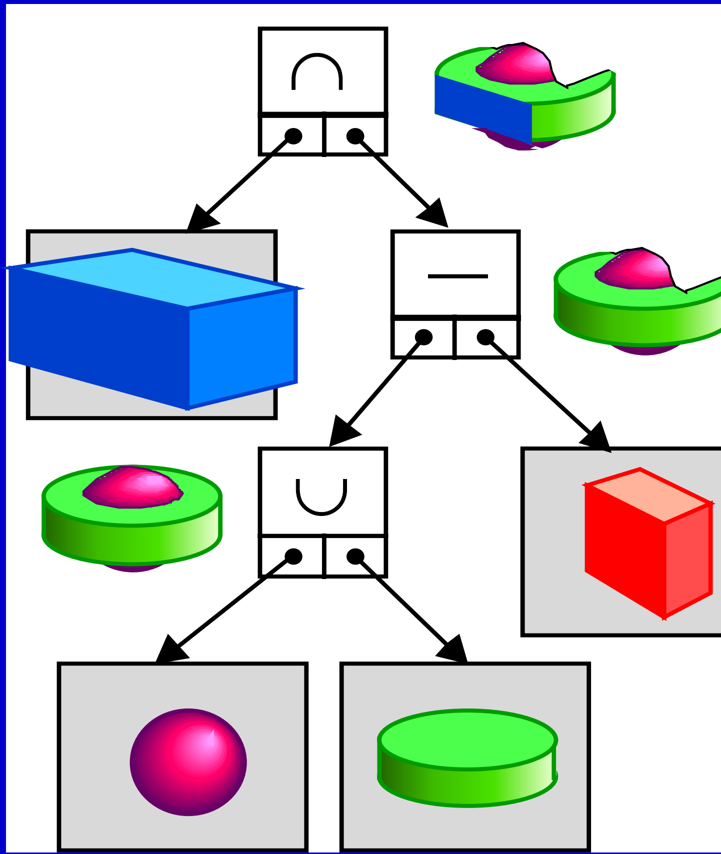
- Union $\cup$ *
- Intersection $\cap$ *
- Subtraction -*



$A \cap B$ $\qquad$ $A \cap^* B$

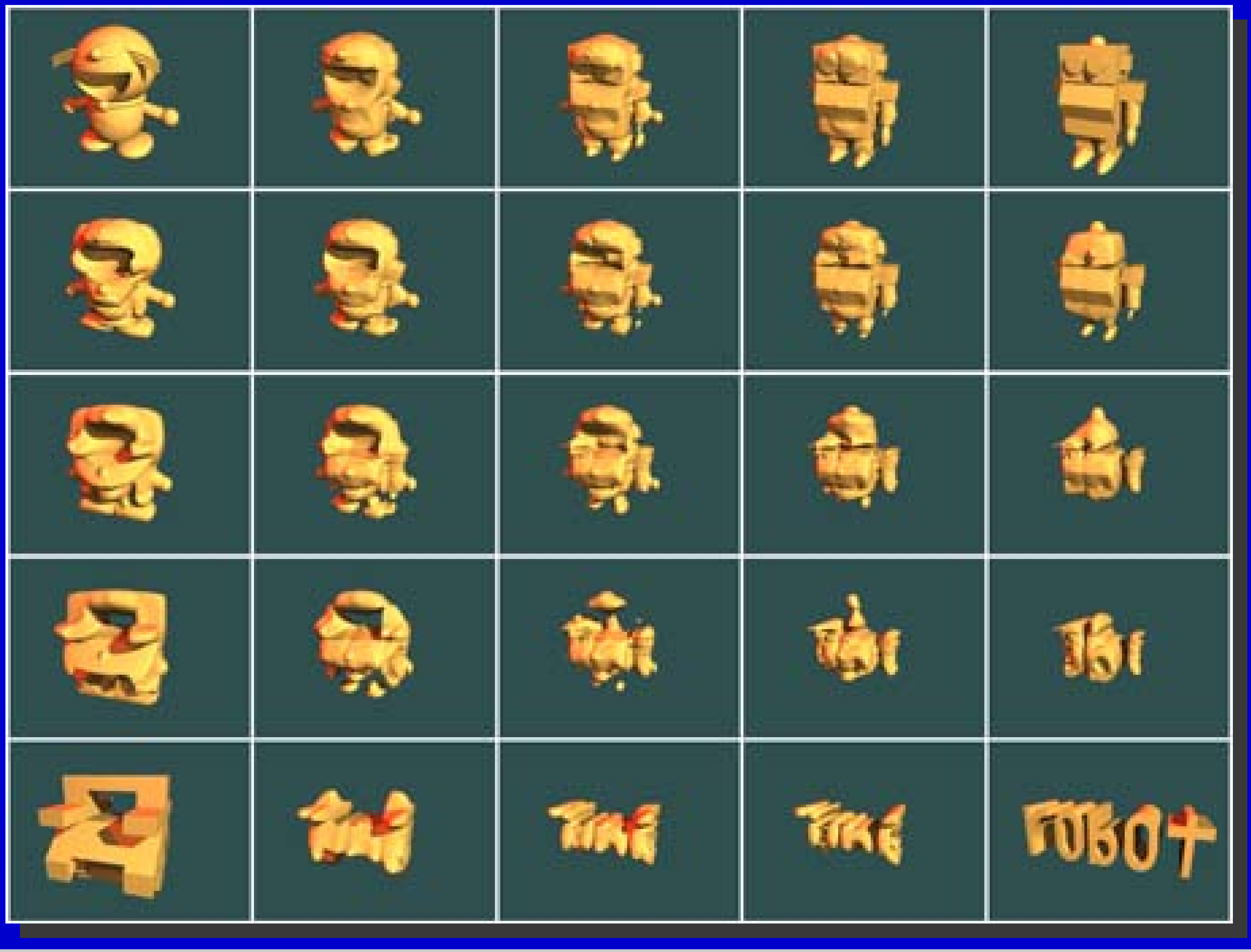
Constructive Solid Geometry

- **Advantages**
  - Very low memory cost
  - Simple combinations
  - Exact representation of curved surfaces
- **Disadvantages**
  - Rendering methods slow and complicated



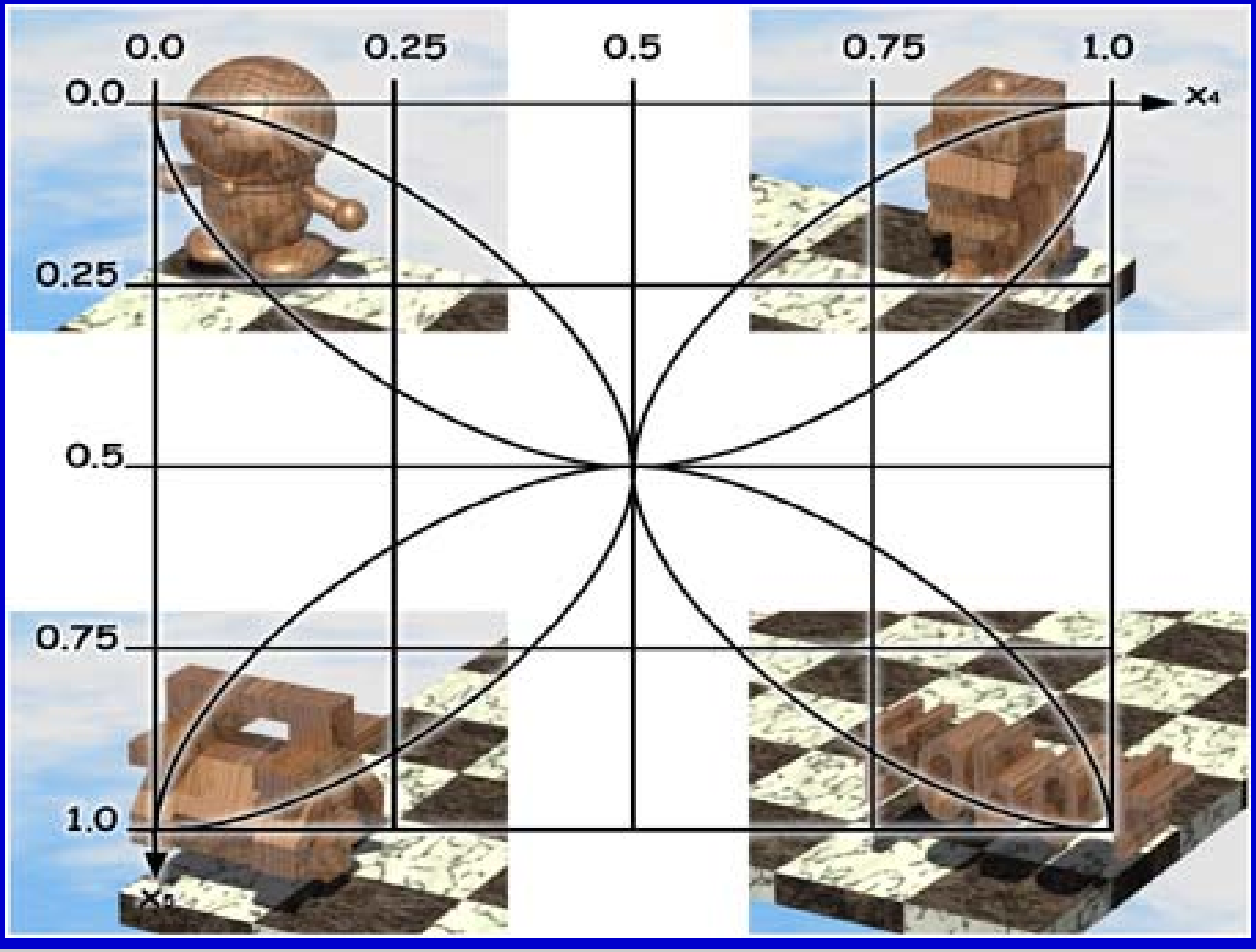
Spreadsheet Rendering

by Alexander PASKO, www.hyperfun.org



Animation Path in t_1t_2 Plane

by A. PASKO, www.hyperfun.org



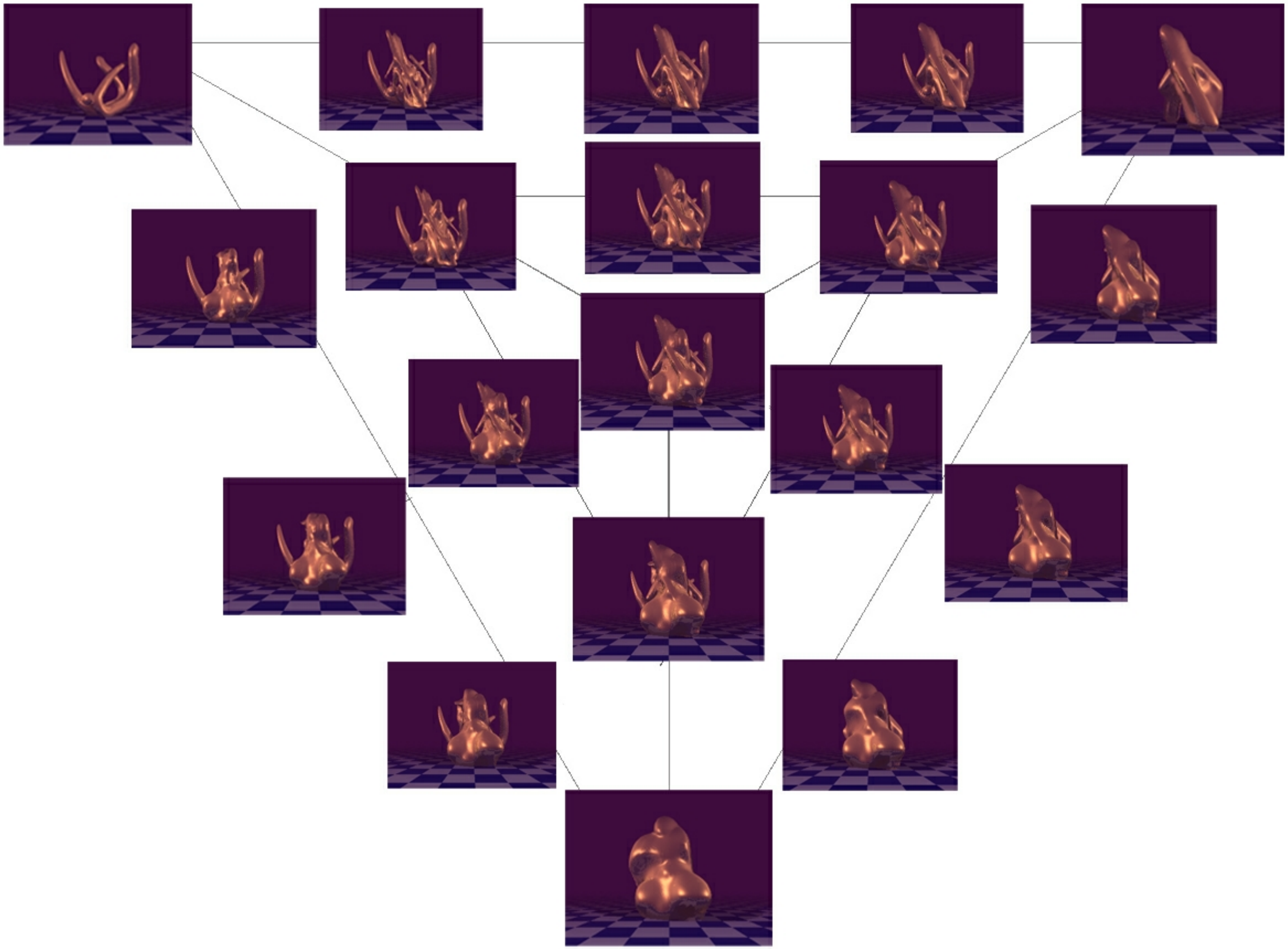
Spreadsheet Rendering

- Spreadsheet animation images from "Homotopic Fun in 5D space" by E. Fausett, A. Pasko, V. Adzhiev
- http://wwwcis.k.hosei.ac.jp/~F-rep/Homotopic.html

