L-Systems

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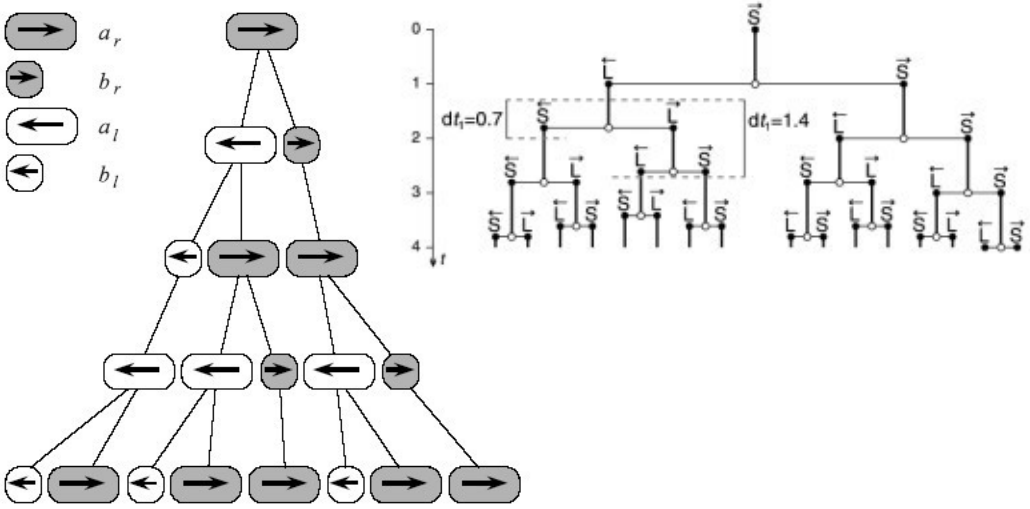
L-Systems: A Language for Modeling Growth

- Aristid Lindenmayer (1925-1989)
  - Anabaena catenula
  - 1968 – formalism for simulating the development of multicelullar organisms
  - Lindenmayer systems (L-systems) – parallel string rewriting systems



Development of a filament (*Anabaena catenula*)

L-Systems: A Language for Modeling Growth

- Subsequently applied to investigate higher plants and plant organs
- L-systems conceived as mathematical theory of plant development

- Przemyslaw Prusinkiewicz
  - extended L-systems in various ways

D0L-Systems

- Deterministic context-free L-systems
- Defined by a formal grammar G = <V, ω, P>

  V is the alphabet of the system

  $\omega \in V^+$ is a nonempty world called the axiom

  $P \subset V \times V^*$ is a finite set of productions

  V^+ is the set of all nonempty words over V

  V* is the set of all words over V

  if no production is specified for a given letter, the identity production a $\rightarrow$ a is assumed

