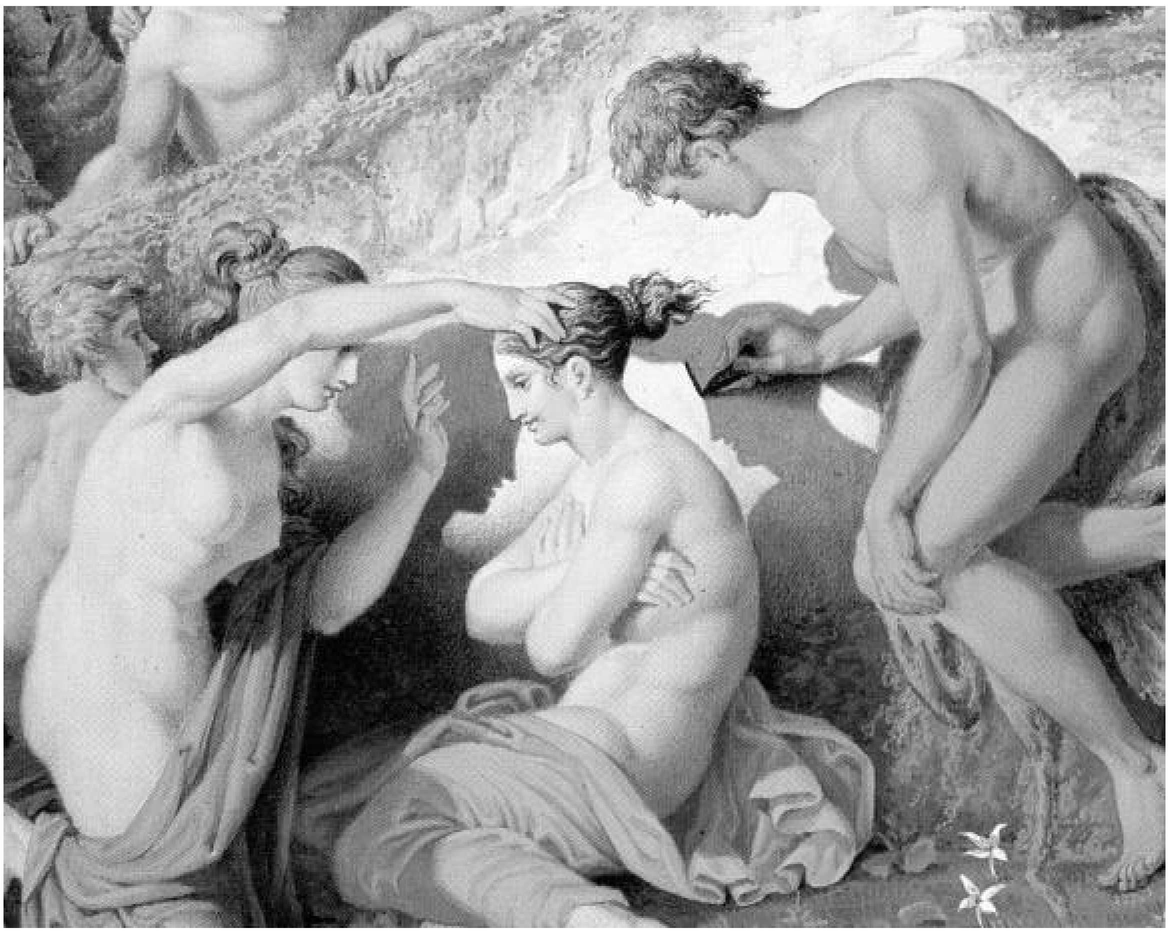
From 3D to 2D: Orthographic and Perspective Projection—Part 1

- History
- Geometrical Constructions
- Types of Projection
- Projection in Computer Graphics

Drawing as Projection

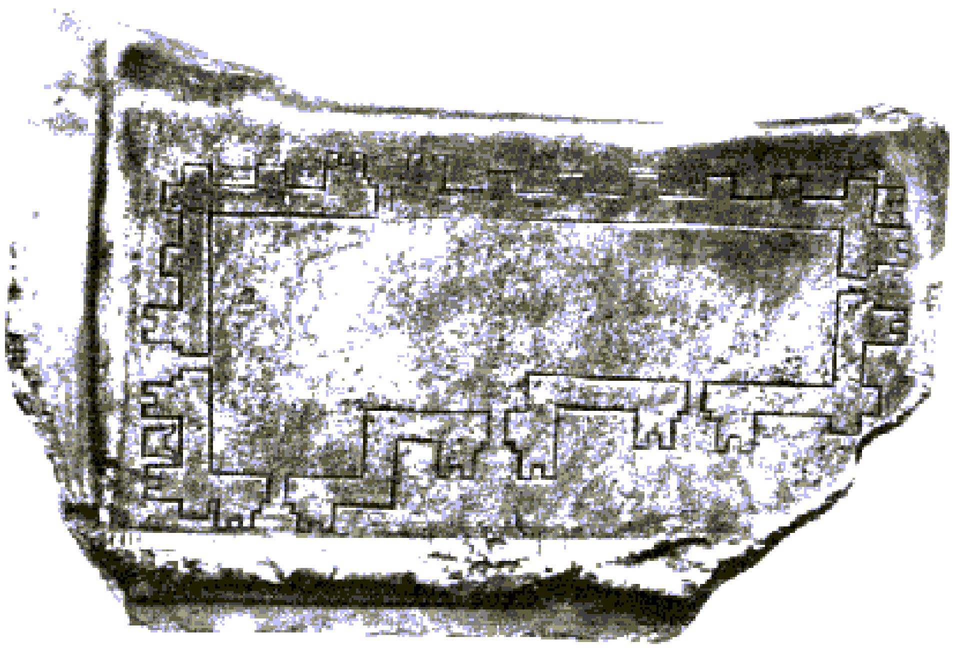
- A painting based on a mythical tale as told by Pliny the Elder: a Corinthian man traces the shadow of his departing lover



detail from **The Invention of Drawing**, 1830: Karl Friedrich Schinkle (Mitchell p.1)

Early Examples of Projection

- Plan view (orthographic projection) from Mesopotamia, 2150 BC is earliest known technical drawing in existence



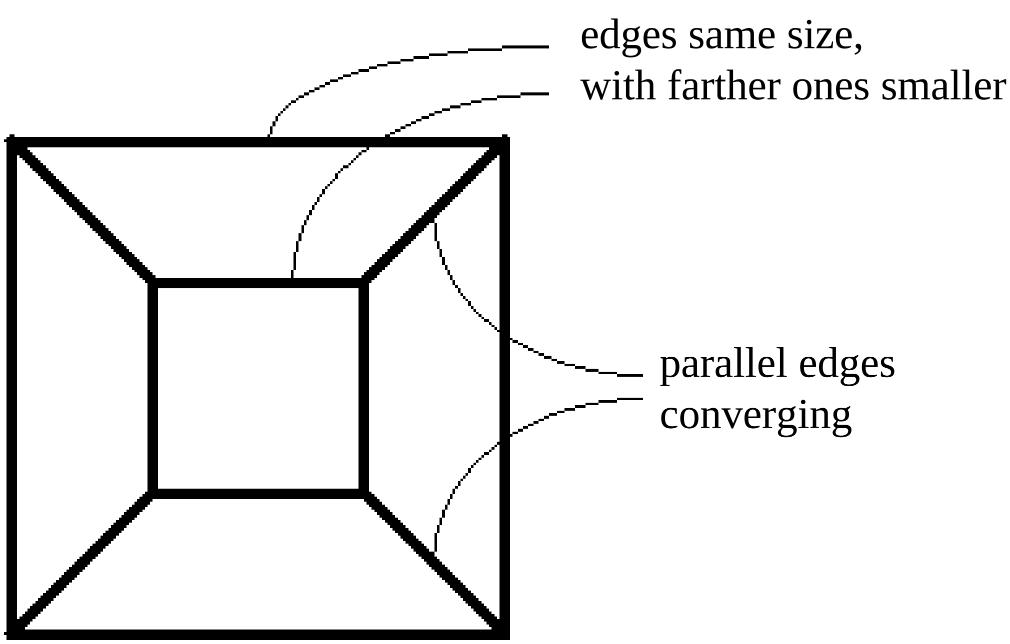
Carlbom Fig. 1-1

- Greek vases from late 6th century BC show perspective(!)