- Inbetween statues in the triangle by M. Kazakov, A. Pasko, V. Adzhiev

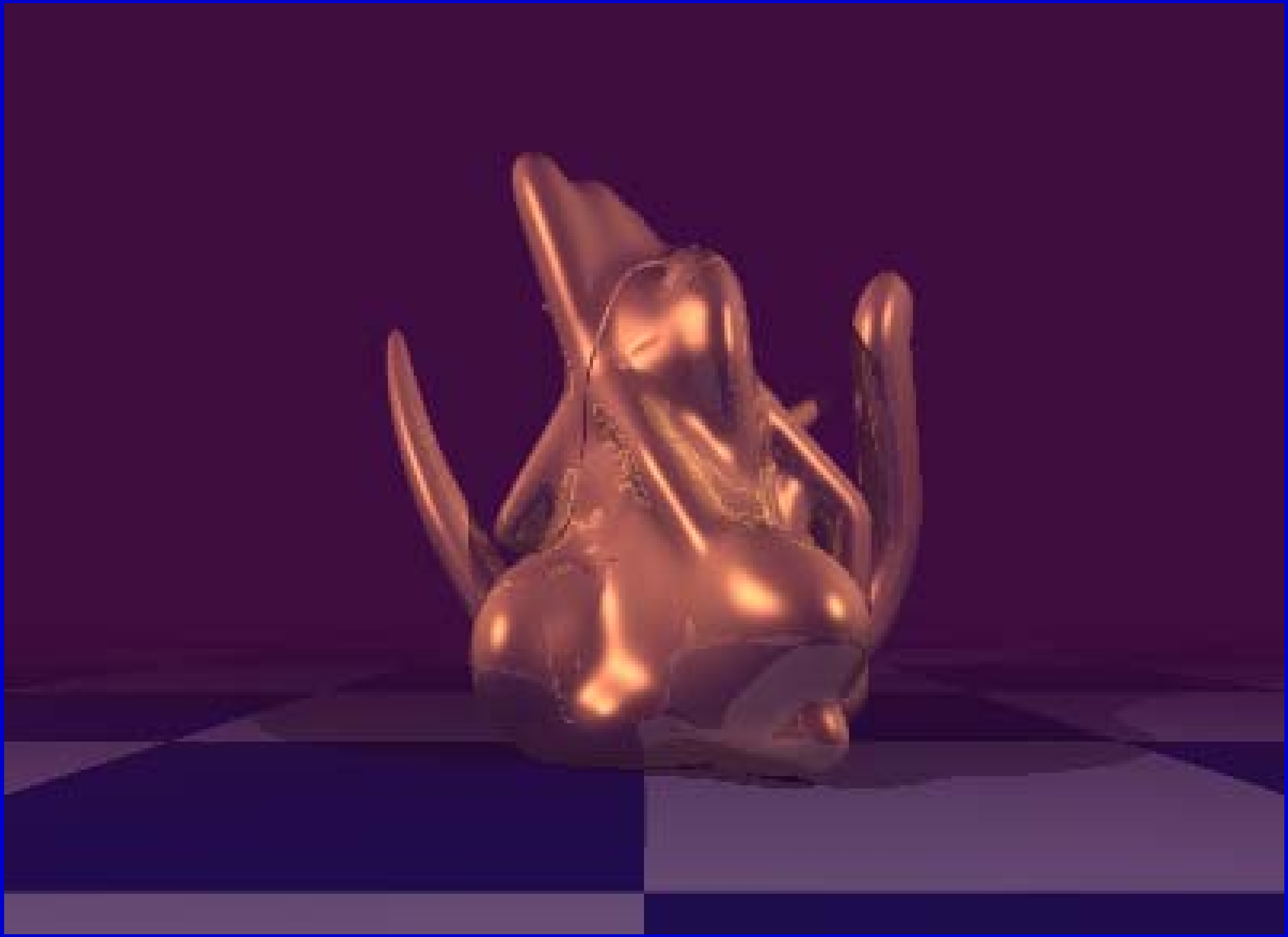
- F-rep completization of CG theory & practice

by Alexander PASKO, www.hyperfun.org



The Shape in the Triangle Center

by Alexander PASKO, www.hyperfun.org



Math Language Ruptures

- *Elementary Arithmetics*
- *Synthetic Geometry*
- *Algebra*
- *Analytic Geometry*
- *Infinitesimal Calculus*
- *Iterative Geometry*
- *Predicate Calculus*
- *Set Theory*

- *(based on Kvasz´s epistemologic research, 1996)*

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- Functional Representation
  - Definitions
  - HyperFun
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CompGeom Methodology

- Name coined in PhD thesis by M. I. SHAMOS, 80s
- Synthesis and analysis of efficient geometric algorithms, book by SHAMOS-PREPARATA (1985)
- Synthesis - algorithmic strategies
- Analysis – model of computation, problem complexity, brute force algorithm, efficient algorithms, optimal solution (LEDA)
- Real RAM model and worst-case complexity today

CompGeom Assumptions I

- Euclidean d-dimensional space (d = 2, 3, …), sets S
- Typically, a set is given by linear equation
- $a1*x1 + \ldots + ad*xd = b$
- … and other conditions (e.g. polyhedra, mesh, halfplanes
- Set operations
- 1. MEMBER(u,S)
- 2. INSERT(u,S)
- 3. DELETE (u,S)

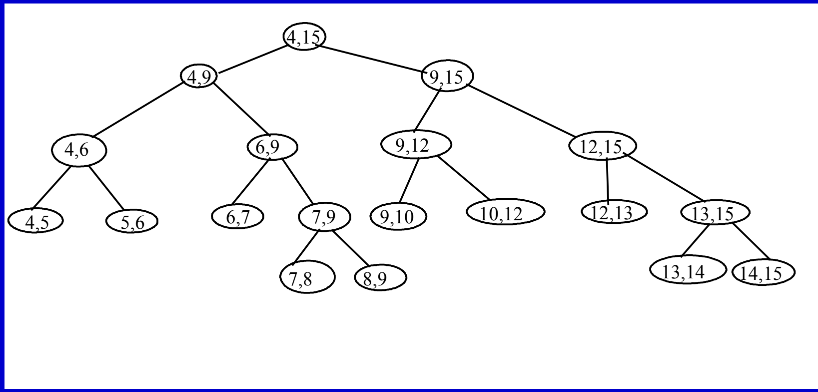
CompGeom Assumptions II

- Let {S1, S2, ..., Sk} is a system of pairwise disjoint set

- 4. FIND(u) in {S1, S2, ..., Sk}
- 5. UNION(Si, Sj; Sk)

- For ordered sets:

- 6. MIN(S)
- 7. SPLIT(u,S) S2 = S - S1
- 8. CONCATENATE(S1, S2)



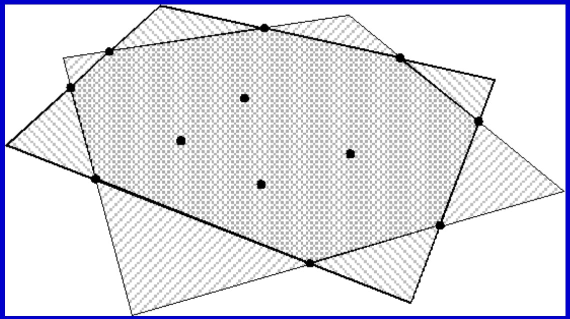
- E.g. VOCABULARY supports MEMBER, INSERT, DELETE
- PRIORITY QUEUE supports MIN, INSERT, DELETE

CompGeom Methodology

- Real RAM model, unit cost operations, real numbers
- Worst-case complexity - the usual (Knuth) notation:
- · $O(f(N))$ means the set of all functions $g(N)$ such that there exist positive constants C and M with
  - $| g(N) | < Cf(N)$ for all $N > M$.
- · $\Omega(f(N))$ means the set of all functions $g(N)$ such that there exist positive constants C and M with
  - $| g(N) | > Cf(N)$ for all $N > M$.
- · $\Theta(f(N))$ means the set of all functions $g(N)$ such that there exist positive constants C, D and M with
  - $Cf(N) < | g(N) | < Df(N)$ for all $N > M$.
-
- Note. $O(\ )$ and $\Omega(\ )$ are used to describe upper and lower bounds, $\Theta(\ )$ we use for "optimal" algorithms. N is the measure of input size (number of points, bits, edges...).

CompGeom Methodology

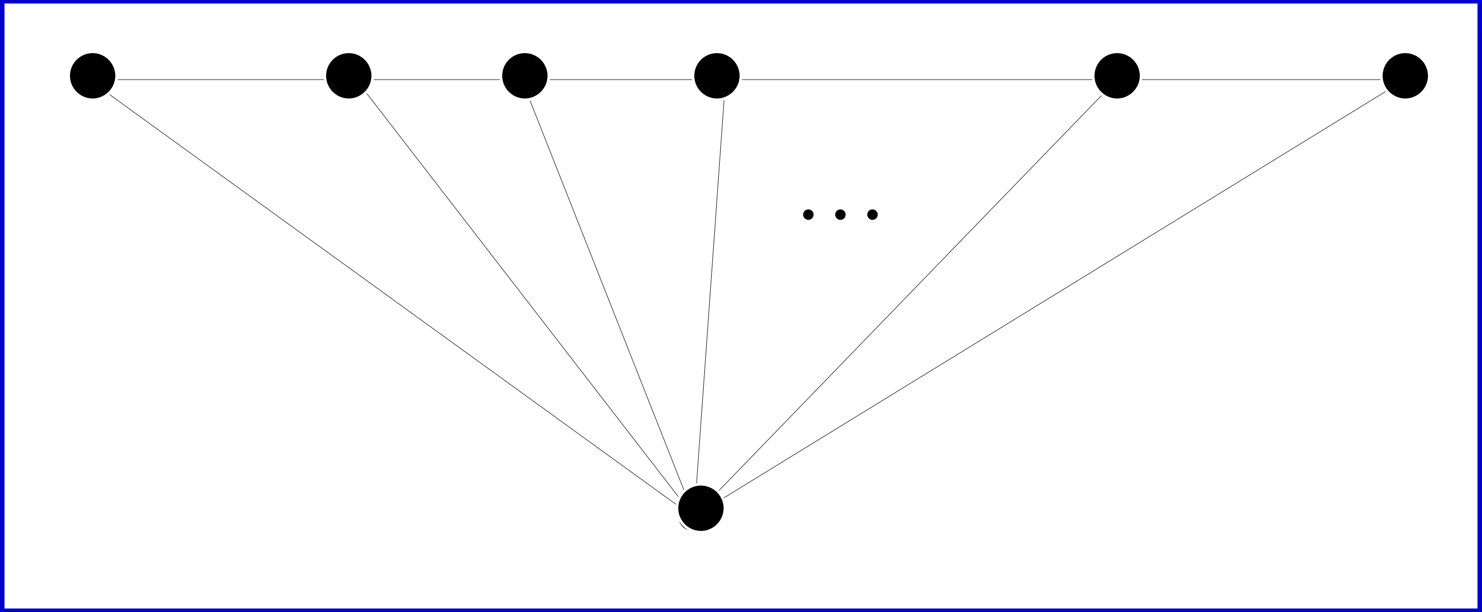
- Lower bound – has to be proven, hard, usually by reduction to another known problem (e.g. sorting)
- Upper bound – any algorithm



- Efficient algorithm
- Optimal algorithm achieves the lower bound
- Complexity measures are time, memory, preprocessing time and memory, query time, (time of programming, output sensitivity, on-line and off-line problems… average complexity)

CompGeom Methodology

- *Lower bound – has to be proven, hard, usually by reduction to another known problem (e.g. sorting)*



- *Triangulation sorts real numbers => Ω(NlogN)*

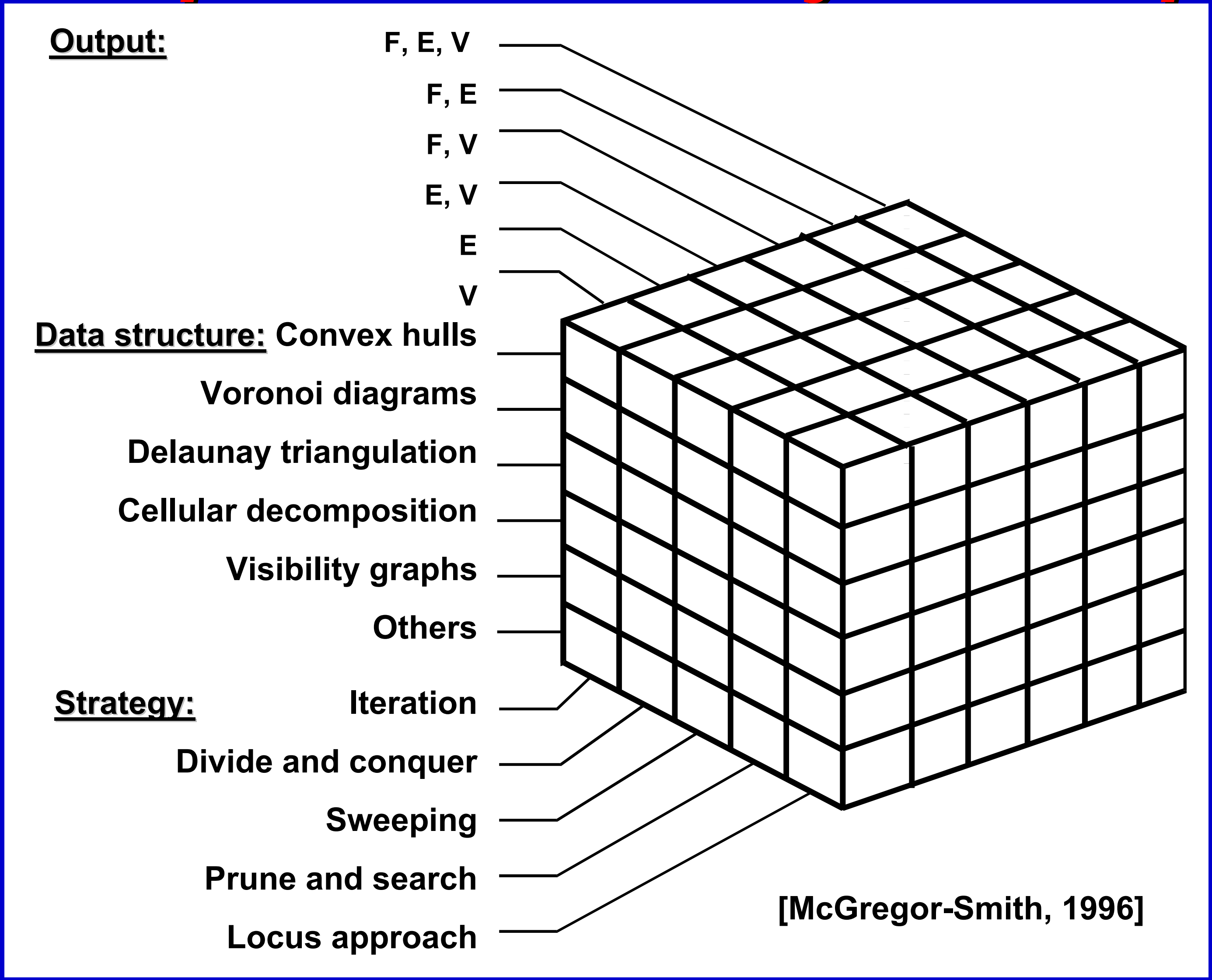
Algorithmic Strategies

- 1. Iteration
- 2. Sweeping
- 3. Sorting
- 4. Divide & Conquer
- 5. Locus Approach
- 6. Duality
- 7. Combinatorial Analysis