D0L-Systems

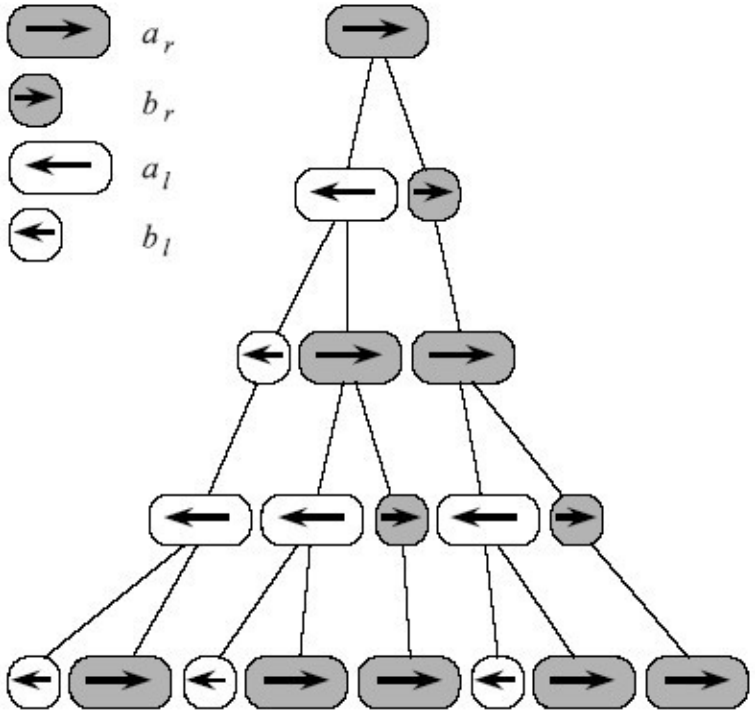
$\omega : a_r$

$p_1 : a_r \rightarrow a_l b_r$

$p_2 : a_l \rightarrow b_l a_r$

$p_3 : b_r \rightarrow a_r$

$p_4 : b_l \rightarrow a_l$



a_r

$a_l b_r$

$b_l a_r a_r$

$a_l a_l b_r a_l b_r$

$b_l a_r b_l a_r a_r b_l a_r a_r$

Development of a filament (*Anabaena catenula*)

Turtle Graphics

- Geometric interpretation of L-systems

  F move forward by a fixed step of length d and draw a line from the old to the new position

  f move forward (as F) but do not draw the line

  + turn left (counterclockwise) by a fixed angle δ

  − turn right (clockwise) by the angle δ

  Using trigonometry the following table can be deduced:

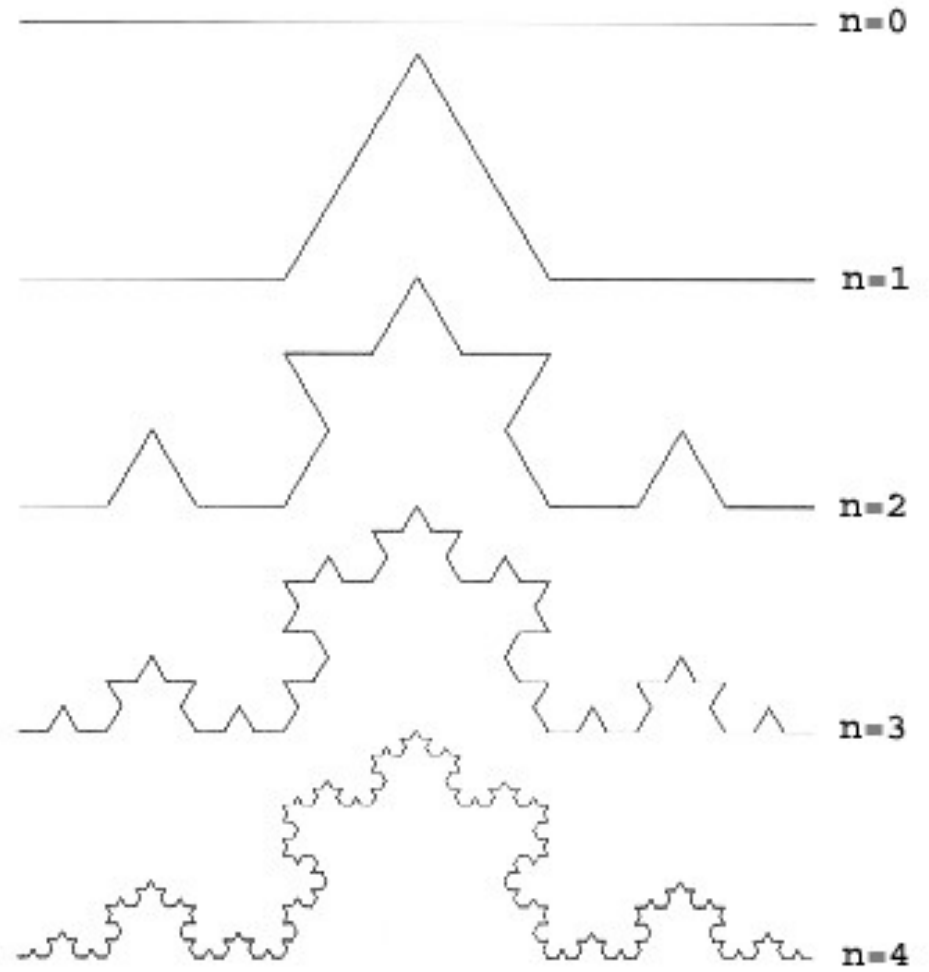
command	the state of the turtle (x, y, α) is changed to:
F	(x+dcosα, y+dsin α, α)
f	(x+dcosα, y+dsin α, α)
+	(x, y, α−δ)
−	(x, y, α+δ)