- Vitruvius, a Roman architect, published specifications of plan and elevation drawings, and perspective. Illustrations for these writings have been lost

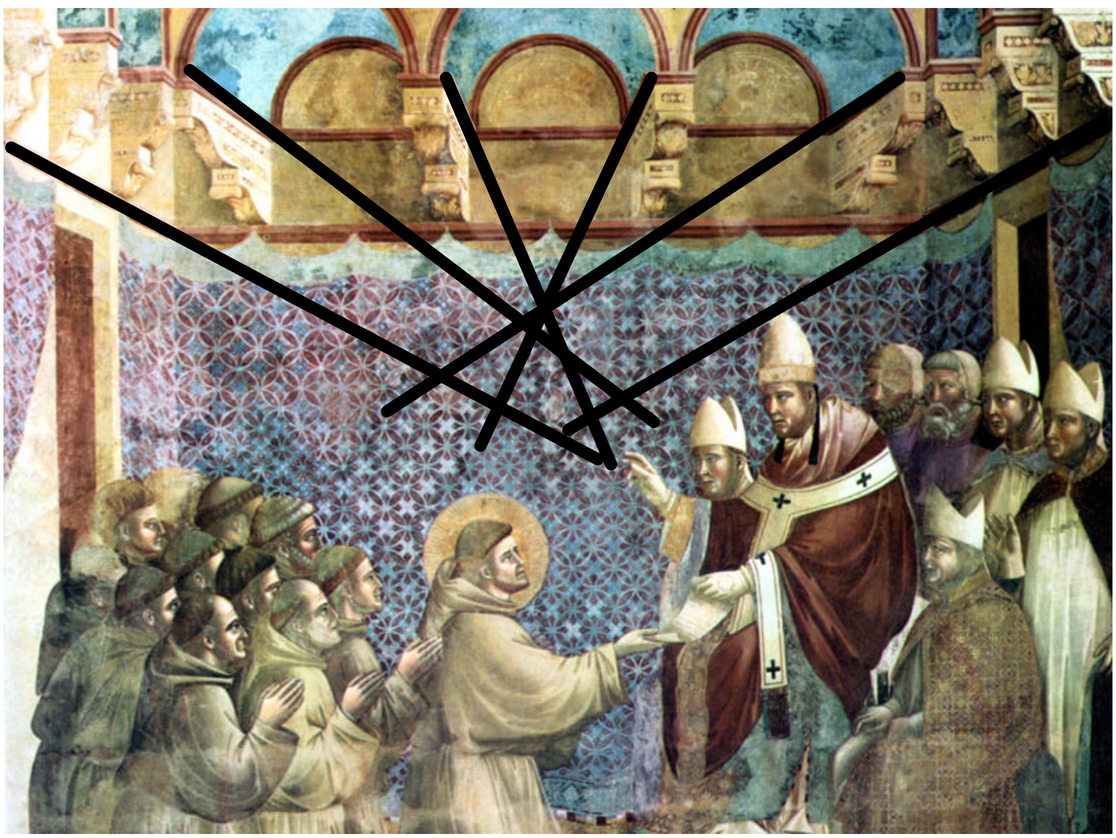
Most Striking Features of Linear Perspective

- || lines converge (in 1, 2, or 3 axes) to a *vanishing point*
- Objects farther away are more *foreshortened* (i.e., smaller) than closer ones
- Example: perspective cube



Early Perspective

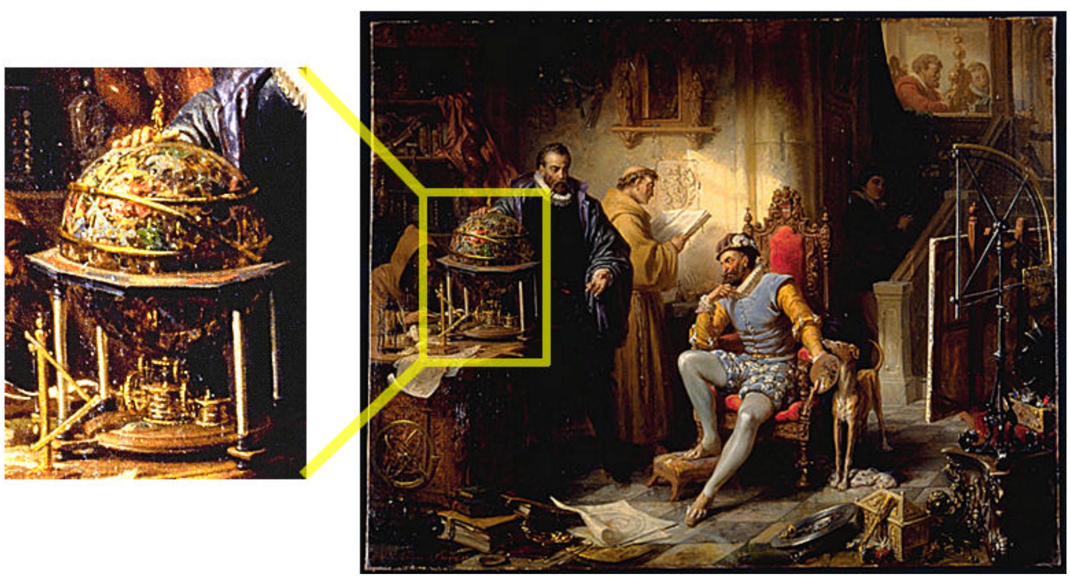
- Ways of invoking three dimensional space: rounded, volumetric forms suggested by shading, spatial depth of room suggested by converging lines
- Not systematic—lines do not converge to a single vanishing point



Giotto, **Franciscan Rule Approved**, Assisi, Upper Basilica, c.1295-1300

Setting for “Invention” of Perspective Projection

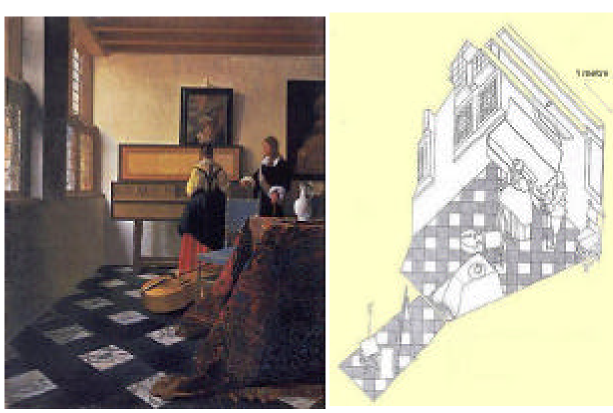
- The Renaissance: new emphasis on importance of individual point of view and interpretation of world, power of observation—particularly of nature (astronomy, anatomy, botany, etc.)
  - Massaccio
  - Donatello
  - Leonardo
  - Newton
- Universe as clockwork: intellectual rebuilding of universe along mechanical lines

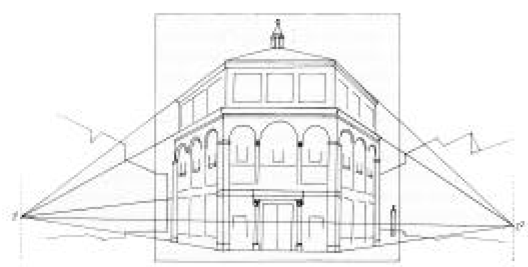


Ender, **Tycho Brahe and Rudolph II in Prague** (detail of clockwork), c. 1855
url: http://www.mhs.ox.ac.uk/tycho/catfm.htm?image10a

Brunelleschi and Vermeer

- Brunelleschi invented systematic method of determining perspective projections in early 1400’s. Evidence that he created demonstration panels, with specific viewing constraints for complete accuracy of reproduction. Note the perspective is accurate only from one POV (see Last Supper)
- Vermeer probably used *camera obscura* to aid painting his masterpieces; also created *perspective boxes* where picture, when viewed through a viewing hole, had correct perspective
- Vermeer on the web:
  - http://www.grand-illusions.com/vermeer/vermeer1.htm
  - http://www.vermeerscamera.co.uk/home.htm

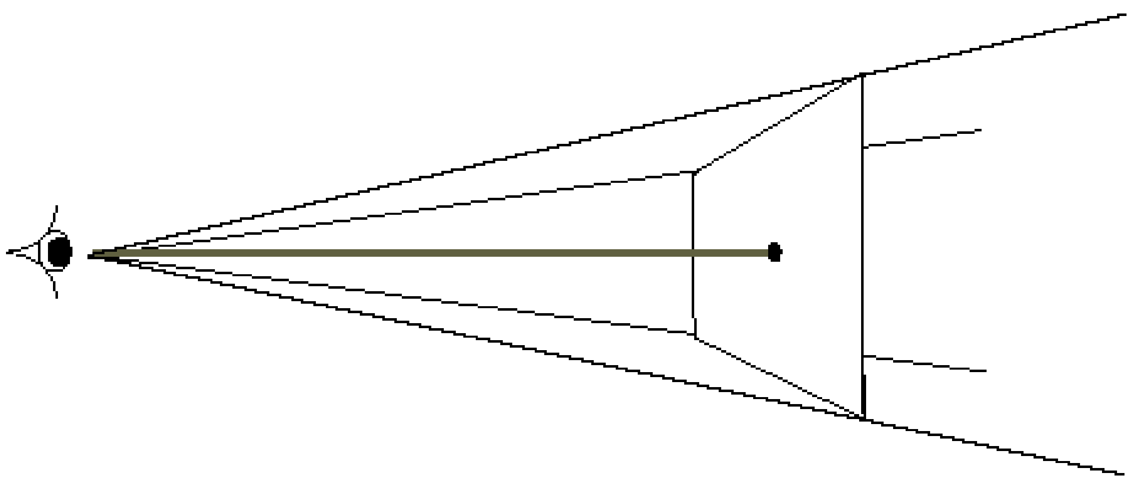




Vermeer, **The Music Lesson**, c.1662-1665 and reconstruction (right)