Algorithmic Strategies

- 8. Prune & Search
- 9. Dynamic Programming
- 10. ASA
- 11. Genetic Algorithms
- 12. Memetic Algorithms
- 13. DNA Computation, Neural Networks...
- 14. Darwish Camel, New Paradigms

CompGeom - 3 Ways to Explain



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  - Voronoi diagram Vor(P)
  - Delaunay triangulation
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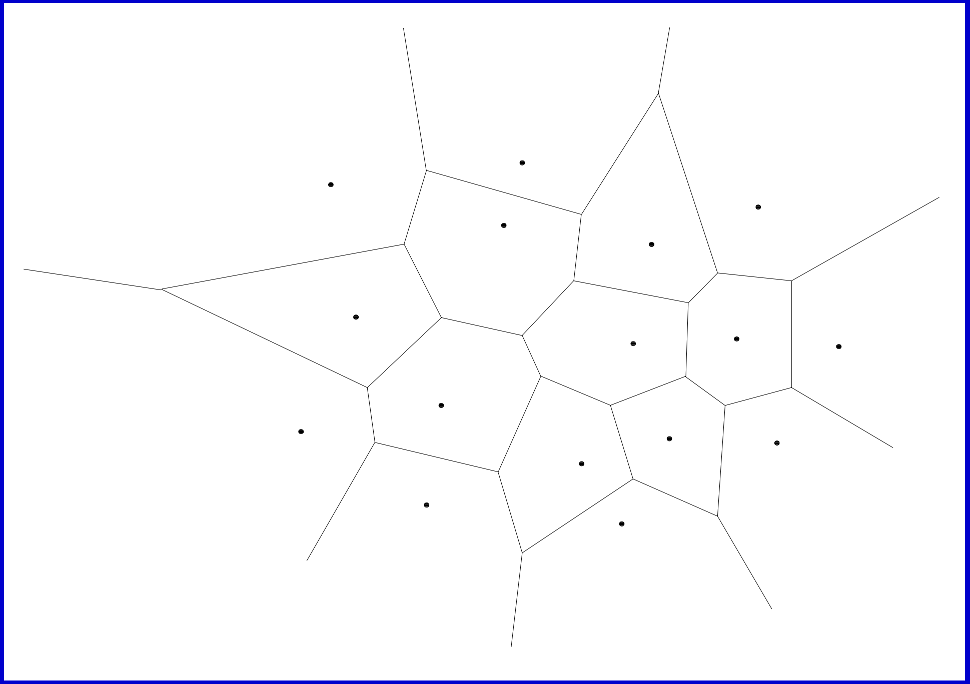
Voronoi Diagram – Many Names

- *Model of both organic and anorganic originals: fences, lattices, spider web, soap bubbles, mineral crystals, honeycomb hexagons, tilings, Escherian space subdivisions… most fundamental geometric structure*
- 

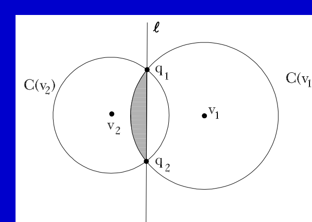
Voronoi Tesselation Problem

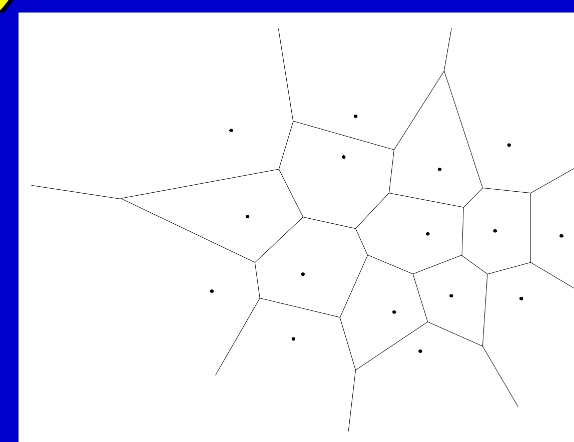
INPUT: Given a set P of N points in the plane, N finite, in general position

OUTPUT: Subdivide the plane according to proximity of points (regions of closest points)

REQUIREMENTS and MODIFICATIONS:

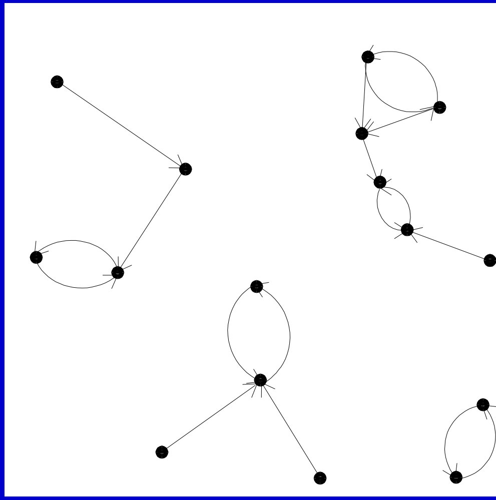
- 2D, 3D (no four cocircular)
- Various metrics
- Robotics (V-edges motion plan)
- Higher order
- Power diagrams

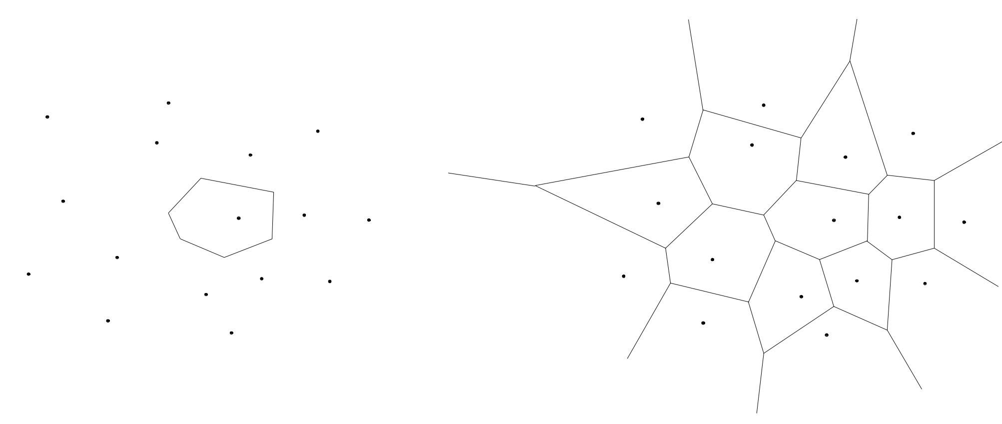




Related Problems

- Closest Pair
- All Nearest Neighbours, clusters
- Point Set Triangulation, paths
- Convex Hull
- Medial Axis, Blum
- TSP Heuristics
- …

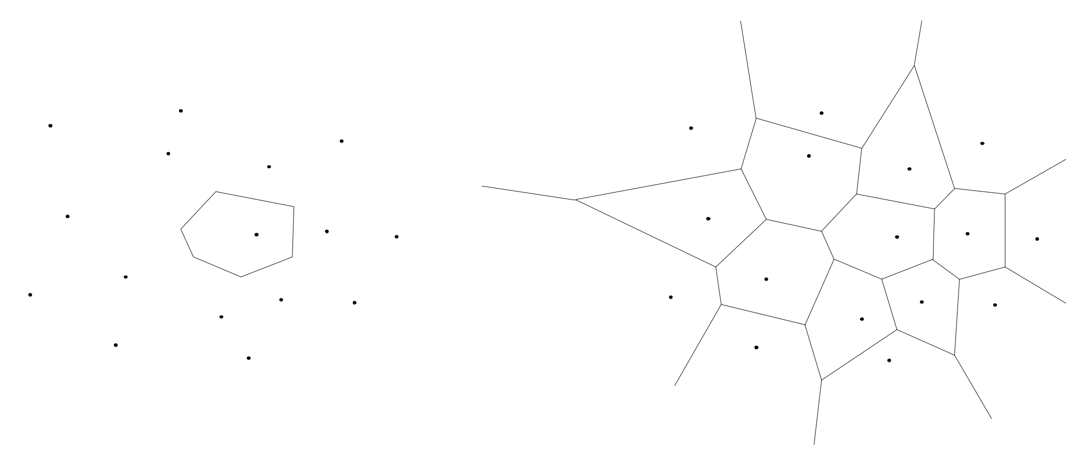




(a) (b)

Vor(P) Definitions

- *Voronoi diagram (Dirichlet tesselation) is a union of Voronoi polygons (tiling, no covering)*
- *Voronoi polygon, reg(p), is a locus of closest points, & a convex set, shares an edge with another one*
- *Voronoi point*
- *Generator*
- *Separator*



(a) (b)

- *Specialised monograph: Okabe at al. 1997 (TUG Lib.)*

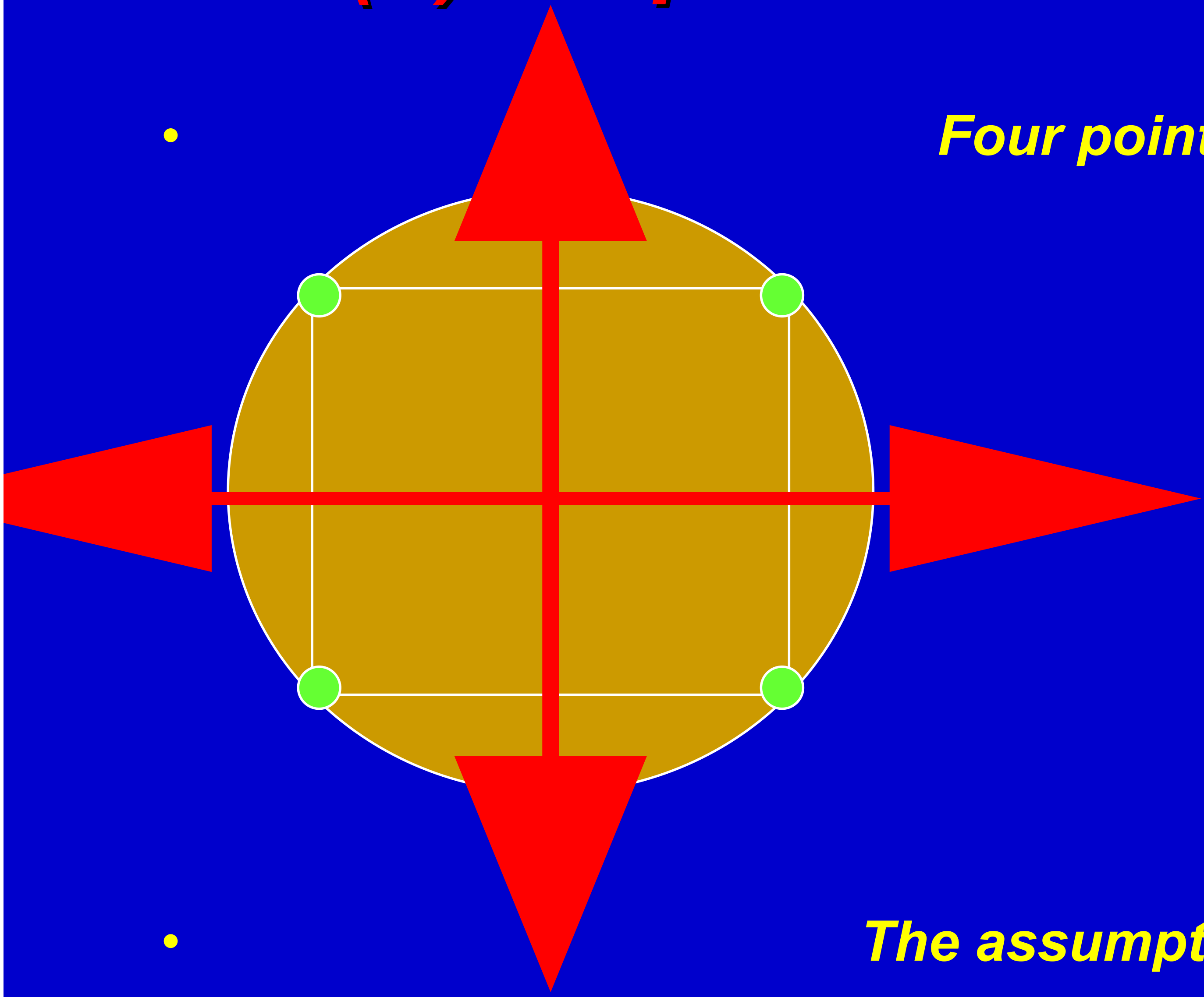
Vor(P) Properties

- Planar graph (Euler‘s formula) $V - E + F = 2$, everything LINEAR, $O(N)$

- N separators => N faces (some unbounded)
- No vertices for collinear input points
- Each vertex belongs to 3 edges and each edge has 2 vertices => $2E \geq 3V$, $E \leq 3N-6$, $V \leq 2N-4$
- Average number of edges for V-polygon is 5 or 6
- If a vertex p is closest to q then reg(p), reg(q) share an edge
- Each reg(p) is nonempty
- Unbounded regions contain extremal points