x, y define position on the plane, angle α orientation

Growing Classical Fractals with L-Systems

- Koch curve

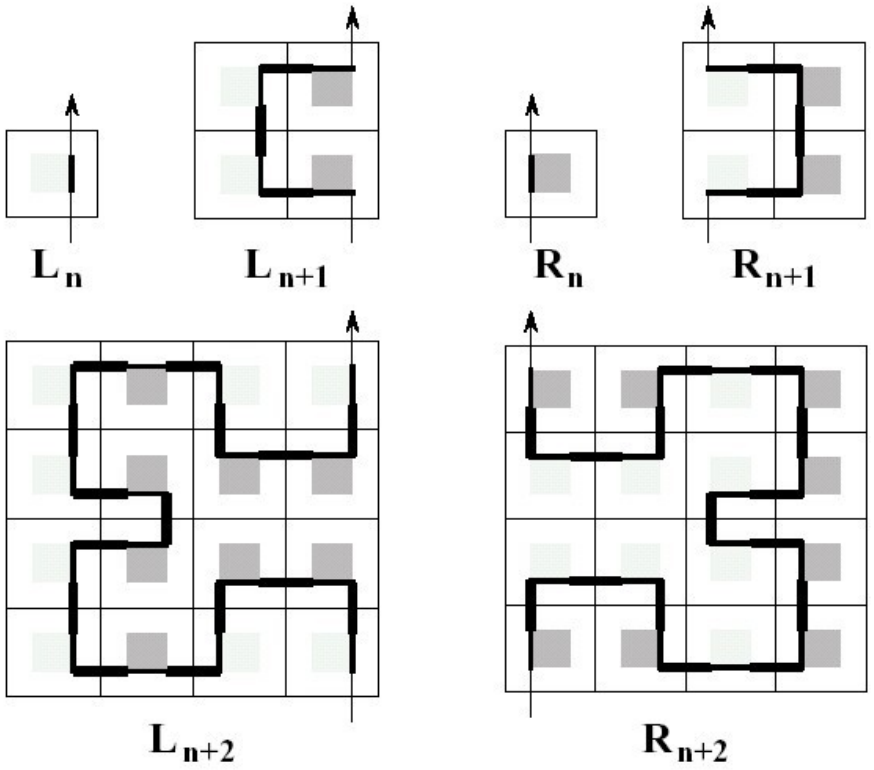
L-system:	Koch curve
Axiom:	F
Production rules:	F → F+F−−F+F
	+ → +
	− → −
Parameter:	δ = 60°



Growing Classical Fractals with L-Systems

- Space filling curves (Hilbert curve)
- Using additional symbols which are ignored by the turtle to control derivation

L-system:	Hilbert curve
Axiom:	L
Production rules:	L → +RF−LFL−FR+
	R → −LF+RFR+FL−
	+ → +
	− → −
Parameter:	δ = 90°