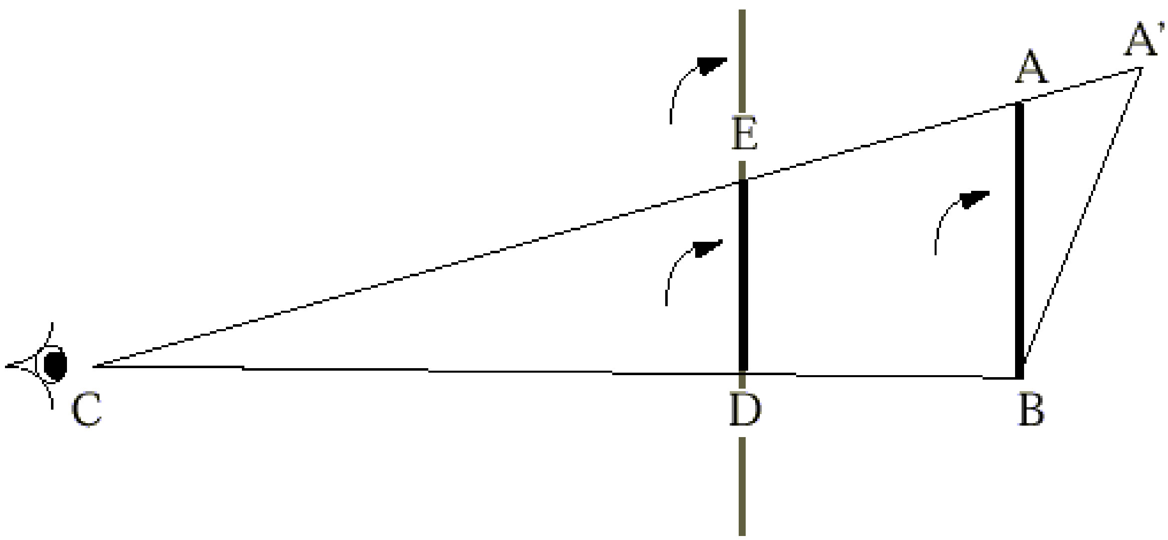
Alberti

- Published first treatise on perspective, *Della Pittura*, in 1435
- "A painting [the projection plane] is the intersection of a visual pyramid [view volume] at a given distance, with a fixed center [center of projection] and a defined position of light, represented by art with lines and colors on a given surface [the rendering]." (Leono Battista Alberti (1404-1472), *On Painting*, pp. 32-33)



The Visual Pyramid and Similar Triangles

- Projected image is easy to calculate based on
  - height of object (AB)
  - distance from eye to object (CB)
  - distance from eye to picture (projection) plane (CD)
  - and using the relationship CB: CD as AB: ED

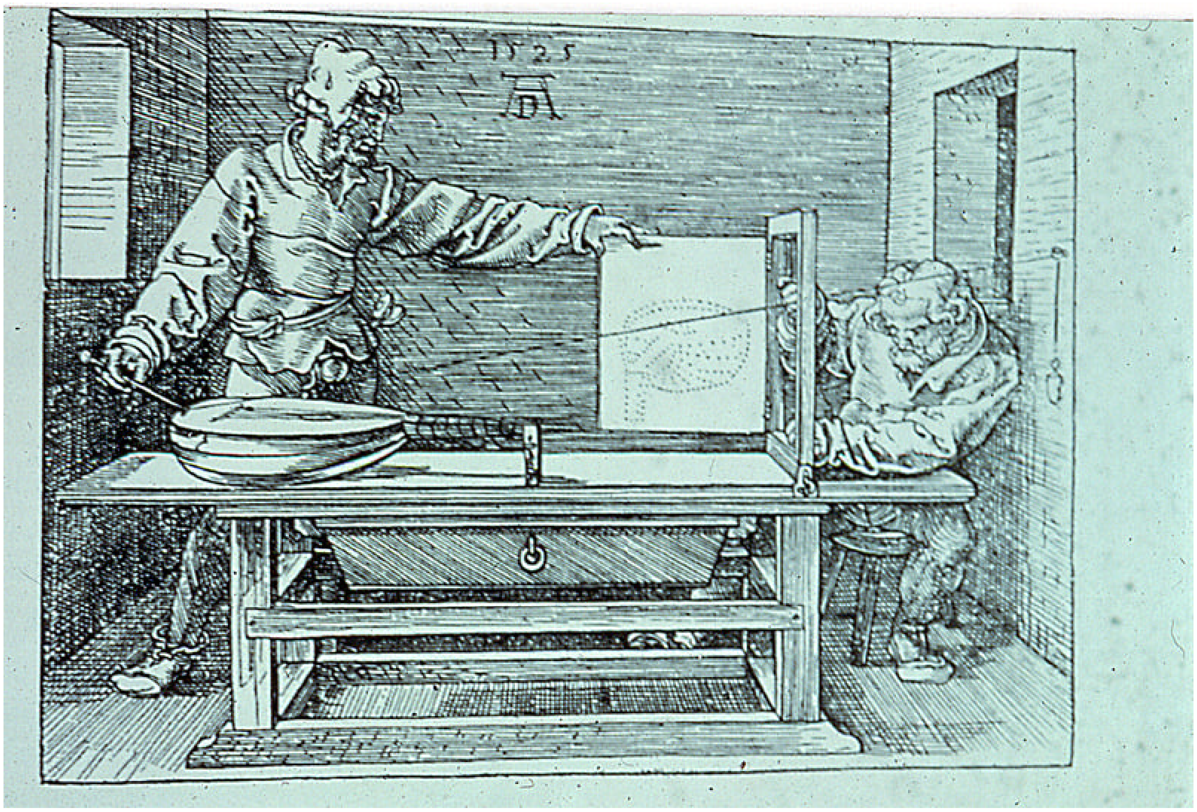


CB : CD as AB : ED

- AB is the component of A' in the plane of the projection

Dürer

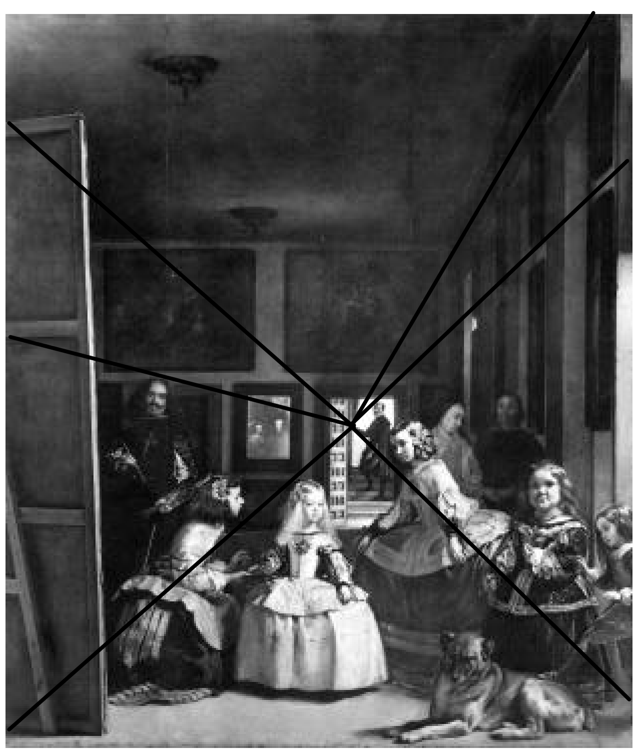
- Concept of similar triangles described both geometrically and mechanically in widely read treatise by Albrecht Dürer (1471-1528)



Albrecht Dürer, **Artist Drawing a Lute**, 1525

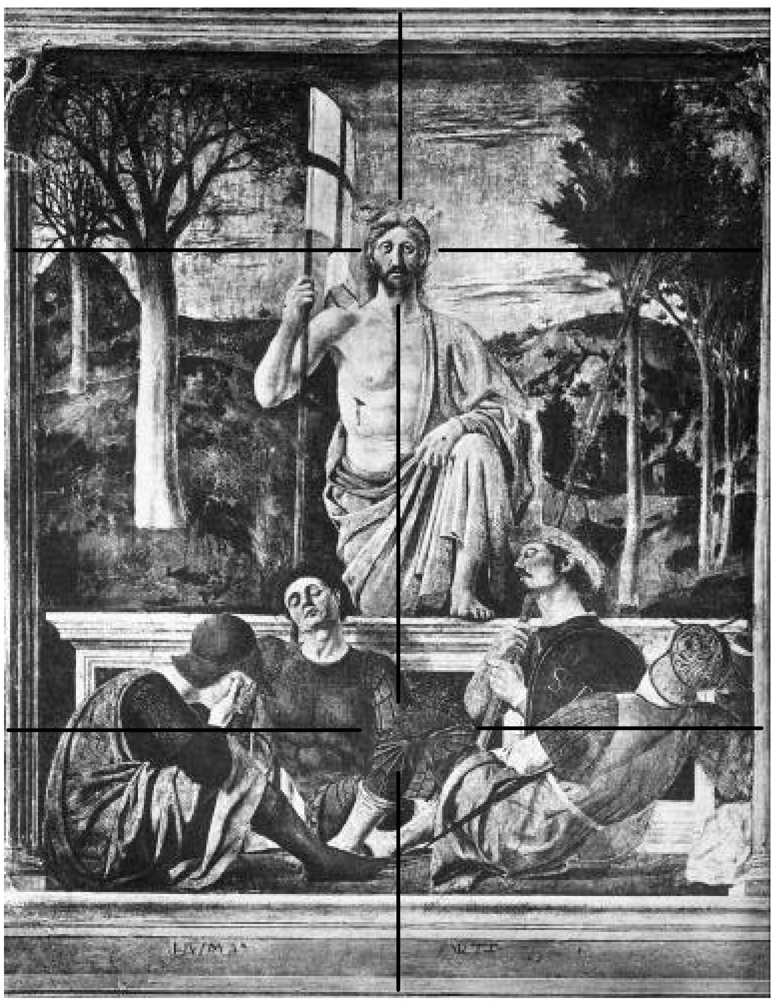
Las Meninas (1656) by Diego Velàzquez

- Point of view influences content and meaning of what is seen
- Are royal couple in mirror about to enter room? Or is their image a reflection of the painting on the far left?
- Analysis through computer reconstruction of the painted space
  - verdict: royal couple in mirror is a reflection from canvas in foreground, not a reflection of actual people (Kemp pp. 105-108)