Vor(P) Properties for a Square

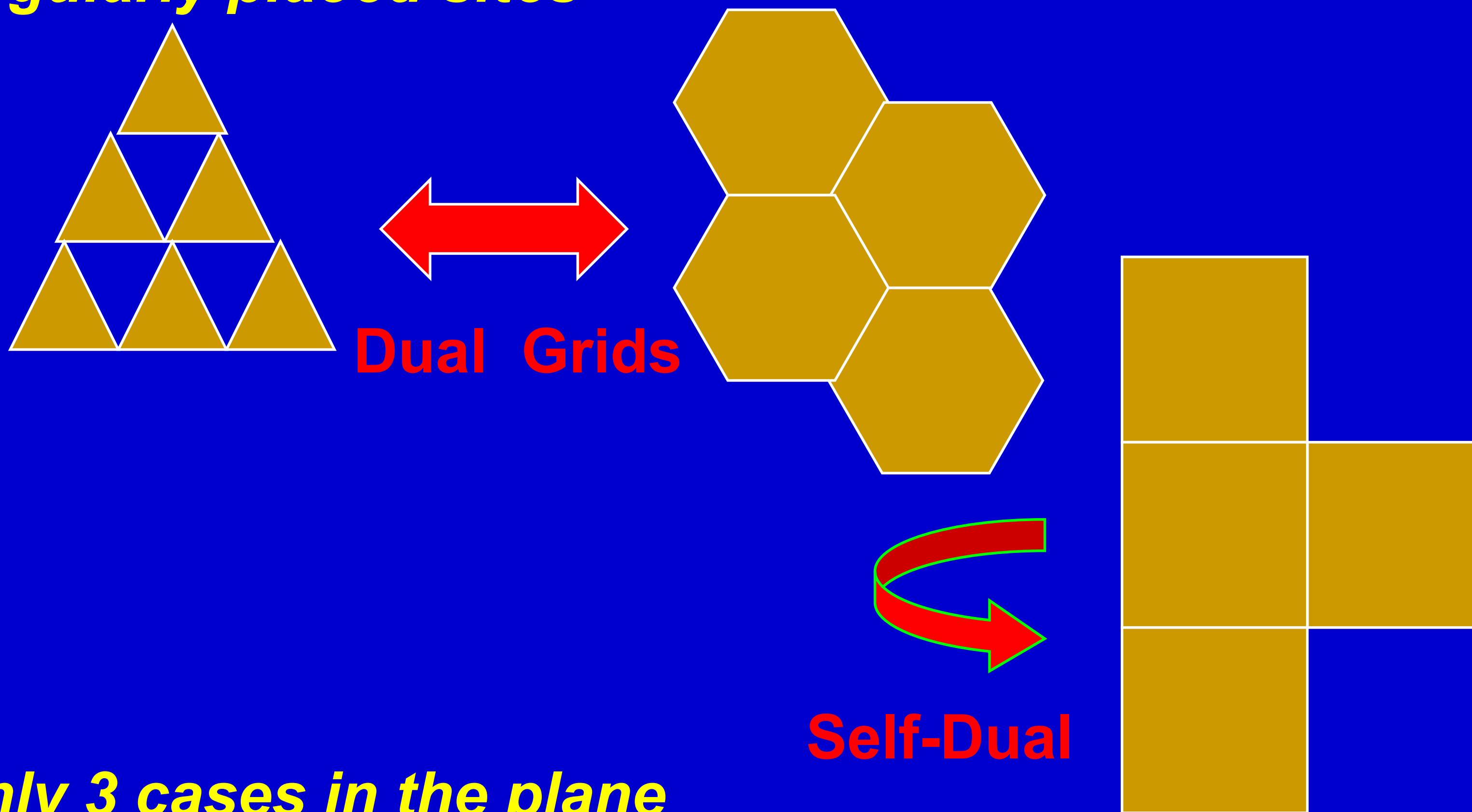
- Four points cocircular



- The assumption is not crucia

Vor(P) Special Cases, 2D

- *Regularly placed sites*
-



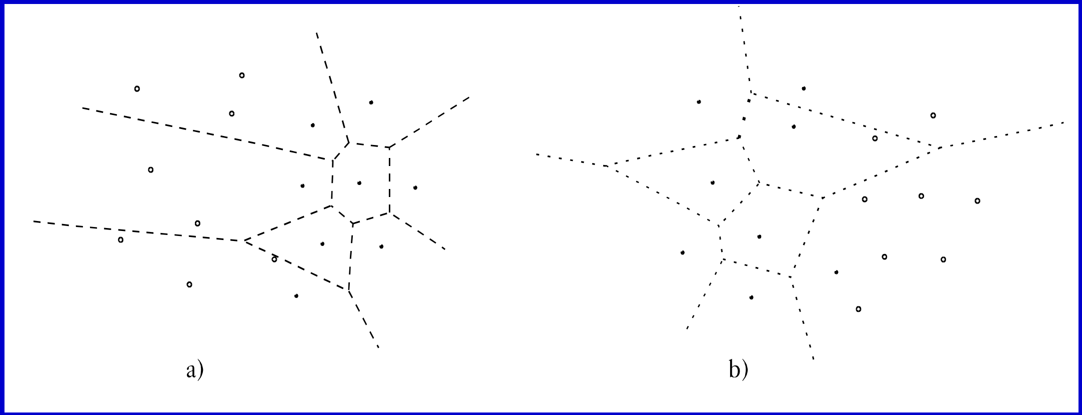
- *Only 3 cases in the plane*
- *Proof by integer division of 360 degrees*

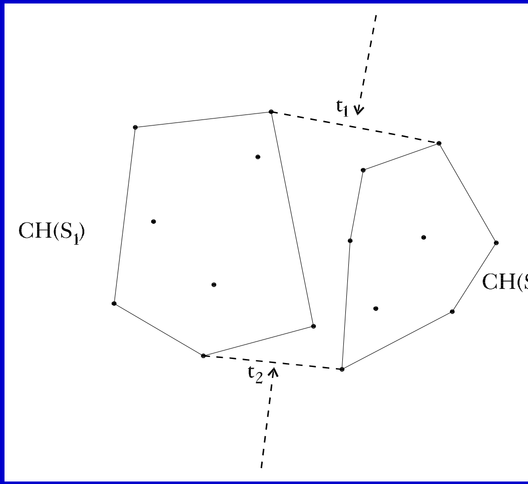
Vor(P) History

- Voronoi, M. G. 1908. Nouvelles Applications des Parametres Continus a la Theorie des Formes Quadratiques. J. Reine Angew. Math. 134. Pp 198-287
- Gauss 1840 – quadratic forms (QF)
- Dirichlet 1850 – simple proof on irreducibility of QF
- Voronoi 1908 – generalization for d>2
- Thiessen 1911 - geography
- Horton 1917 – Thiessen polygons...
- Blum 1967 – new shape descriptors, Gestalt psychology
- ... crystallography, databases, biology...
- Aurenhammer, ACM Surveys
- Okabe et al.
-

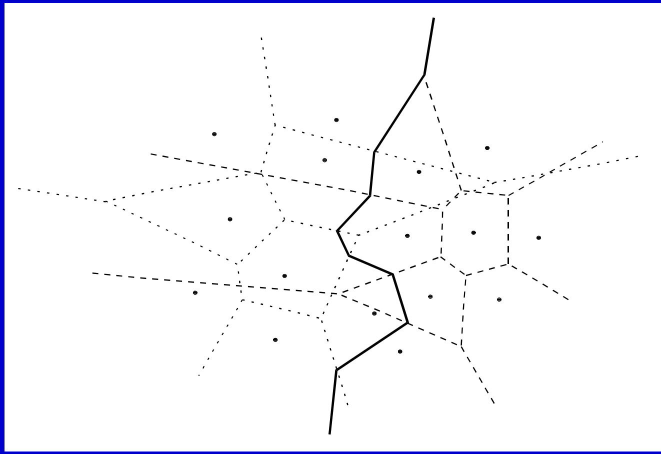
Divide and Conquer Proof Sketch

- *Left and right parts*





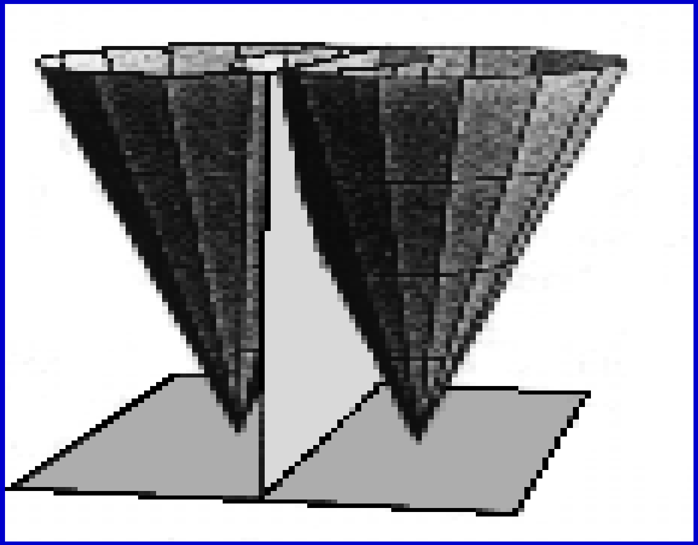
- *Merge solutions*

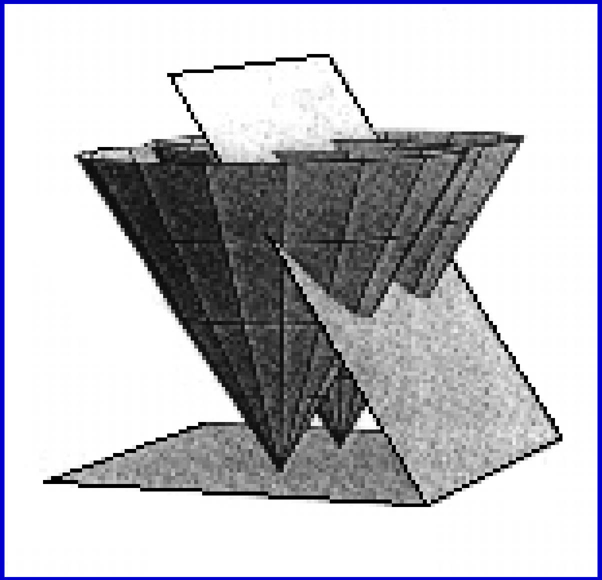


- *O(NlogN)*
- *Optimal, but unstable*
- *Lower bound proof*
- *- too complex for today*

Sweepline Construction

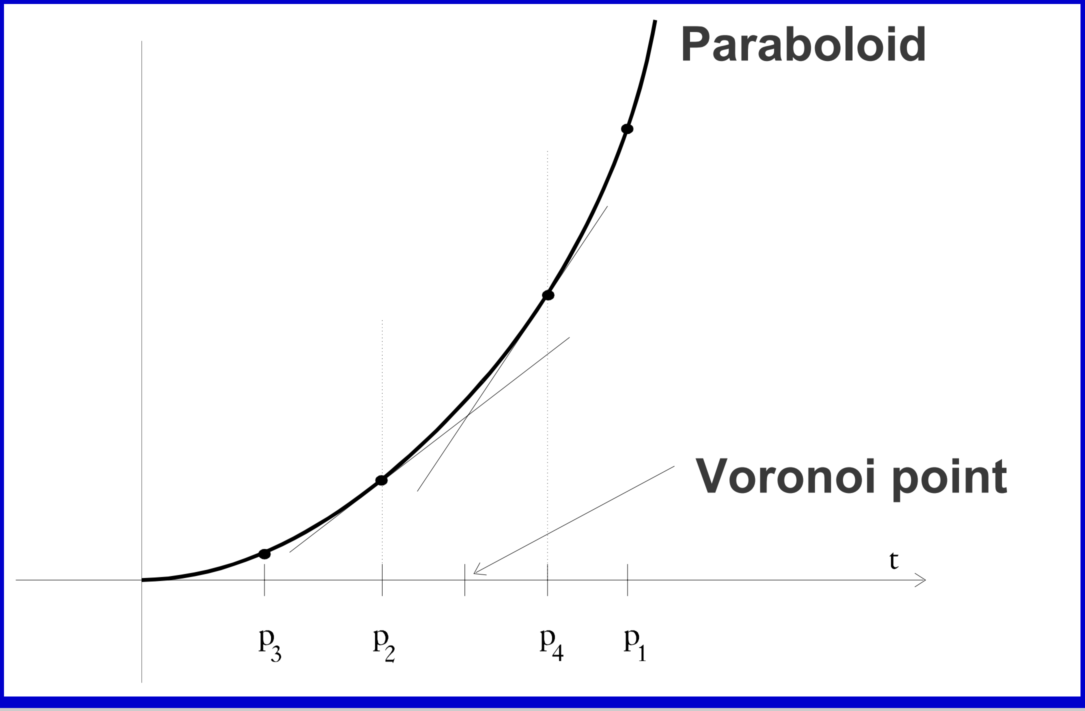
- S. Fortune
- Geometric Interpretation by Guibas and Stolfi