L-System Trees and Bushes

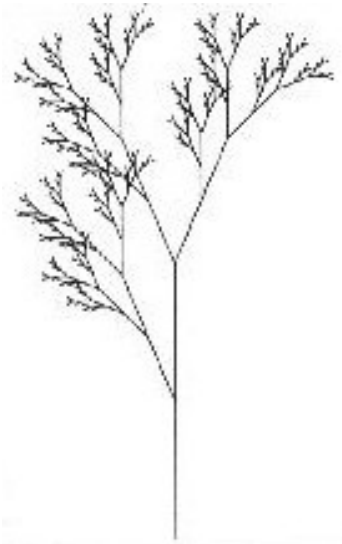
Stacking of turtle states

[save the current state of the turtle at the top of the stack

] pops the top state from the from the stack and put the turtle into this state

Axiom: F
Rules: F → F[+F]F[−F]F
Parameter: δ = 25.7°

Axiom: F
Rules: F → FF+[+F−F−F]−[−F+F+F]
Parameter: δ = 25°



Axiom: B
Rules: F → FF
B → F[+B]F[−B]+B
Parameter: δ = 20°

Non Deterministic L-Systems

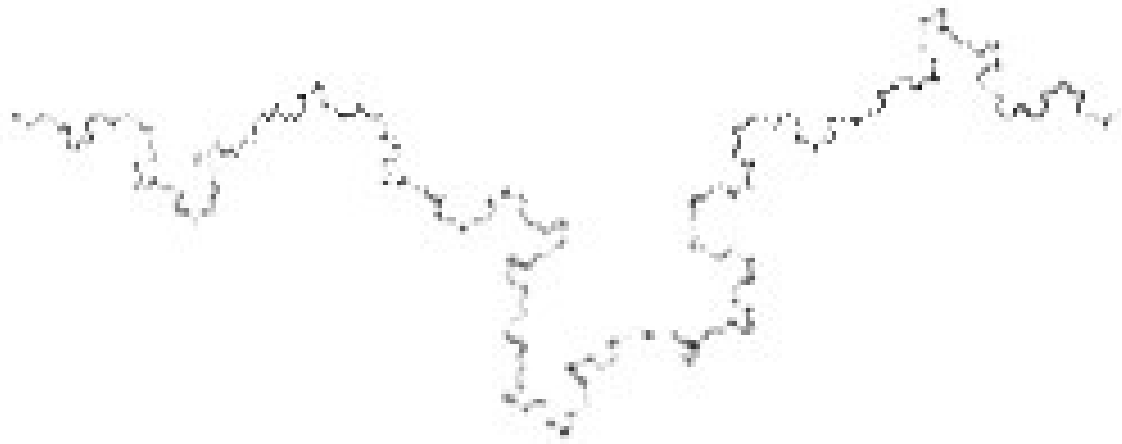
- At least one symbol has more than one production
- There must be a mechanism which selects one of the productions for each symbol during derivation
  - Stochastic L-systems
  - Context-sensitive L-systems
  - Parametric L-systems

Stochastic L-Systems

- Different productions for a symbol are selected randomly
- Used to generate variation among individuals

L-system: Stochastic Weedlike Plant
Axiom: F
Rules: F → F[+F]F[−F]F (probability 1/3)
F → F[+F]F (probability 1/3)
F → F[−F]F (probability 1/3)
Parameter: δ = 25.7°

Stochastic L-Systems



L-system: Random Koch Curve
Axiom: F
Rules: F → F–F++F–F (probability 1/2)
F → F– –F+F (probability 1/2)
Parameter: δ = 60°

Context-sensitive L-Systems

- The selection of a production for a symbol depends on the adjacent symbols in the current string
  - $lc < A > rc \rightarrow B$
- 1L-systems, 2L-systems, IL-systems

ω: baaaaaaaa
p_1: b < a → b
p_2: b → a

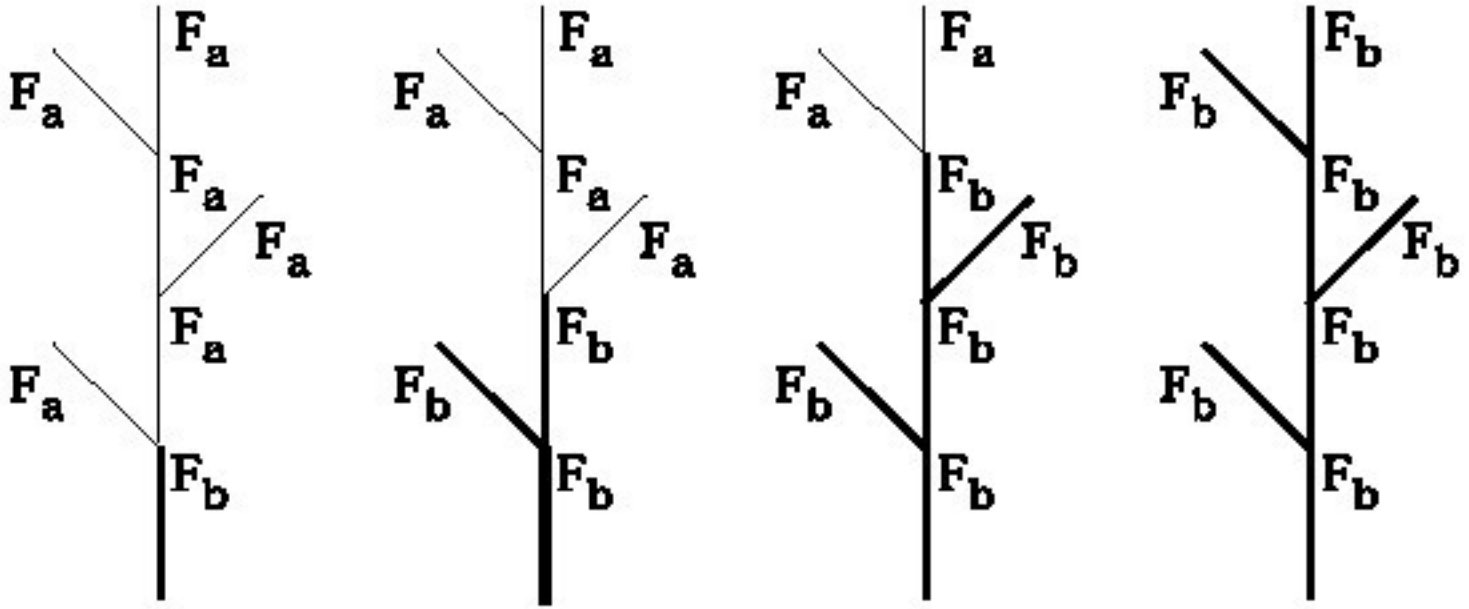
baaaaaaaa
abaaaaaaa
aabaaaaaa
aaabaaaaa
aaaabaaaa
...