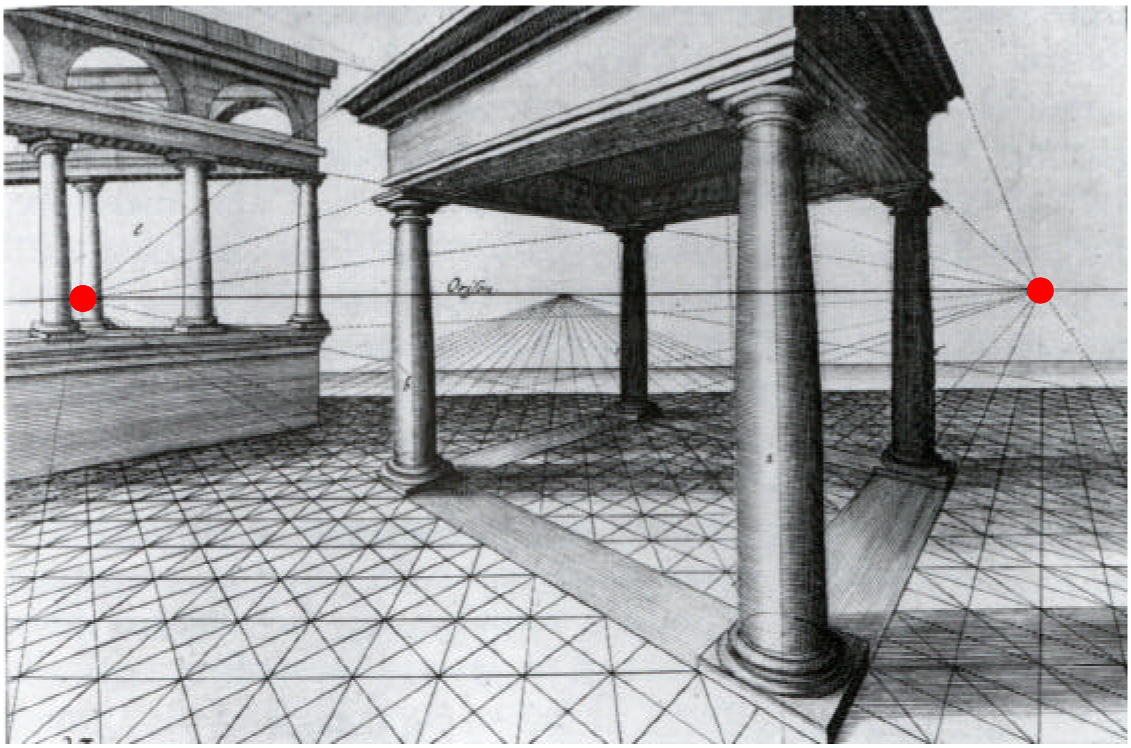


Piero della Francesca
The Resurrection (1460)

- Perspective can be used in an unnatural way to control perception
- Use of two points of view concentrates viewer's attention alternately on face of Christ and sarcophagus



Geometrical Construction of Projections

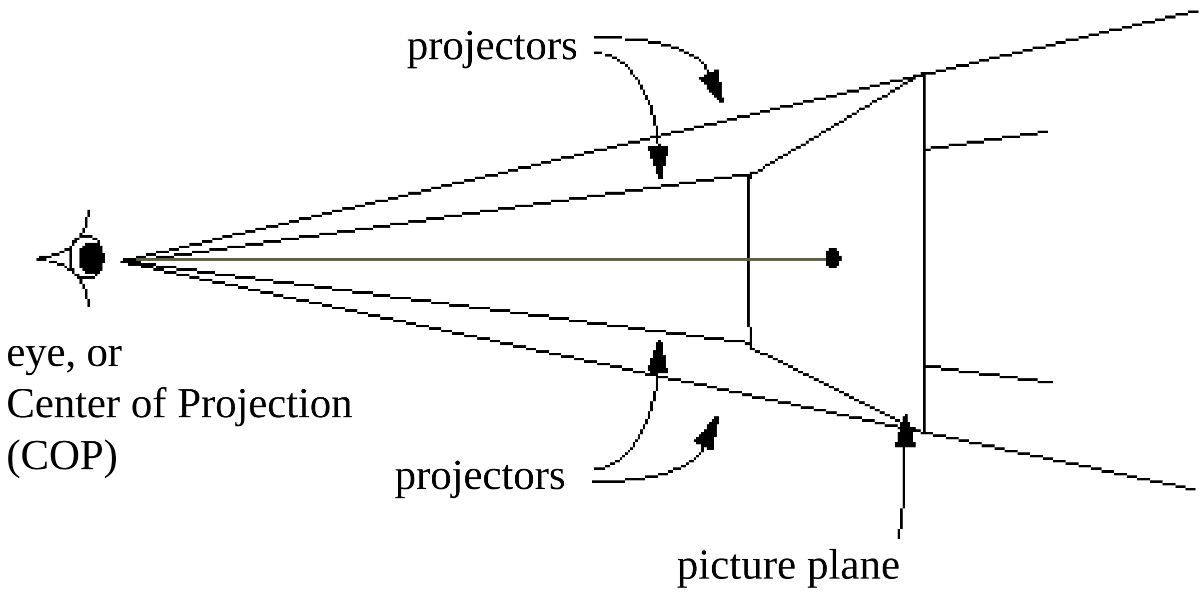


from Vredeman de Vries's **Perspective**, Kemp p.117

- 2 point perspective—two vanishing points

Planar Geometric Projection

- Projectors are straight lines
- Projection surface is a plane (picture plane, projection plane)

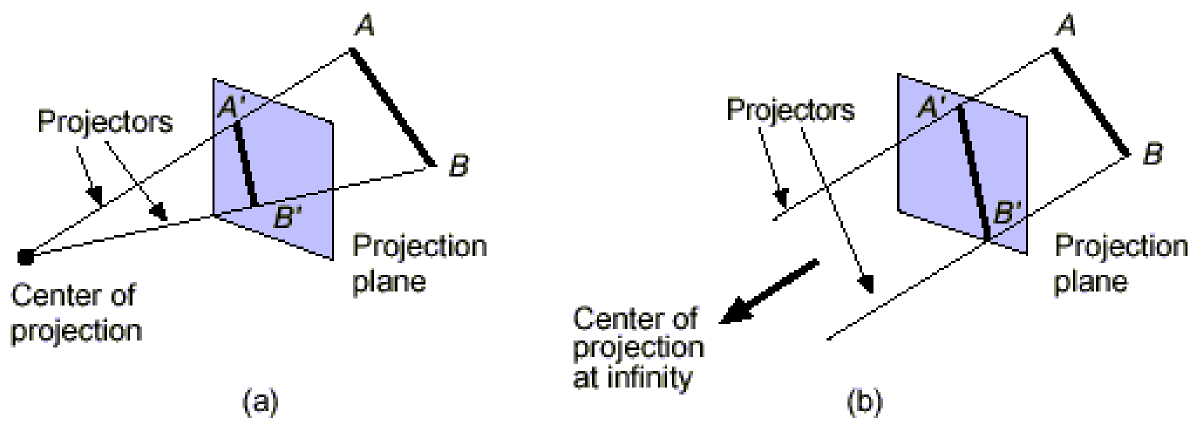


- This drawing itself is a perspective projection
- What other types of projections do you know?
  - hint: maps