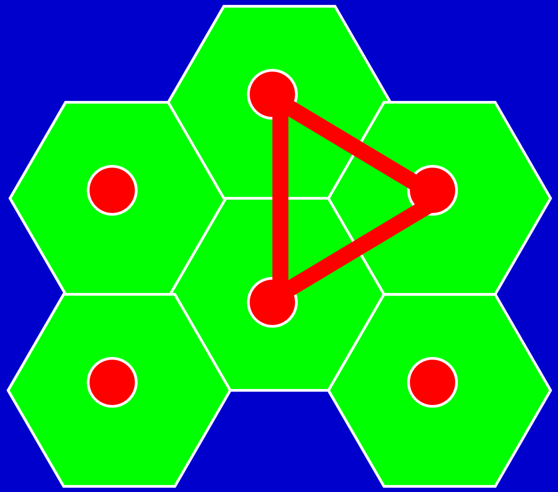
Lifting Transformation

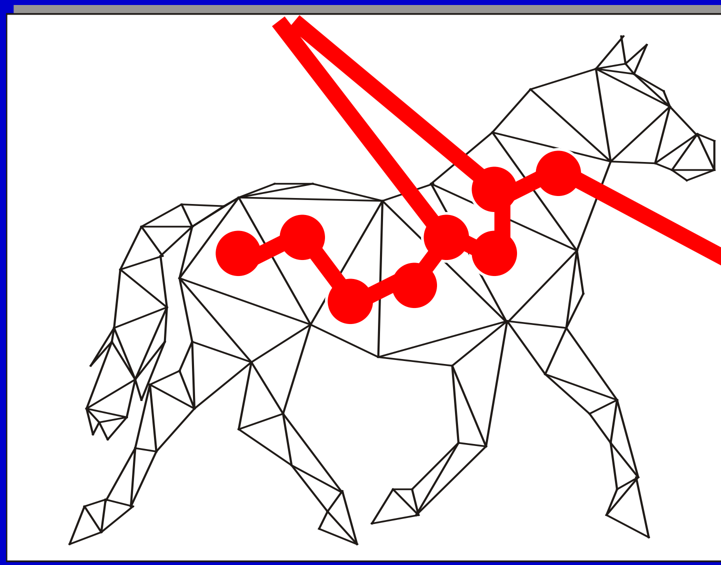
- 1. Elevate to paraboloid (linear time, O(N))
- 2. Compute convex hull (O(NlogN) even in 3D)
- 3. Return to the 2D plane (linear time, O(N))



Constructions using Triangulation

- *Local optimality and global optimality*
- *Hint: Vor(P) is dual with some triangulation*
- *Prof. Aurenhammer, TU Graz, ACM Survey*





Agenda

- Motivation
- Computational Geometry
  - Methodology, Algorithmic Strategies
  - Voronoi diagram Vor(P)
  - Delaunay triangulation
- Functional Representation
  - Definitions
  - HyperFun
  - Examples

Triangulation Problem

INPUT: Given a set P of N points in the plane, N finite, in general position

OUTPUT: Subdivide the interior into O(N) non-overlapping triangles

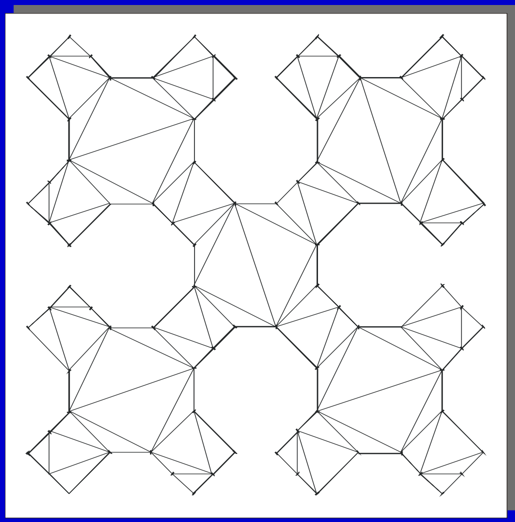
REQUIREMENTS and MODIFICATIONS:

- *2D, 2.5D, 3D (tetrahedralization)*
- *Maximal planar graph*
- *Triangles may share vertex/edge*
- *No Steiner points (mesh)*

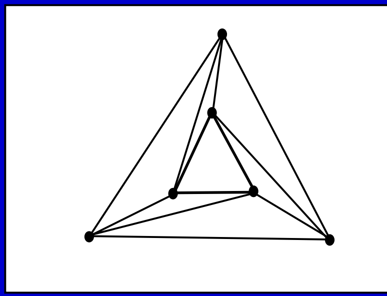
  Steiner points for mesh

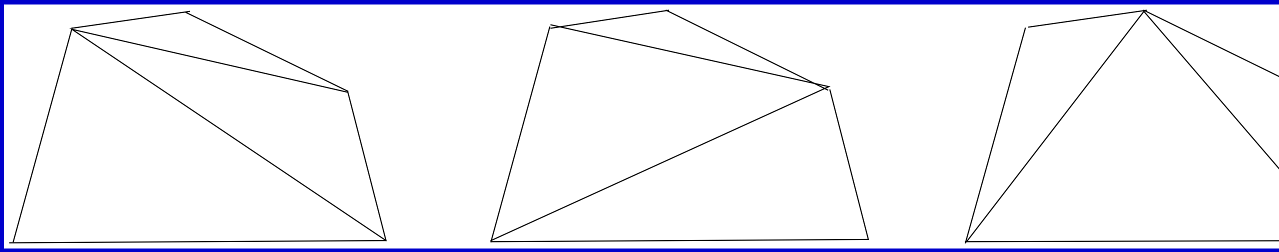
  Prescribed edges for constrained triangulation