xy < a > cde → aaa

zaxyacdef
zaxyaaacdef

Context-sensitive L-Systems

- To simulate the propagation of signals (hormones, nutrients) between parts of a plant



#ignore: + −

ω: $F_b[+F_a]F_a[-F_a]F_a[+F_a]F_a$

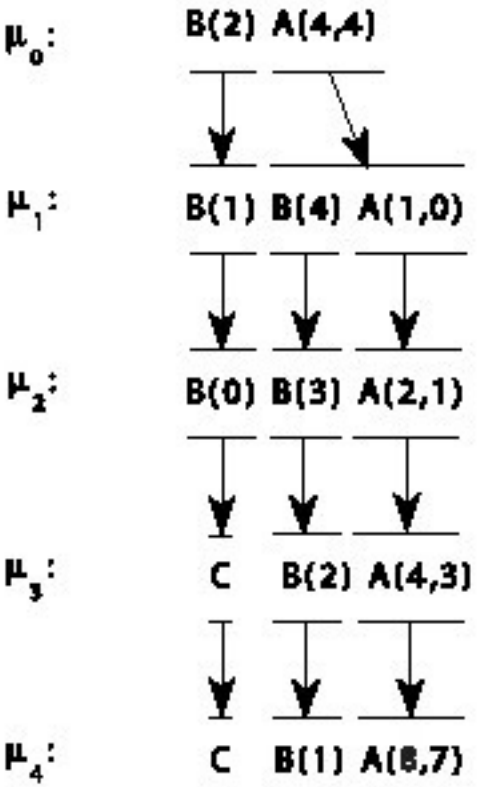
p: $F_b < F_a \rightarrow F_b$

Parametric L-Systems

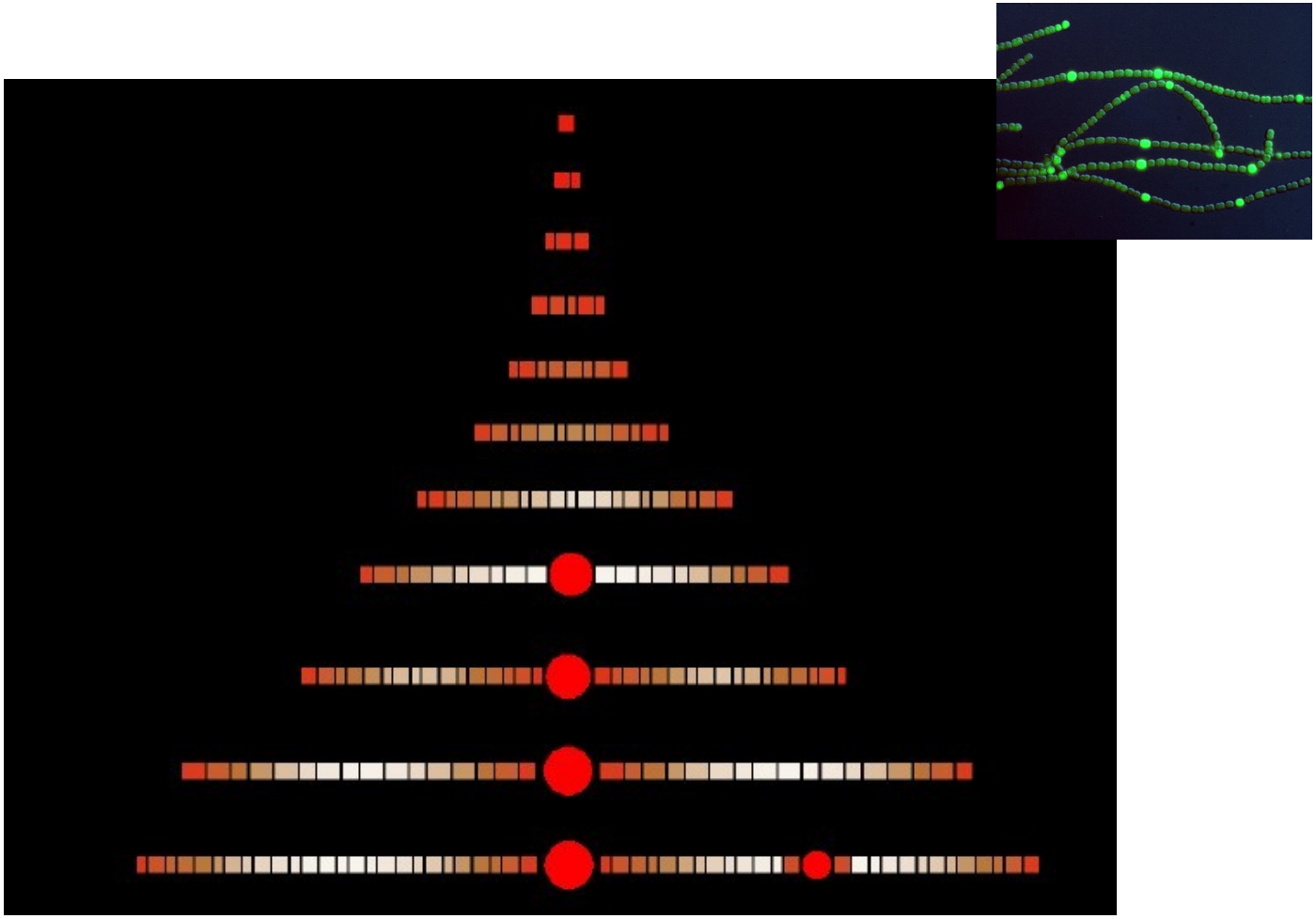
- Symbols are associated with a finite set of parameters
- Parameter values are used to select productions and to control turtle geometry
  - $id : lc < pred > rc : cond \rightarrow succ : prob$
- Important for plant modeling, because the geometry of a plant is the result of its developmental process

Parametric L-Systems

$$
\begin{array}{lll}
\omega : & B(2)A(4,4) & \\
p_1 : & A(x,y) : y <= 3 & \rightarrow \quad A(x*2, x+y) \\
p_2 : & A(x,y) : y > 3 & \rightarrow \quad B(x)A(x/y, 0) \\
p_3 : & B(x) : x < 1 & \rightarrow \quad C \\
p_4 : & B(x) : x >= 1 & \rightarrow \quad B(x-1)
\end{array}
$$

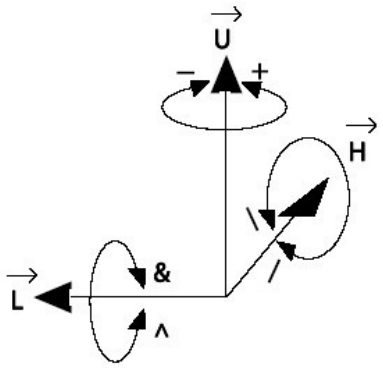


Parametric L-Systems



Development of a filament (*Anabaena catenula*)