Main Classes of Planar Geometrical Projections

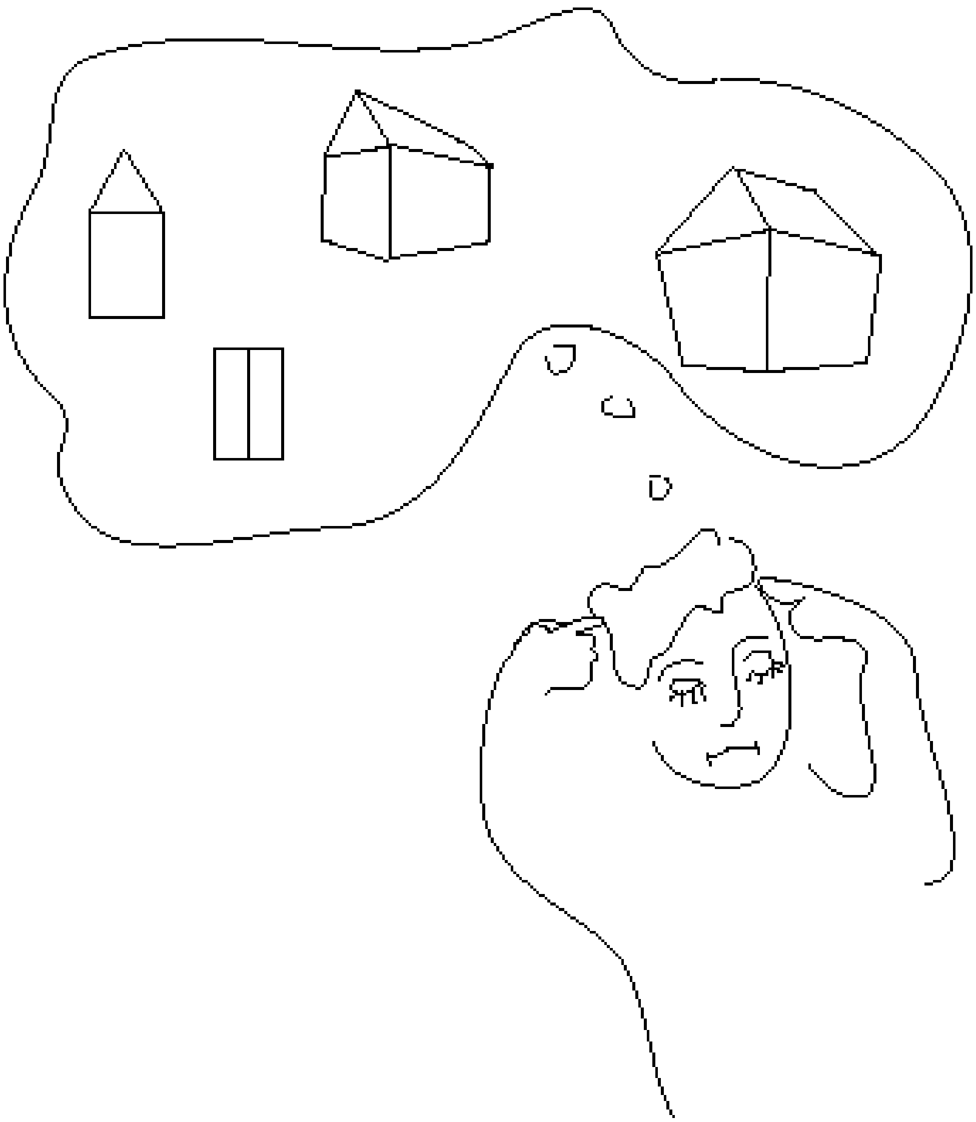
a) Perspective: determined by Center of Projection (COP) (in our diagrams, the "eye")

b) Parallel: determined by Direction of Projection (DOP) (projectors are parallel—do not converge to an "eye" or COP). Alternatively, COP is at ∞

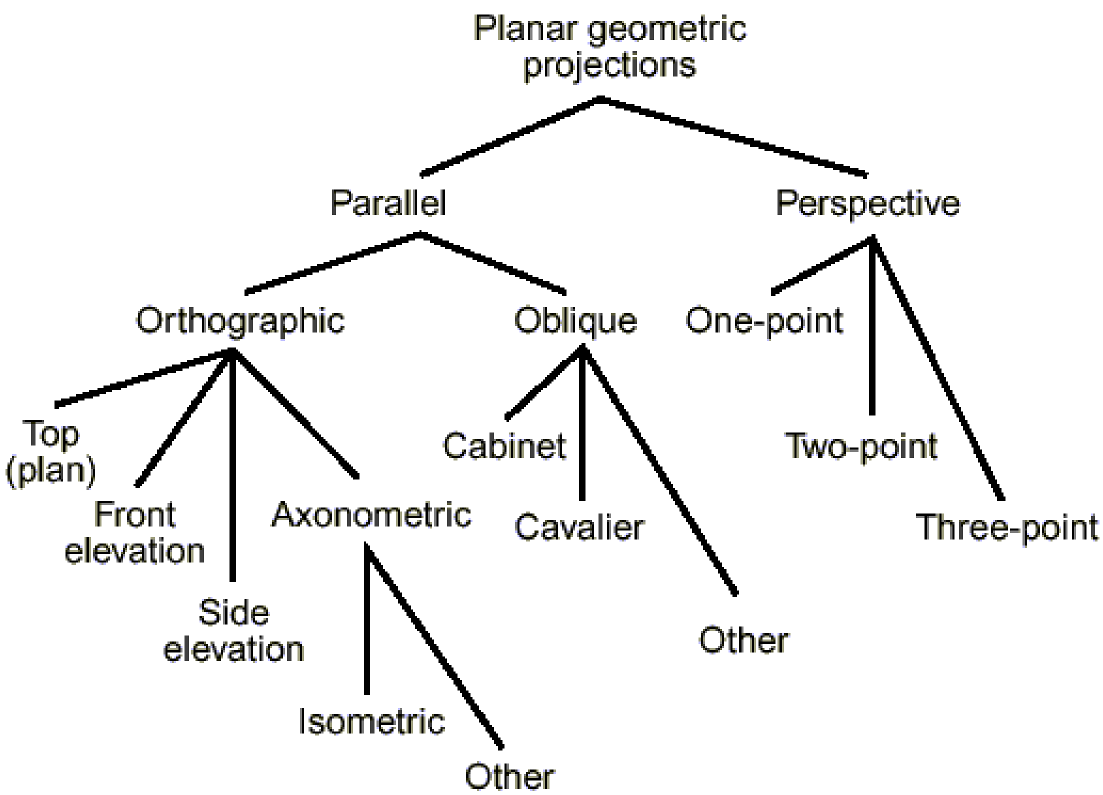


- In general, a projection is determined by where you place the projection plane relative to principal axes of object, and what angle the projectors make with the projection plane

Types of Projection



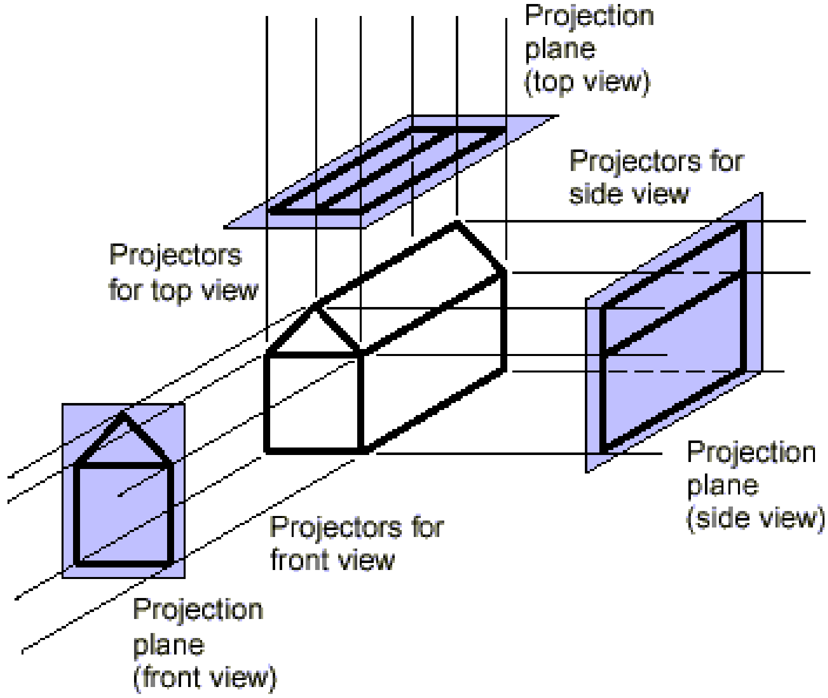
Logical Relationship Between Types of Projections



- Parallel projections used for engineering and architecture because they can be used for measurements
- Perspective imitates our eyes or a camera and looks more natural

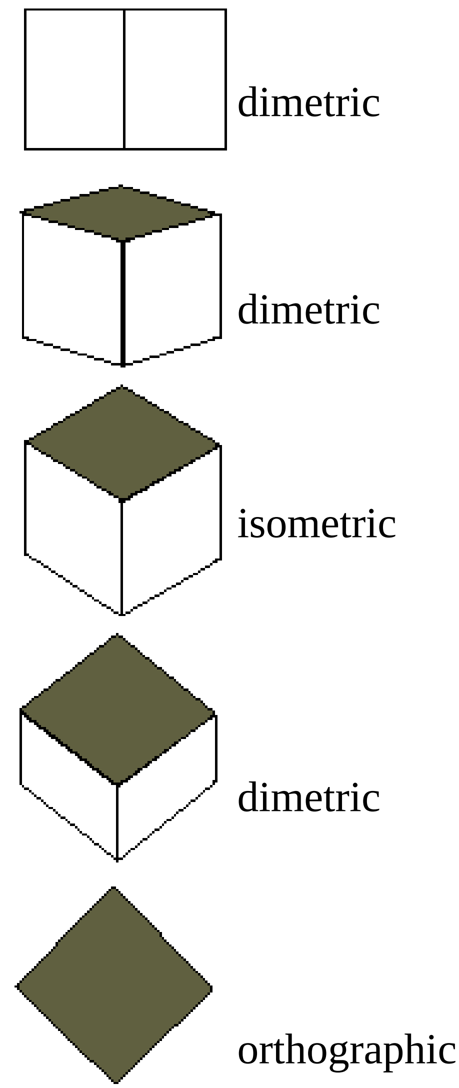
Multiview Orthographic

- Used for:
  - engineering drawings of machines, machine parts
  - working architectural drawings
- Pros:
  - accurate measurement possible
  - all views are at same scale
- Cons:
  - does not provide “realistic” view or sense of 3D form
- Usually need multiple views to get a three-dimensional feeling for object