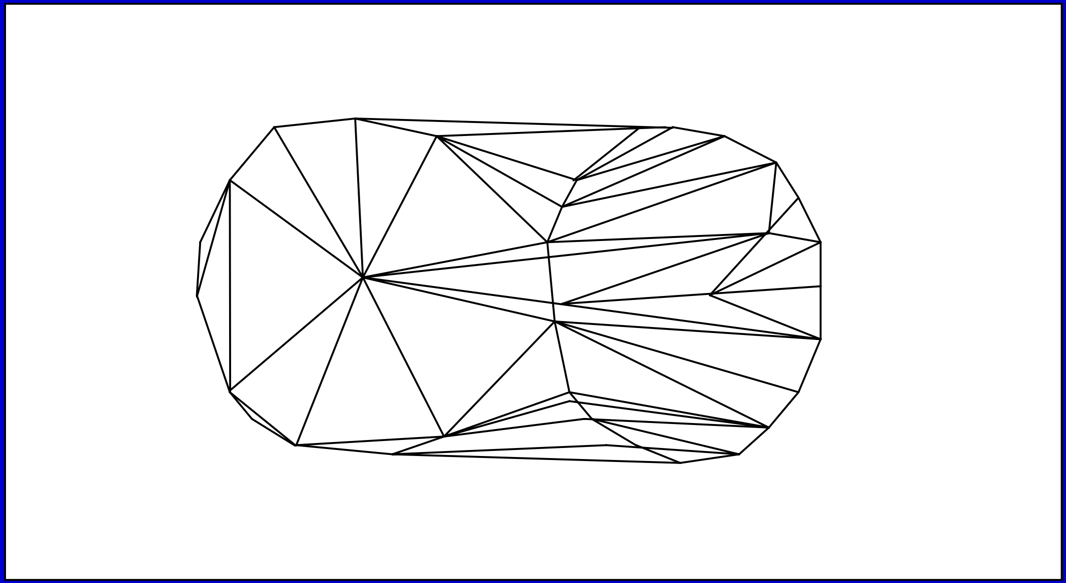
Planar Triangulations

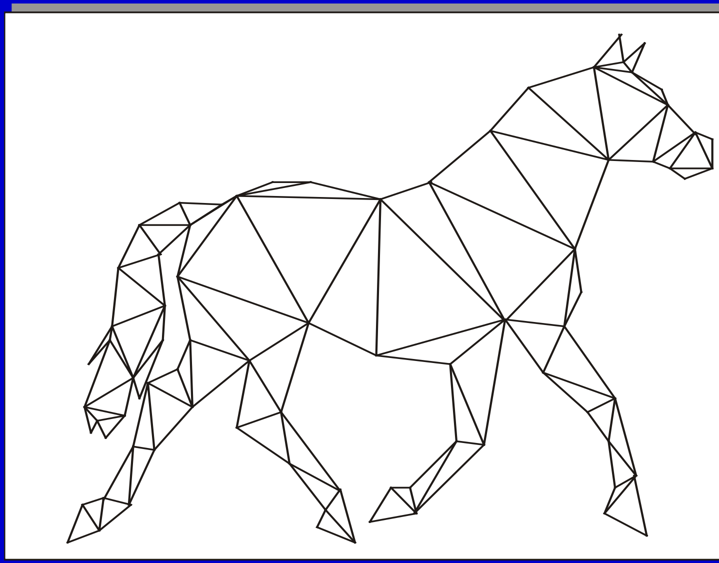


Optimisation criteria, triangle ordering, art gallery…



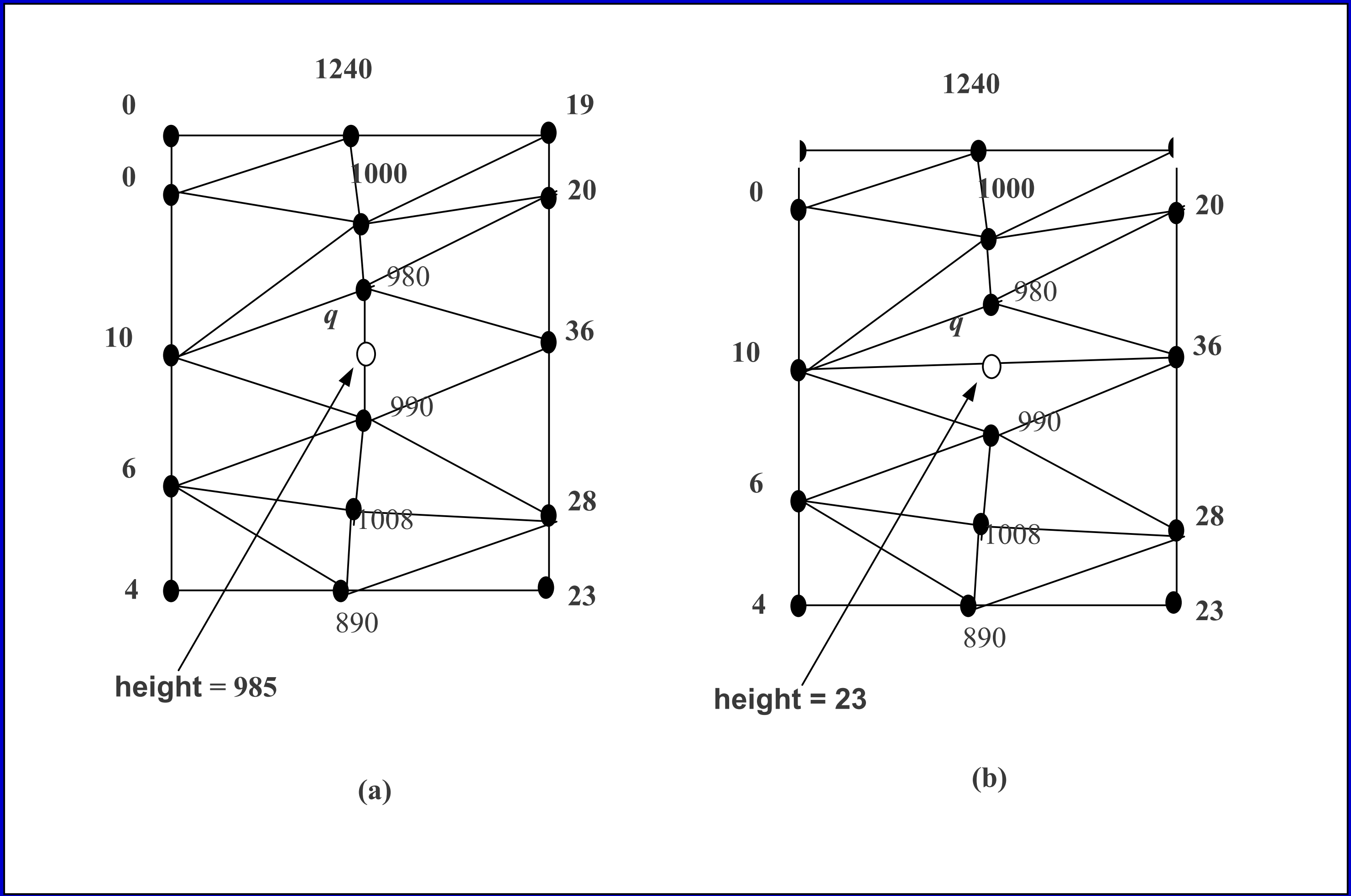






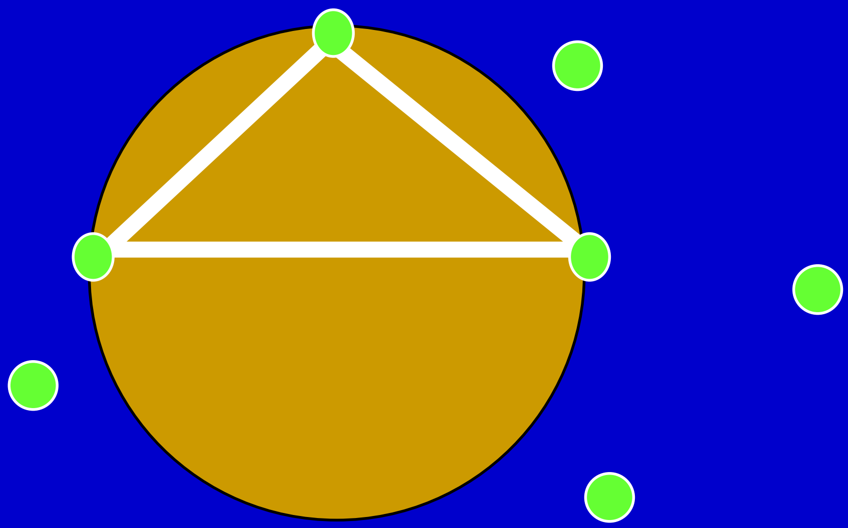
Terrain Interpolation

Approximation, minimum roughness property…



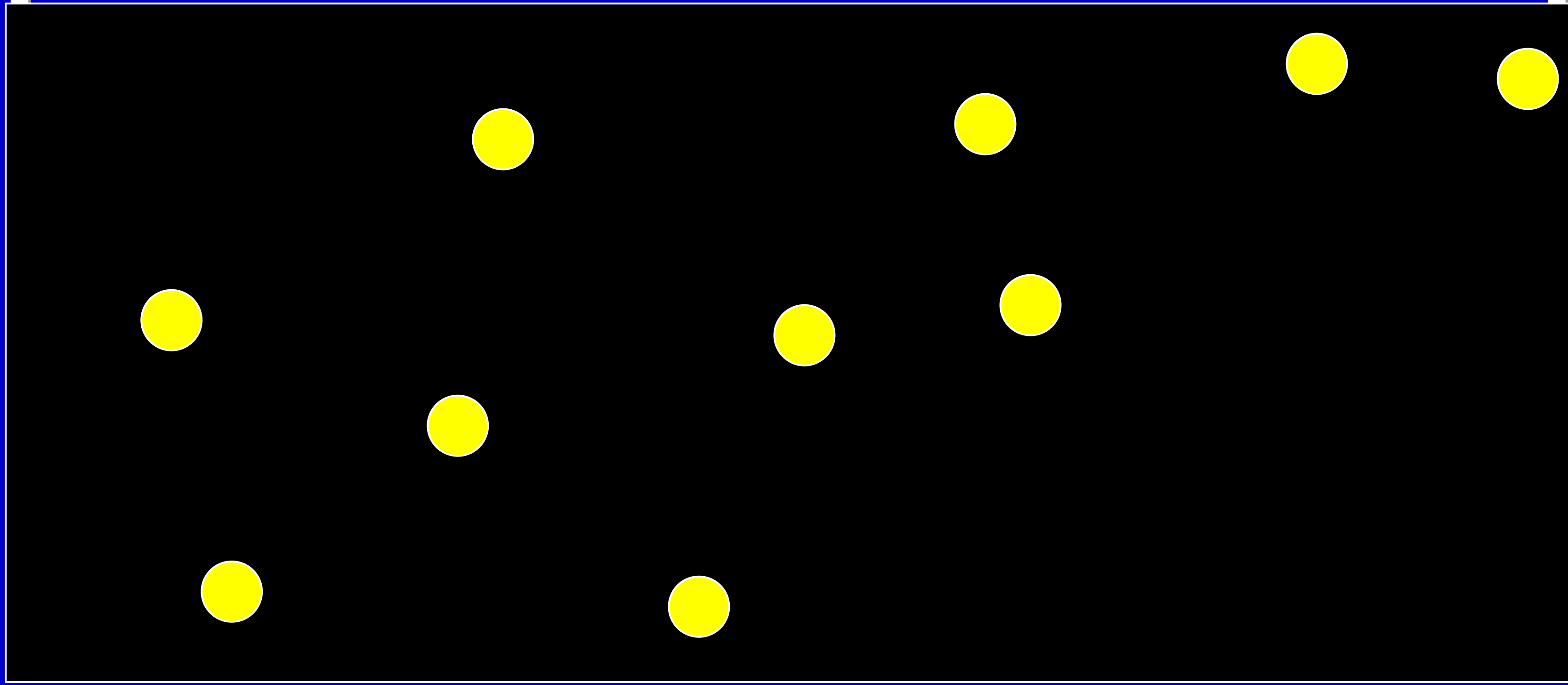
Lower bound and Optimal algorithm

Lower bound of triangulation problem:
O(NlogN) by reduction to sorting
achieved by Delaunay triangulation DT(P)



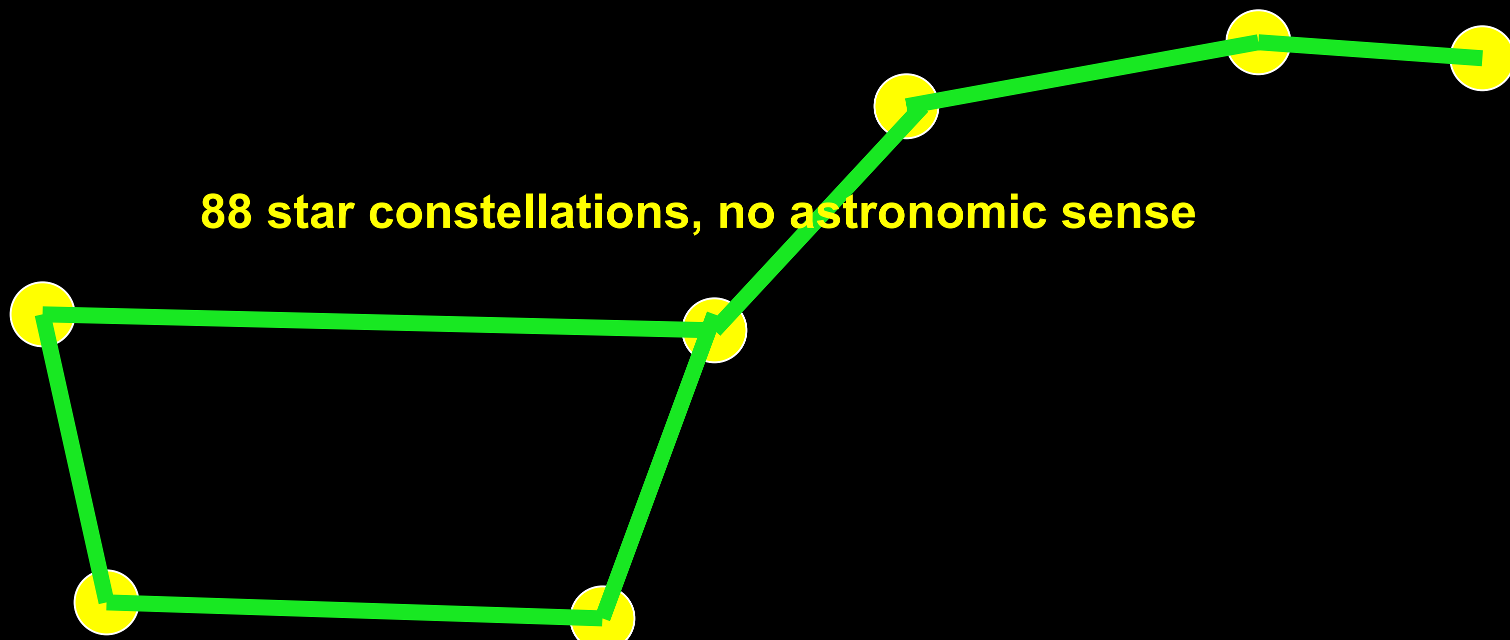
Point Constellations

Aurenhammer (2001): 14 309 547 sets of 10 points with respect to the different crossing properties



Star Constellations – DT edges

- 88 star constellations, no astronomic sense



DT(P) History

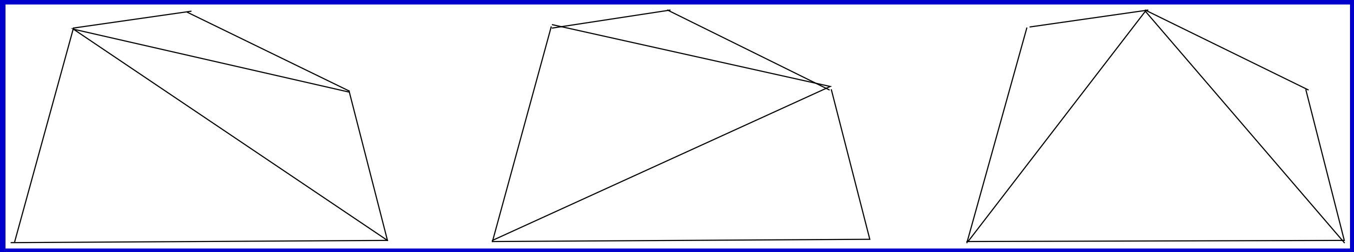
- Voronoi, M. G. 1908. Nouvelles Applications des Parametres Continus a la Theorie des Formes Quadratiques. J. Reine Angew. Math. 134. Pp 198-287
- Delaunay B. 1932. – student of Voronoi
- Numerical Interpolation and Finite Elements
- Wrong claim that DT(P) is optimal
- Delaunay refinement in mesh generation
- ...
- Aurenhammer, ACM Surveys
- Okabe et al.
- Bern, M. – Eppstein, B. Mesh Generation and Optimal Triangulations, a chapter in Computing in Euclidean Geometry, and another chapter in Sack-Urrutia. 2000.
- Fortune, S. in Goodman-O´Rourke. 1999.

DT(P) Properties (A Selection)

- DT(P) is unique under no 4-cocircularity assumption
- Graph theoretic dual to Vor(P) and again close/intimate relation to convexity in 3D
- Planar graph with prominent subgraphs: EMST (US telekom law), RNG, GG, ...
- Cheapest triangulation
- Optimizes several criteria concerning triangle quality
- Extremely popular
- Alpha shapes, generalization of convex hull (non-convex)
- Beta skeletons
- ... Many others – refer to CompGeom handbooks

DT(P) Constructions (A Selection)

- Brute force bi-quadratic algorithm
- Incremntal Insertion
- Edge Flipping Procedure



- Cheapest starting triangulation, Fortunes sweepline
- ... Many others – refer to CompGeom handbooks and CompGeom software repositories (randomized, parallel, constrained DT, Steiner DT, 3D...

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Functional Representation

- *Prehistory: An old paper by RVACHEV, V. L. 1963. (in Russian on expressing set operations functionally)*

- *„The function representation (or F-rep) defines a geometric object by a single real continuous function of several variables as*

$$F(X) >= 0.$$

- *It combines many different models like classic "implicits", skeleton based "implicits", set-theoretic solids, sweeps, volumetric objects, parametric models, procedural models…"*

- *Key reference: PASKO A.* et al. 1995. ***Function representation in geometric modeling: concepts, implementation and applications**. The Visual Computer, vol. 11, No. 8, 1995, pp. 429-446.*

Geometric Concepts

Geometric concepts of a functionally based modeling environment:

$$(M, \Phi, W),$$

where

M is a set of *geometric objects*,
Φ is a set of *geometric operations*,
W is a set of *relations* on the set of objects.

Mathematically this triplet is a sort of

algebraic system.

F-rep Objects

Let $\boldsymbol{X = (x_1, x_2, \ldots, x_n)}$ is a point in E^n.

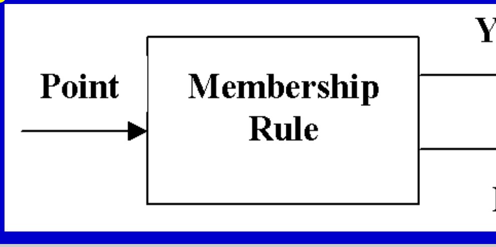
Objects are closed subsets of the n-dimensional Euclidea
space E^n defined by

$$F(x_1, x_2, \ldots, x_n) \geq 0,$$

where F is a real-valued continuous function defined on E

Convention:

- $F(\boldsymbol{X}) > 0$ - point X is inside the object
- $F(\boldsymbol{X}) = 0$ - for point X on the boundary
- $F(\boldsymbol{X}) < 0$ - X outside



F-rep Objects Requirements

The boundary of such an object is called ***"implicit" surface in 3D***.

The function can be defined by:
1) analytical expression;
2) function evaluation algorithm;
3) tabulated values and an appropriate interpolation procedure.

The major requirement to the function is to have at least **C^0 continuity**

F-rep Operations

The same.

$$f_1 | f_2 = \frac{1}{1+\alpha}\left(f_1 + f_2 + \sqrt{f_1^2 + f_2^2 - 2\alpha f_1 f_2}\right.$$

$$f_1 \,\&\, f_2 = \frac{1}{1+\alpha}\left(f_1 + f_2 - \sqrt{f_1^2 + f_2^2 - 2\alpha f_1 f_2}\right.$$

$$f_1 \setminus f_2 = f_1 \,\&\, (-f_2)$$

Rvachev 1972. R-functions.

F-rep Operations II

⇨ The same.

If C^m continuity is to be provided, one may use another set of R-functions:

$$f_1 | f_2 = \left(f_1 + f_2 + \sqrt{f_1^2 + f_2^2}\right)\left(f_1^2 + f_2^2\right)^{\frac{m}{2}}$$

$$f_1 \& f_2 = \left(f_1 + f_2 - \sqrt{f_1^2 + f_2^2}\right)\left(f_1^2 + f_2^2\right)^{\frac{m}{2}}$$

Another alternative.

Union is Maximum, Intersection is Minimum.