3D Graphics Representation



Turtle orientation in space

$\vec{H}$ the turtle's heading
$\vec{L}$ the direction to the left
$\vec{U}$ the direction up

rotation of the turtle: $\left[\vec{H}'\vec{L}'\vec{U}'\right] = \left[\vec{H}\vec{L}\vec{U}\right]\mathbf{R}$
R rotation matrix

\+ turn left by angle δ, using $\mathbf{R_U}(\delta)$
− turn right by angle δ, using $\mathbf{R_U}(-\delta)$
& pitch down by angle δ, using $\mathbf{R_L}(\delta)$
^ pitch up by angle δ, using $\mathbf{R_L}(-\delta)$
\ roll left by angle δ, using $\mathbf{R_H}(\delta)$
/ roll right by angle δ, using $\mathbf{R_H}(-\delta)$
| turn around, using $\mathbf{R_U}(180°)$

$$\mathrm{R_U}(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{R_L}(\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$\mathrm{R_H}(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

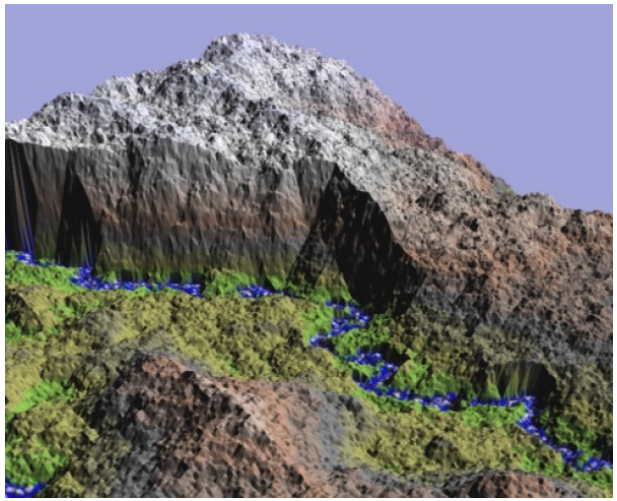
3D Graphics Representation

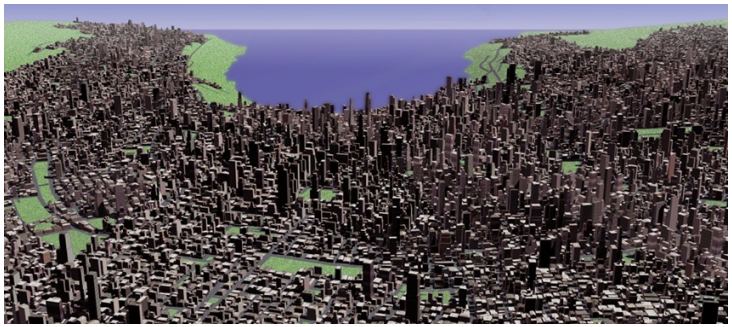
n=7, δ=22,5°
ω: A
p_1: A → [&FL!A]/////'[&FL!A]///////'[&FL!A]
p_2: F → S/////F
p_3: S → FL
p_4: L → [’’’^^{–f+f+f–|–f+f+f}]

Application of L-Systems

- Simple multicellular organisms
- Plants, development of plants
- Rivers
- Seashells
- City models (street maps, buildings)
- Development of human organs
- Classical fractal forms
- Generating of music (3D L-systems as MIDI files)

Application of L-Systems





References

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- H.-O. Peitgen, H. Jürgens, D. Saupe. *Chaos and fractals*. New York : Springer-Verlag, 1992. pp. 984.
- J. Žára, B. Beneš, J. Sochor, P. Felkel. *Moderní počítačová grafika*. Brno : Computer Press, 2004. pp. 609.

References

Pictures:

Music:
http://www.geocities.com/Athens/Academy/8764/lmuse/lmuse.html

Cities:
http://terra.cs.nps.navy.mil/DistanceEducation/online.siggraph.org/2001/Pa

Seashells:
http://old.cpsc.ucalgary.ca/Research/bmv/papers/sigcourse.2003/2-39-diffe

Stomach: http://www.sccg.sk/~durikovic/projects/livstom/anim.html

Plants: http://algorithmicbotany.org/papers/#papers
http://algorithmicbotany.org/papers/ecosys.sig98.pdf

Mountains: http://algorithmicbotany.org/papers/mountains.gi93.pdf