Axonometric Projections

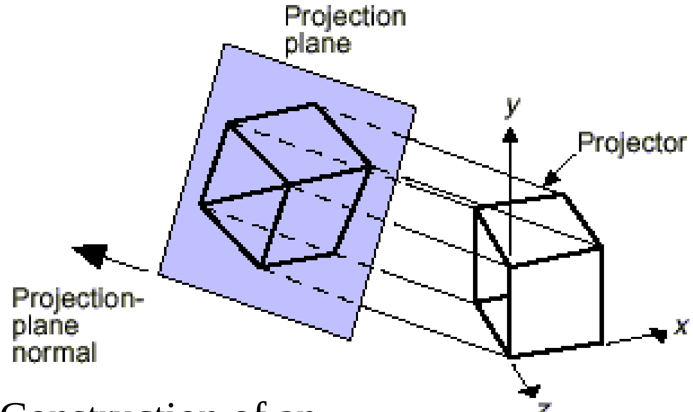
- Same method as multiview orthographic projections, except projection plane not parallel to any of the **coordinate planes**; parallel lines are equally foreshortened
- *Isometric*: Angles between all three principal axes are equal (120º). The same scale ratio applies along each axis
- *Dimetric:* Angles between two of the principal axes are equal; need two scale ratios
- *Trimetric:* Angles different between the three principal axes; need three scale ratios
- Note: different names for different views, but all part of a continuum of parallel projections of the cube; these differ in where the projection plane is relative to its cube



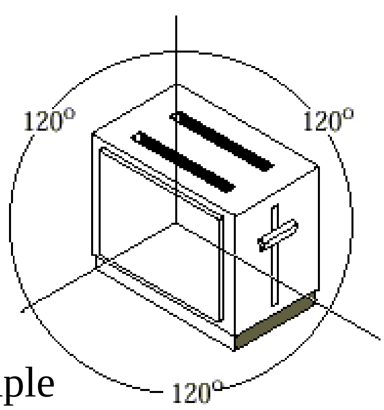
Carlbom Fig. 3-8

Isometric Projection

- Used for:
  - catalogue illustrations
  - patent office records
  - furniture design
  - structural design
- Pros:
  - don't need multiple views
  - illustrates 3D nature of object
  - measurements can be made to scale along principal axes
- Cons:
  - lack of foreshortening creates distorted appearance
  - more useful for rectangular than curved shapes



Construction of an isometric projection: projection plane cuts each principal axis by 45°



Example

Carlbom Fig.2.2

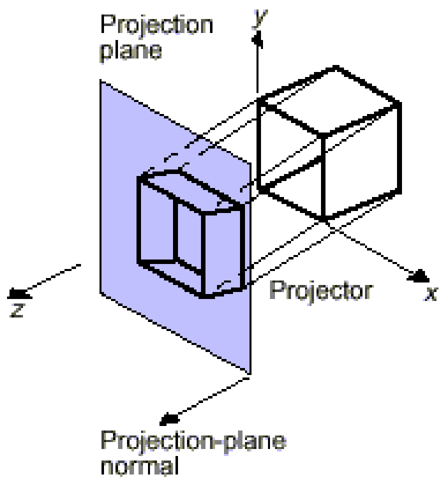
Oblique Projections

- Projectors are at an oblique angle to the projection plane; view cameras have accordion housing, used for skyscrapers
- Pros:
  - can present the exact shape of one face of an object (can take accurate measurements): better for elliptical shapes than axonometric projections, better for "mechanical" viewing
  - lack of perspective foreshortening makes comparison of sizes easier
  - displays some of object's 3D appearance
- Cons:
  - objects can look distorted if careful choice not made about position of projection plane (e.g., circles become ellipses)
  - lack of foreshortening (not realistic looking)

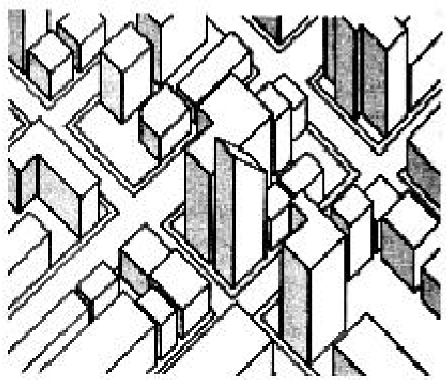
View Camera

source: http://www.usinternet.com/users/rniederman/star01.htm

Examples of Oblique Projections

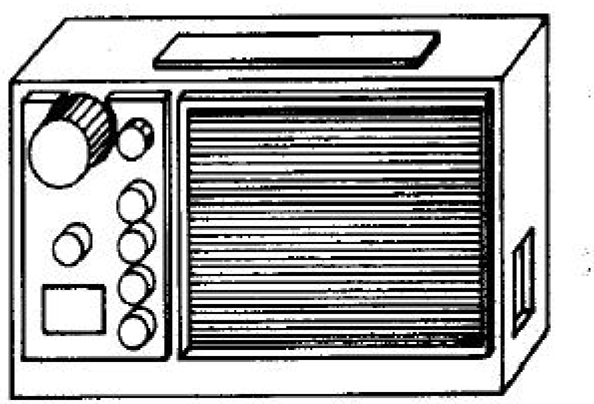


Construction of an
oblique parallel projection



(Carlbom Fig. 2-6)

Plan oblique projection of a city

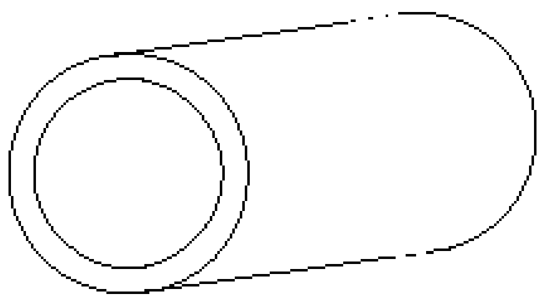


(Carlbom Fig. 2-4)

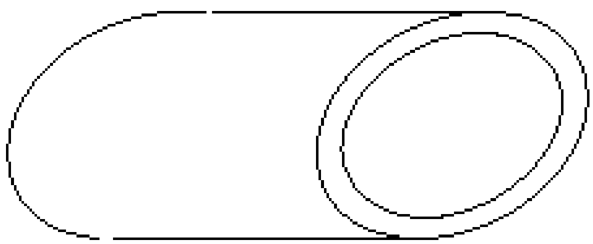
Front oblique projection of a radio

Example: Oblique View

- Rules for placing projection plane for oblique views: projection plane should be chosen according to one or several of the following:
  - it is parallel to the most irregular of the principal faces, or to the one which contains circular or curved surfaces
  - it is parallel to the longest principal face of the object
  - it is parallel to the face of interest



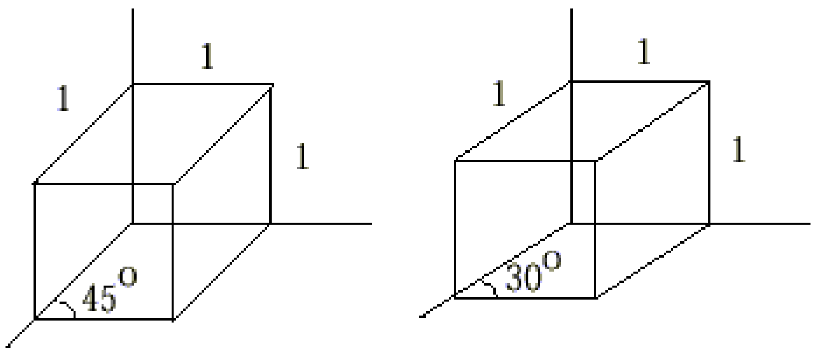
Projection plane parallel to circular face



Projection plane not parallel to circular face

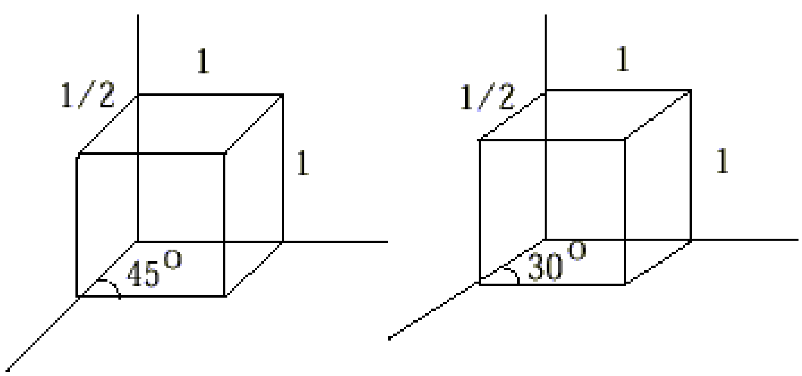
Main Types of Oblique Projections

- *Cavalier:* Angle between projectors and projection plane is 45º. Perpendicular faces are projected at full scale



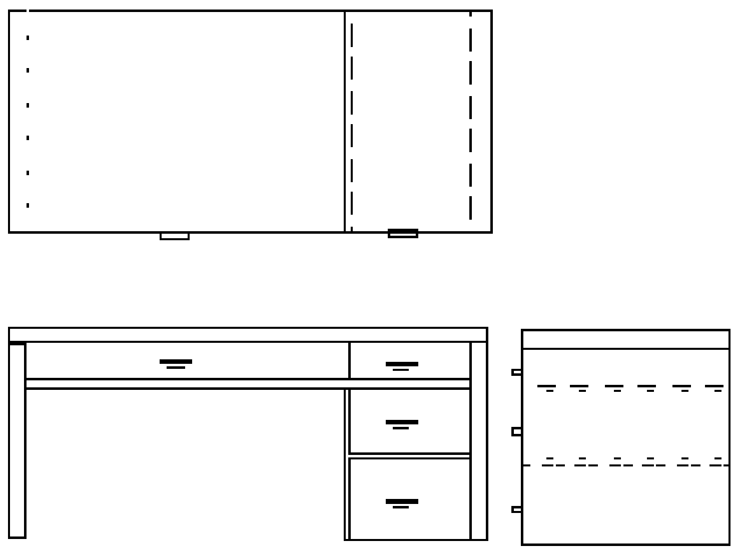
cavalier projection of unit cube

- *Cabinet:* Angle between projectors and projection plane is arctan(2) = 63.4º. Perpendicular faces are projected at 50% scale