F-rep Objects Requirements

Uniform representation of multidimensional solids defined as

$$F(X) \geq 0$$

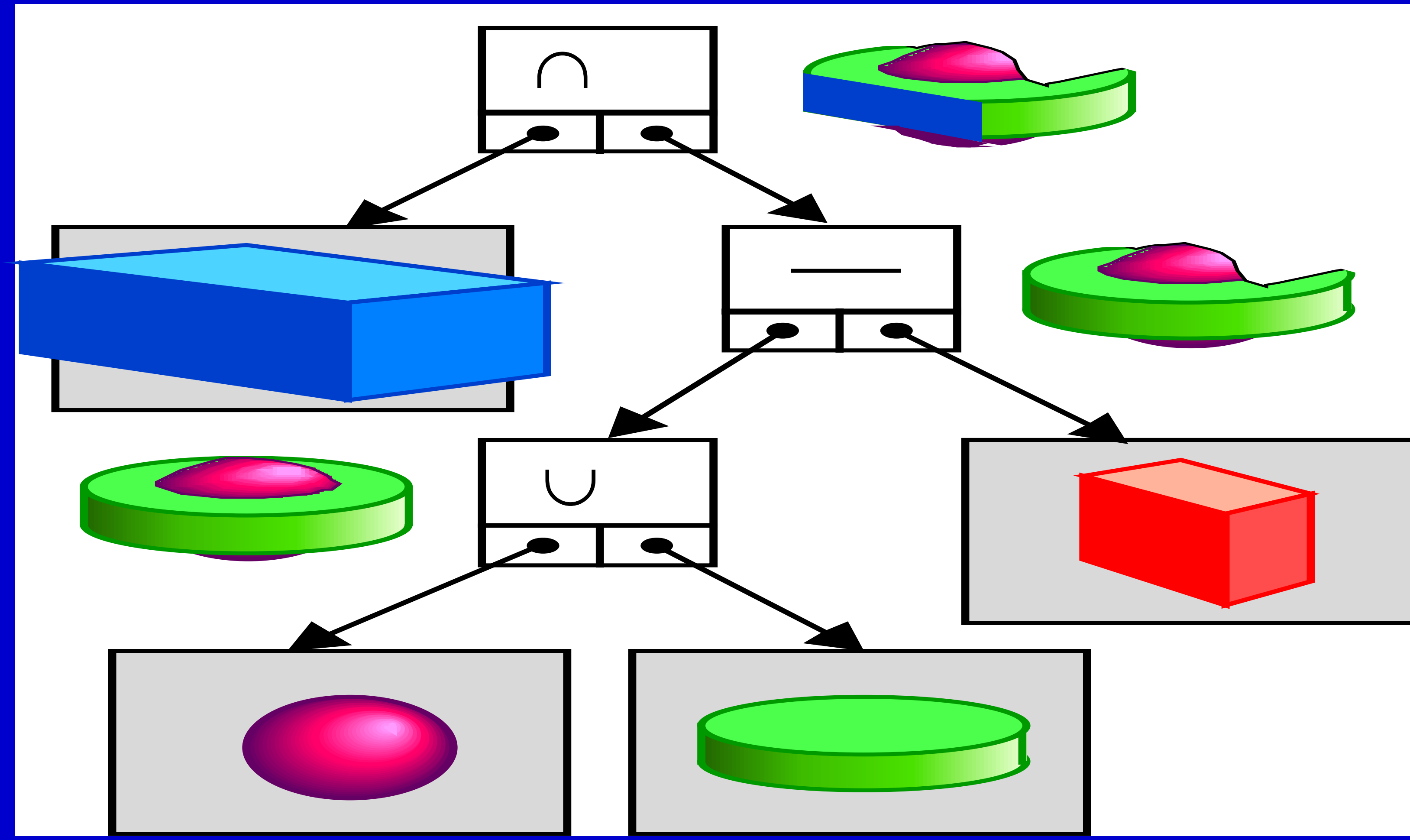
Function ***F(X)*** evaluation procedure traverses the construction tree structure

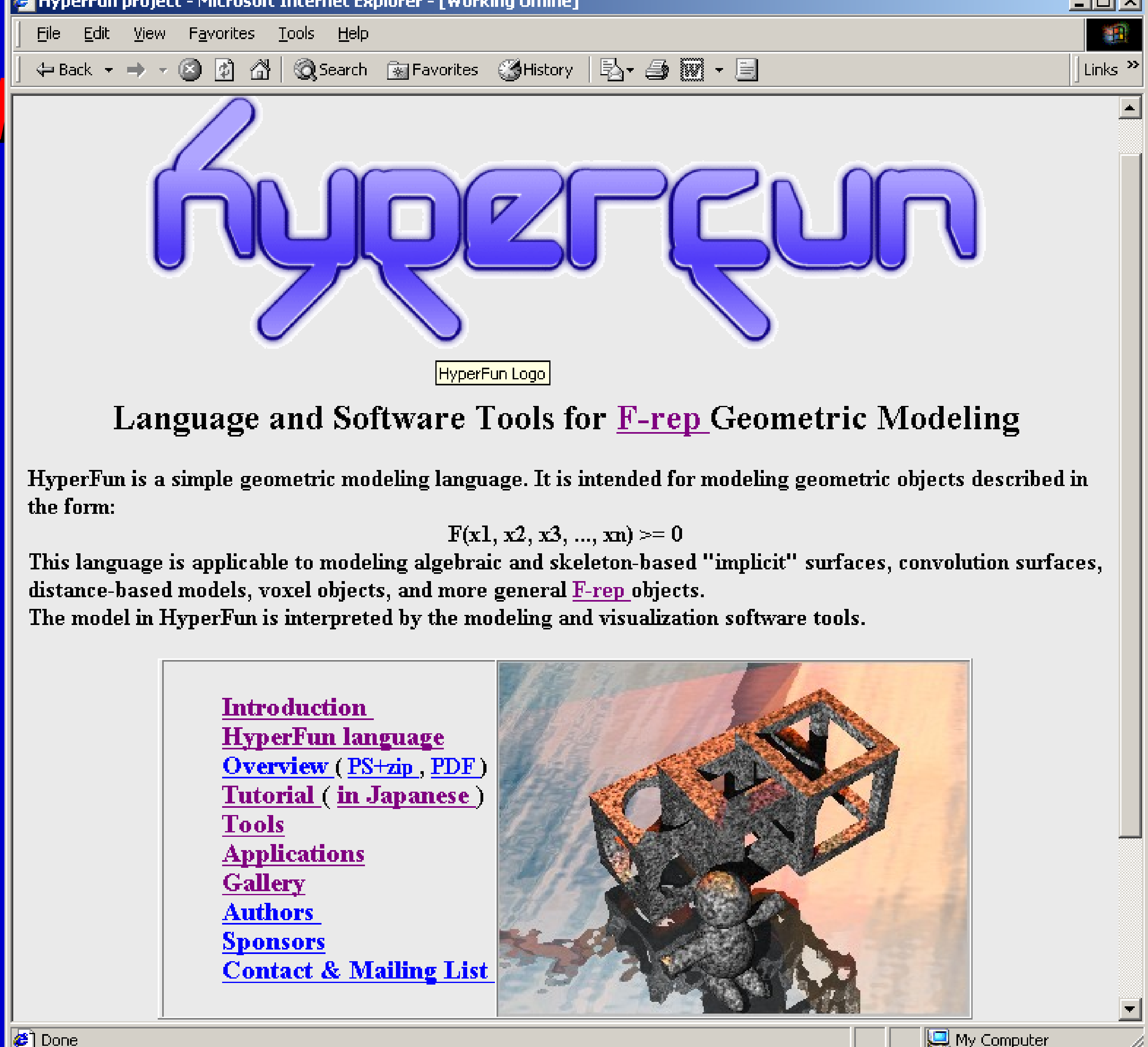
Leaves: primitives

Nodes: operations + relations

“Empty Case” principle and extensibility

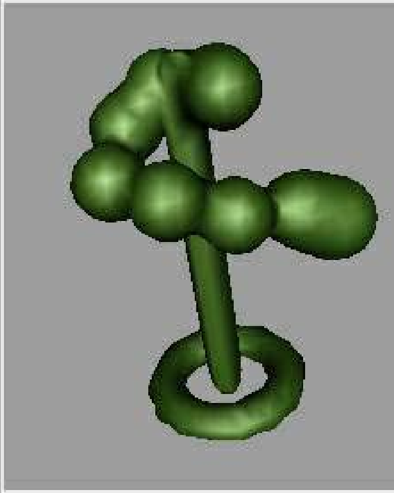
CSG-tree replaced by functions




HyperFun project - Microsoft Internet Explorer - [Working Offline]
File Edit View Favorites Tools Help
Back Search Favorites History Links
HyperFun Logo
Language and Software Tools for F-rep Geometric Modeling
HyperFun is a simple geometric modeling language. It is intended for modeling geometric objects described in the form:
F(x1, x2, x3, ..., xn) >= 0
This language is applicable to modeling algebraic and skeleton-based "implicit" surfaces, convolution surfaces, distance-based models, voxel objects, and more general F-rep objects.
The model in HyperFun is interpreted by the modeling and visualization software tools.
Introduction
HyperFun language
Overview (PS+zip , PDF)
Tutorial (in Japanese)
Tools
Applications
Gallery
Authors
Sponsors
Contact & Mailing List
Done
My Computer

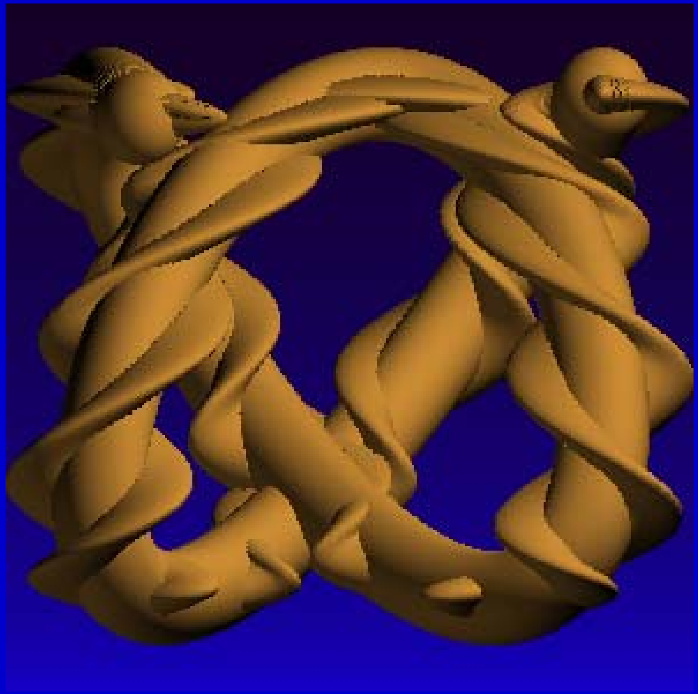
hfExample

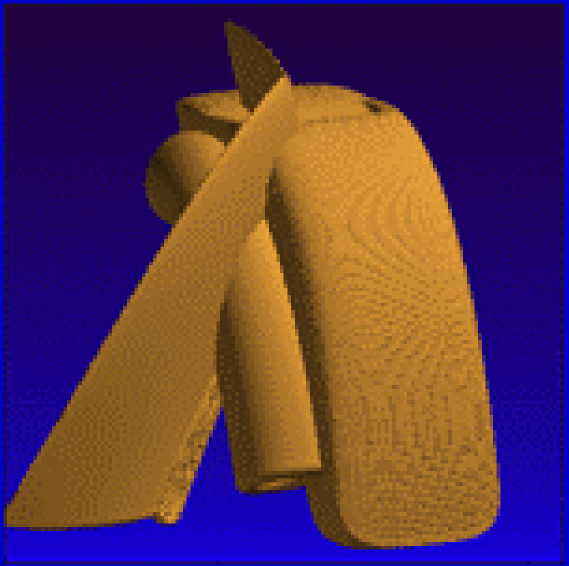
```
--This HyperFun program consists of one object:
--union of superellipsoid, torus and soft object

my_model(x[3], a[1])
{
array x0[9], y0[9], z0[9], d[9], center[3];
x1=x[1];
x2=x[2];
x3=x[3];

-- superellipsoid by formula
superEll = 1-(x1/0.8)^4-(x2/10)^4-(x3/0.8)^4;

-- torus by library function
center = [0, -9, 0];
torus = hfTorusY(x,center,3.5,1);

-- soft object
x0 = [2.,1.4, -1.4, -3, -3, 0, 2.5, 5., 6.5];
y0 = [8, 8, 8, 6.5, 5, 4.5, 3, 2, 1];
z0 = [0, -1.4,-1.4, 0, 3, 4, 2.5, 0, -1];
d = [2.5, 2.5, 2.5, 2.5, 2.5, 2.5, 2.5, 2.7, 3];
sum = 0.;
i = 1;
while (i<10) loop
        xt = x[1] - x0[i];
        yt = x[2] - y0[i];
        zt = x[3] - z0[i];
        r = sqrt(xt*xt+yt*yt+zt*zt);
        if (r <= d[i]) then
                r2 = r*r; r4 = r2*r2; r6 = r4*r2;
                d2 = d[i]^2; d4 = d2*d2; d6 = d4*d2;
                sum = sum + (1 - 22*r2/(9*d2) +
                17*r4/(9*d4) - 4*r6/(9*d6));
        endif;
        i = i+1;
endloop;
soft = sum - 0.2;

-- final model as set-theoretic union
my_model = superEll | torus | soft;
}
```



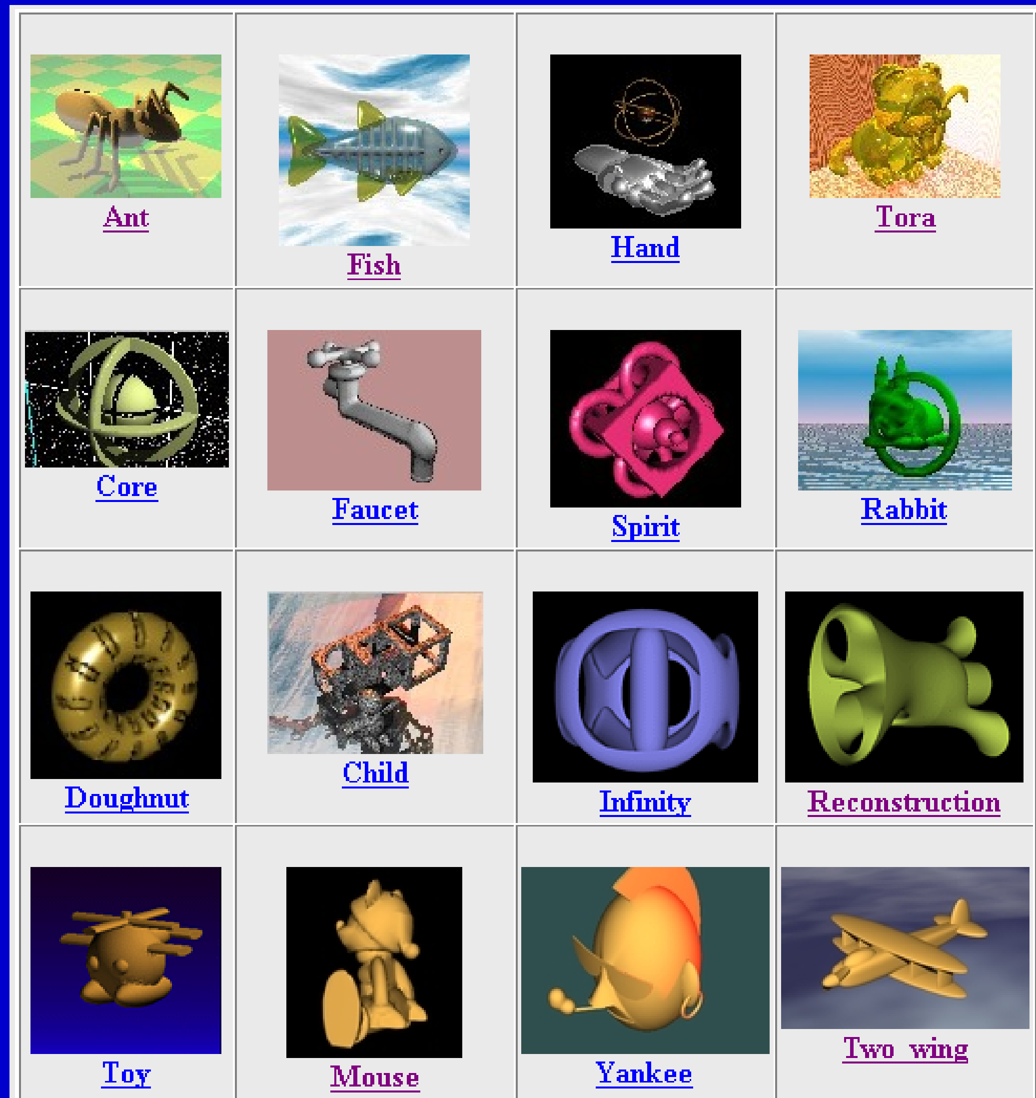
Unary Operations: Sweeping

Images by A. Pasko



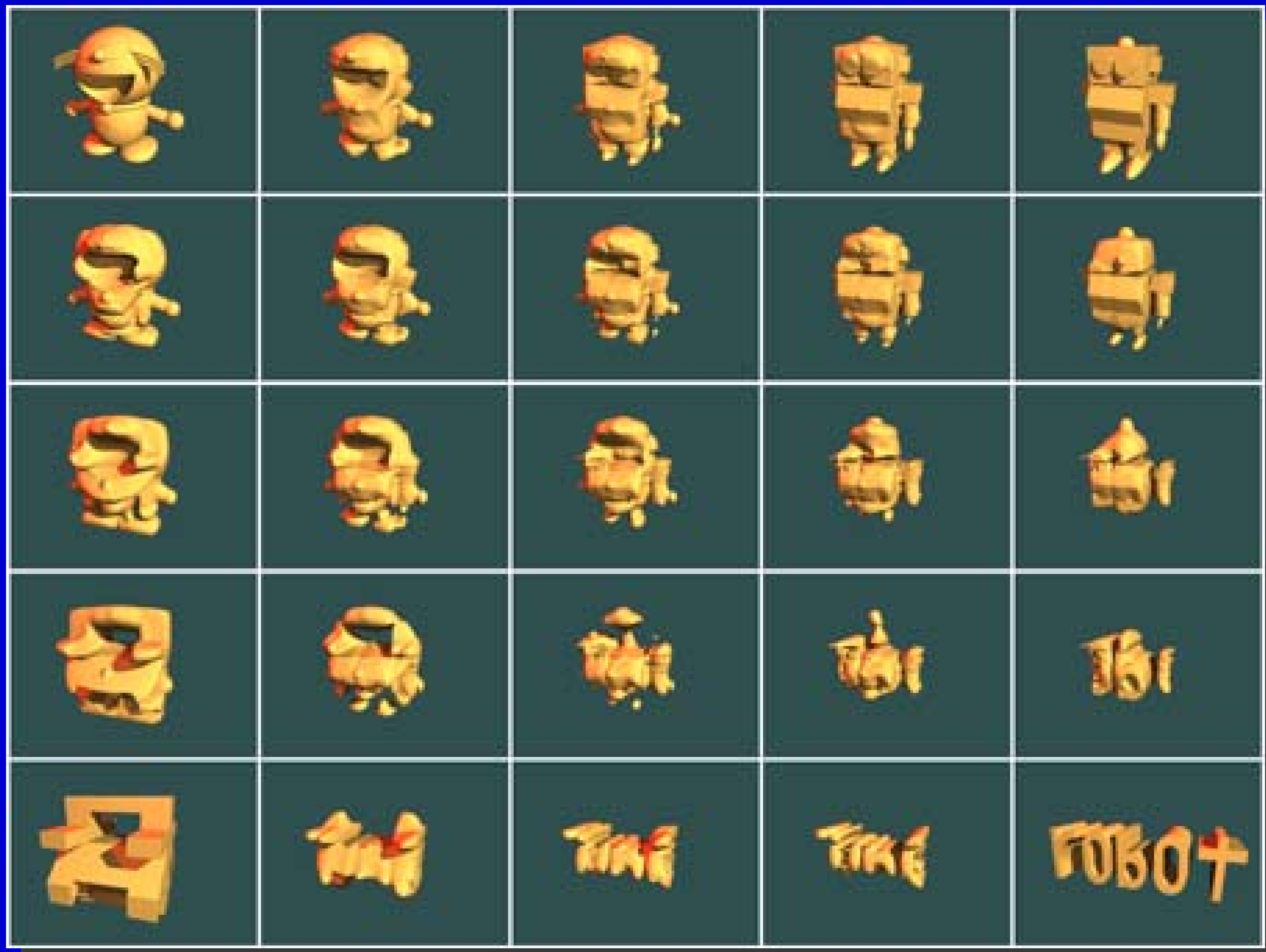


HyperFun Gallery by Students



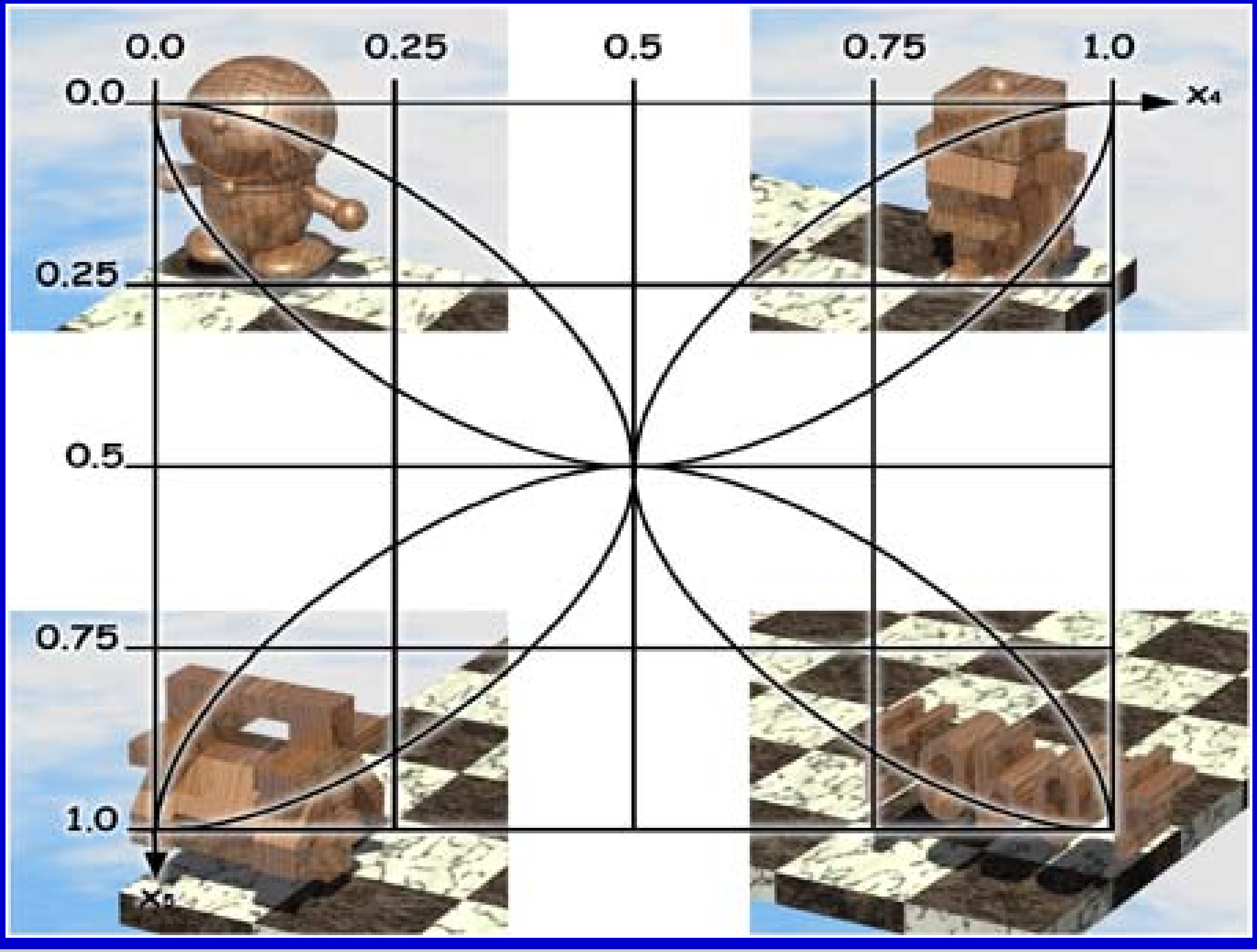
Spreadsheet Rendering

by Alexander PASKO, www.hyperfun.org



Animation Path in $t_1 t_2$ Plane

by A. PASKO, www.hyperfun.org



HyperFun

- HyperFun is a minimalist language
- Supports all notions of FRep and the hypervolume model
- Multiple coordinate variables support: multidimensional modeling
- Functional expressions for geometry and attributes
- Built-in operators for set-theoretic operations: |, &, \, @
- Extendible FRep library of primitives and operations
- POV-Ray High Quality Rendering

F-rep Summary

- *F-rep (function representation)*
- *Implicit surfaces using the functional form of set operations (union, intersection, difference)*
- *Discovery by Rvachev [Rvac63] thus enables for unified language for both CSG tree (or scene graph) and subsequent primitives and any operations*
- BLOOMENTHAL J. et al. 1997. *Introduction to Implicit Surfaces. Morgan Kaufmann 1997. 332 pp.*

Computer Graphics...

- ... must account <u>geometry</u>
- <u>material properties</u>: reflectance/color, refractive index, opacity, and (for light sources) emmisivity
- <u>radiometry</u>
- output for viewing: explicitly or implicitly <u>psychophysics</u>

- by D. Dobkin & S. Teller, 1999

F-rep Hypertextures

- F-rep
- Unified language for objects and attributes
  Schlick et al. 2001
- This historical value result unifies differen
  dialects of mathematical language for
  objects, relationships, and operations AND
  ATTRIBUTES...
- ... DENSITY, TOO (SIMULATION)
- WWW.HYPERFUN.ORG

Constructive Hypervolume Texturing

by Alexander PASKO, www.hyperfun.org

implemented by B. Schmitt

F-rep Importance

- F-rep
- Unified language for both CSG tree (or scene graph) and subsequent primitives
- This historical value result unifies different dialects of mathematical language for objects, relationships, and operations
- Challenge, no special modeling, slow
- Modeling "esperanto"

Further Reading - Papers

- AURENHAMMER, F. 1991. Voronoi Diagrams - A Survey of Fundamental Geometric Data Structure. pp. 345 - 405. ACM Computing Surveys. Vol. 23. No. 3. September 1991
- CHAZELLE, B. et al. 1996. Application Challenges to Computational Geometry. TR-521-96. Princeton University April 1996 http://www.cs.princeton.edu/~Chazelle/taskforce/CGreport.p
- FERKO, A. 2003. Algorithmic Strategies (to appear on www.sccg.sk/ ~ferko)
- PASKO, A. et al. 2003. HyperFun Page.

Further Reading – Selected Books

- BOISSONNAT, J-D. - YVINEC, M. 1998. *Algorithmic Geometry.* 519 p. Cambridge: Cambridge University Press 1998. ISBN 0-521-56529-4.
- **DE BERG, M. et al. 1997. *Computational Geometry, Algorithms and Applications.* 365 p. Berlin: Springer Verlag 1997. ISBN 3-540-61270-X – in TU Graz Library**
- EDELSBRUNNER, H. 1987. *Algorithms in Combinatorial Geometry.* 423 p. Berlin: Springer-Verlag 1987. ISBN 3-540-13722-X ISBN 3-387-13722-X
- GOODMAN, J. E. - O'ROURKE, J., eds. 1997. *Handbook of Discrete and Computational Geometry.* Boca Raton - New York: CRC Press 1997
- O'ROURKE, J. 1994. *Computational Geometry in C.* Cambridge: Cambridge University Press 1994. 346 p. ISBN 0-521-445922
- PREPARATA, F. P. - SHAMOS, M. I. 1985. *Computational Geometry: An Introduction.* 390 p. New York: Springer-Verlag 1985. ISBN 0-387-96131-3 ISBN 0-540-96131-3 – with Konrad Schindler

Conclusions

- Motivation: optimal modeling
- Computational Geometry … optimal…
- Functional Representation … modeling…
- Current Hot Research: Implicit Surfaces to Interpolate (Turk et al.) and Real-time Raytracing (Hasan, 2003), F-rep community @ www.hyperfun.org

Vor(P), DT(P), and F-rep

Andrej Ferko: Thank You for Your Attention

ferko@icg.tu-graz.ac.at