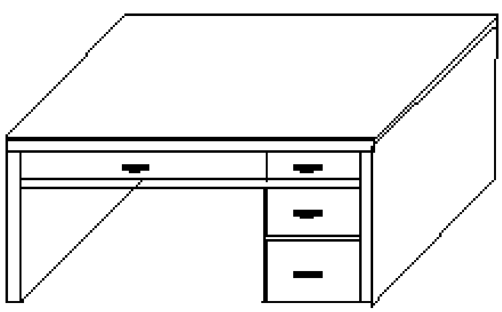


cabinet projection of unit cube

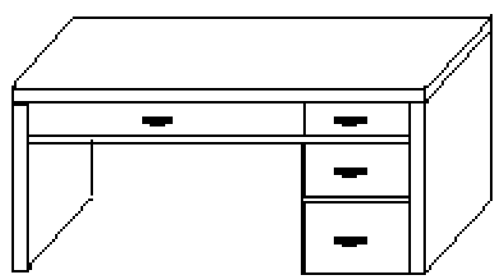
Examples of Orthographic and Oblique Projections



multiview orthographic



cavalier

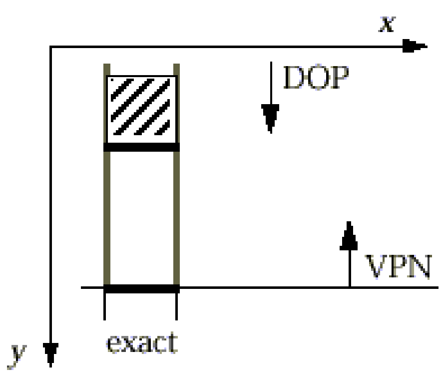


cabinet

Carlbom Fig. 3-2

Summary of Parallel Projections

- Assume object face of interest lies in principal plane, i.e., parallel to *xy*, *yz*, or *zx* planes. (DOP = Direction of Projection, VPN = View Plane Normal)

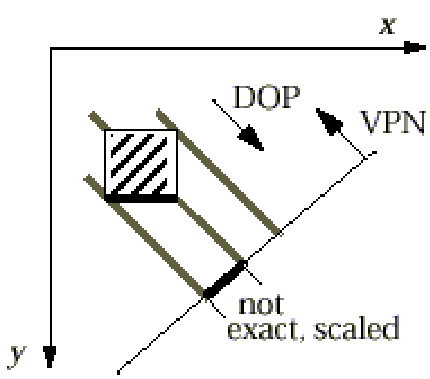


1) Multiview Orthographic

– VPN || a principal coordinate axis

– DOP || VPN

– shows single face, exact measurements

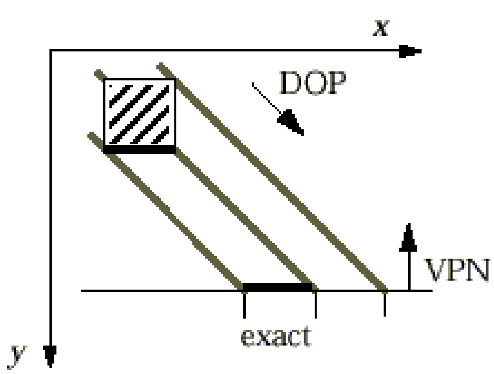


2) Axonometric

– VPN $\nparallel$ a principal coordinate axis

– DOP || VPN

– adjacent faces, none exact, uniformly foreshortened (as a function of angle between face normal and DOP)



3) Oblique

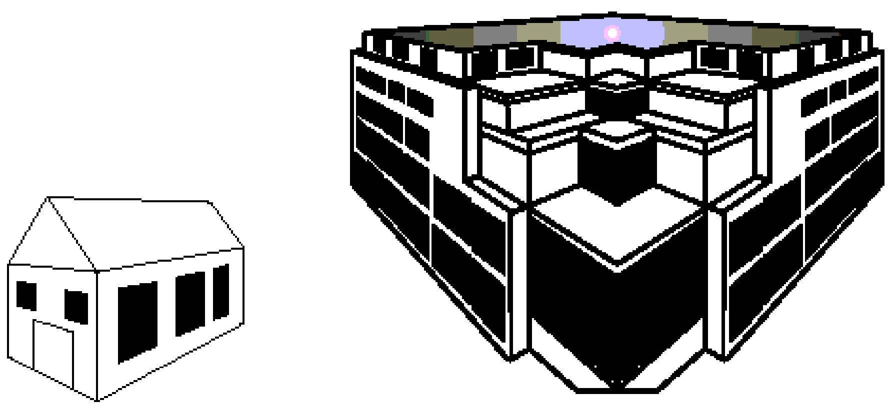
– VPN || a principal coordinate axis

– DOP $\nparallel$ VPN

– adjacent faces, one exact, others uniformly foreshortened

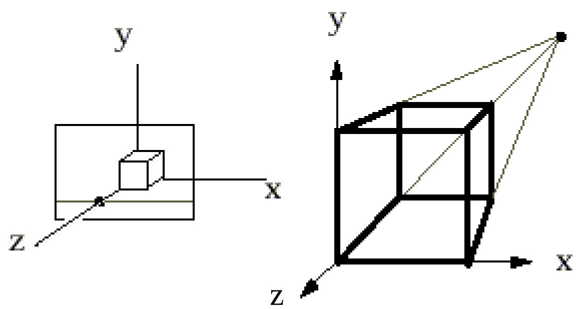
Perspective Projections

- Used for:
  - advertising
  - presentation drawings for architecture, industrial design, engineering
  - fine art
- Pros:
  - gives a realistic view and feeling for 3D form of object
- Cons:
  - does not preserve shape of object or scale (except where object intersects projection plane)
- Different from a parallel projection because
  - parallel lines not parallel to the projection plane converge
  - size of object is diminished with distance
  - foreshortening is not uniform

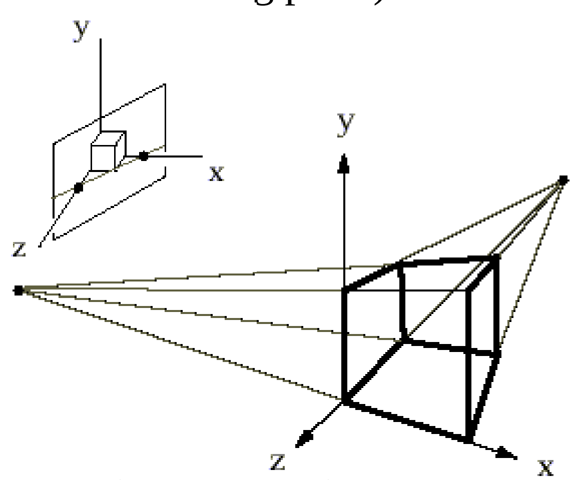


Vanishing Points (1/2)

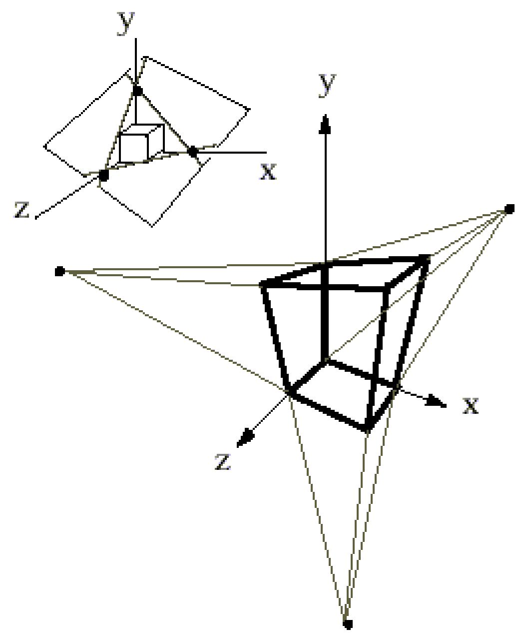
- For right-angled forms whose face normals are perpendicular to the *x*, *y*, *z* coordinate axes, the number of vanishing points = number of principal coordinate axes intersected by projection plane



One Point Perspective
(*z*-axis vanishing point)



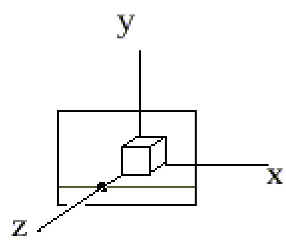
Two Point Perspective
(*z*, and *x*-axis vanishing points)



Three Point Perspective
(*z*, *x*, and *y*-axis vanishing points)

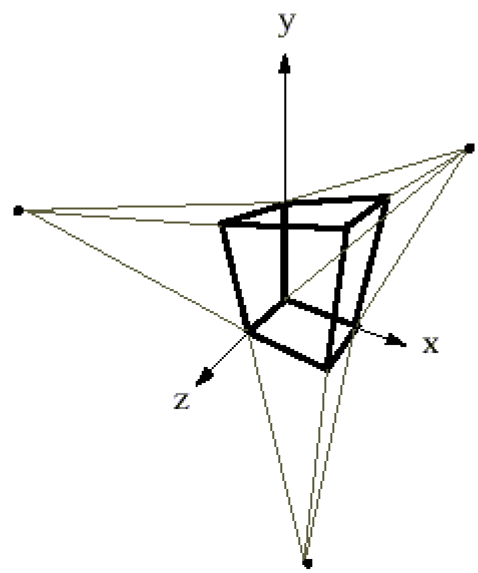
Vanishing Points (2/2)

- What happens if same form is turned so that its face normals are *not* perpendicular to the *x*, *y*, *z* coordinate axes?



- New viewing situation: cube is rotated, face normals are no longer perpendicular to any of the principal axes

- Note: unprojected cube is depicted here with parallel projection



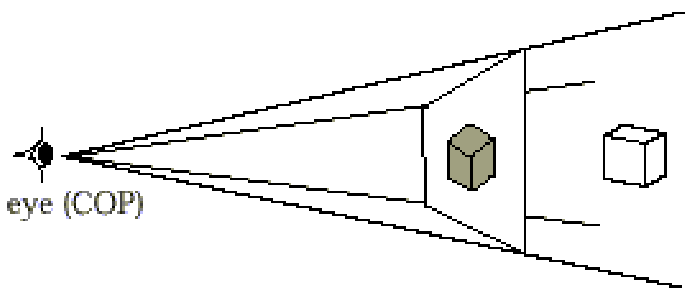
Perspective drawing of the rotated cube

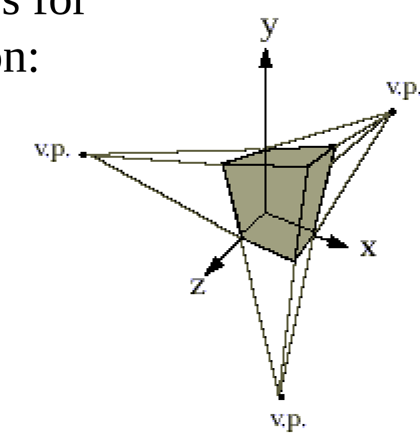
- Although the projection plane only intersects one axis (*z*), three vanishing points were created

- Note: can achieve final results which are identical to previous situation in which projection plane intersected all three axes

Vanishing Points and the View Point (1/3)

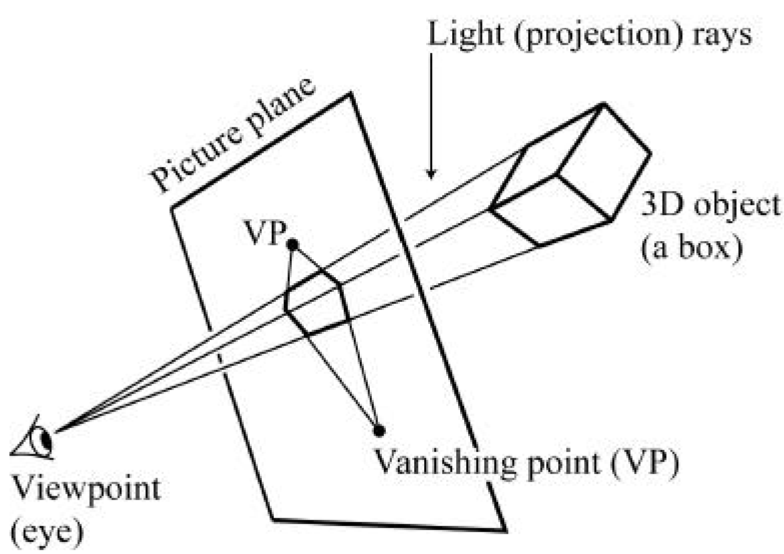
- We've seen two pyramid geometries for understanding perspective projection:





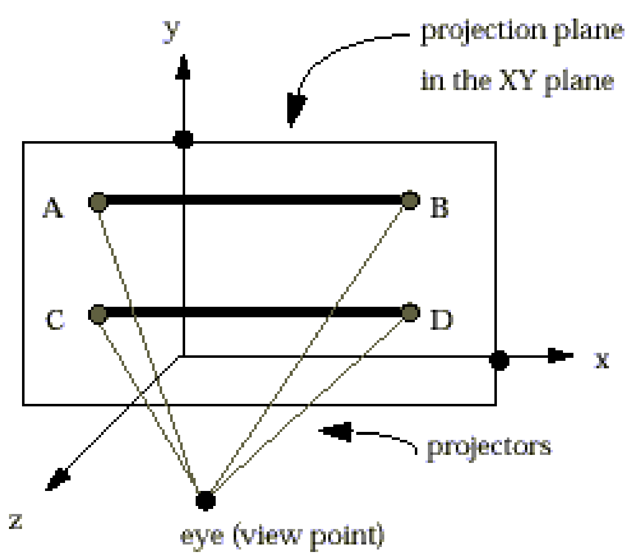
1. perspective image is intersection of a plane with light rays from object to eye (COP)
2. perspective image is result of foreshortening due to convergence of some parallel lines toward vanishing points

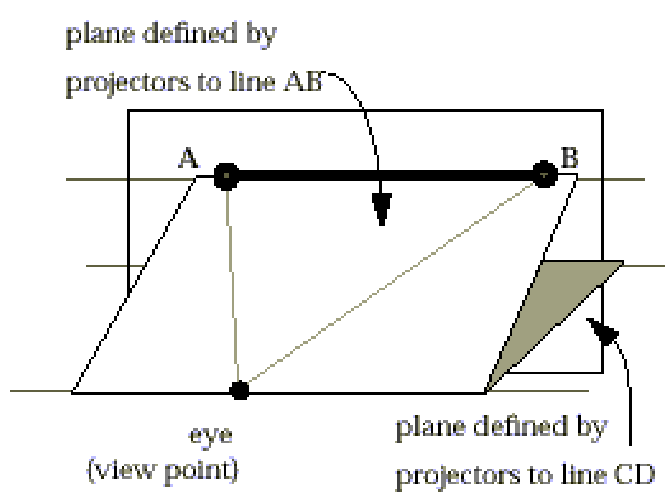
- Combining these 2 views:



Vanishing Points and the View Point (2/3)

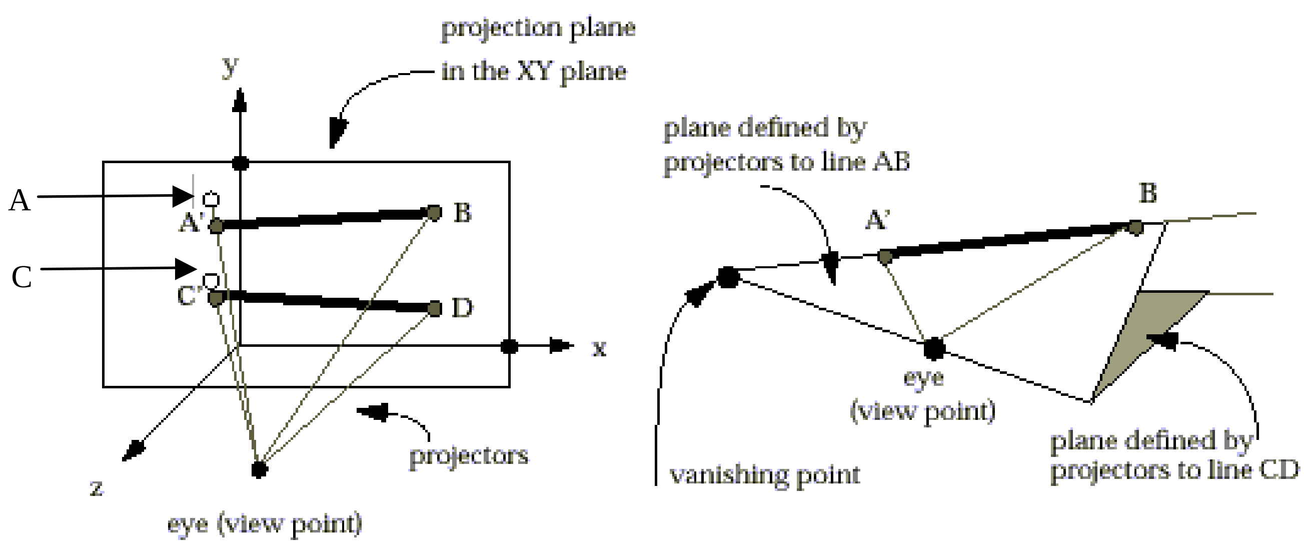
- Project parallel lines *AB*, *CD* on *xy* plane
- Projectors from the eye to *AB* and *CD* define two planes, which meet in a line which contains the view point, or eye
- This line **does not** intersect the projection plane (*XY*), because parallel to it. Therefore there is no vanishing point





Vanishing Points and the View Point (3/3)

- Lines *AB* and *CD* (this time with *A* and *C* behind the projection plane) projected on *xy* plane: *A'B* and *C'D*
- Note: *A'B* not parallel to *C'D*
- Projectors from eye to *A'B* and *C'D* define two planes which meet in a line which contains the view point
- This line **does** intersect the projection plane
- Point of intersection is the vanishing point



Next Time: Projection in Computer Graphics