

Rasterization

LESSON 8

Computer Graphics 1

Outline

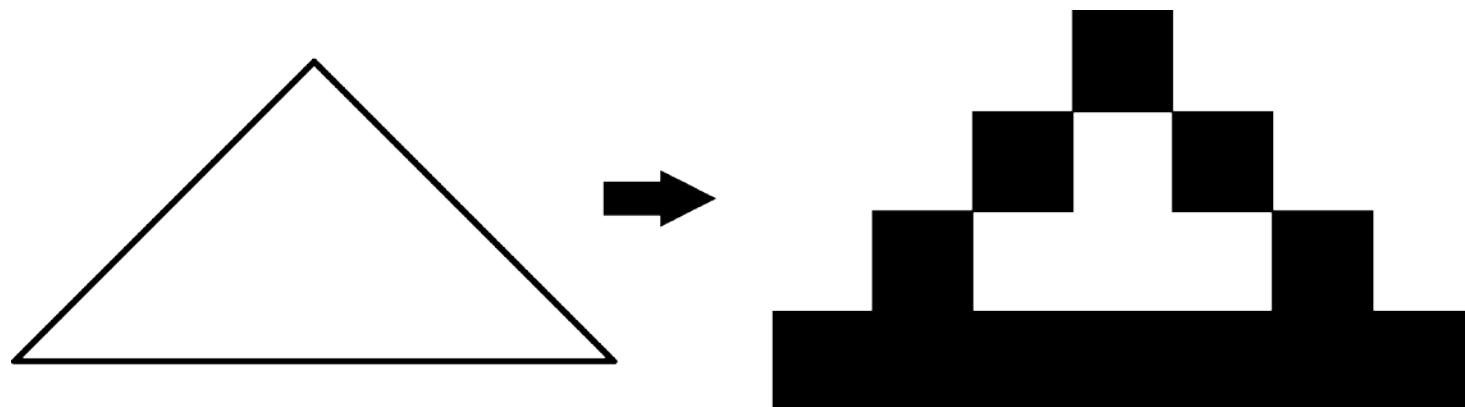
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- What is rasterization
- Line rasterization
- Curve rasterization
- Polygon rasterization
- Filling algorithms

Rasterization

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- One of the most important tasks in computer graphics
- Mapping from continuous space to discrete space
- Converting vector graphics to raster image
- Represent object by a set of pixels
- Output on display or printer (write to frame buffer)



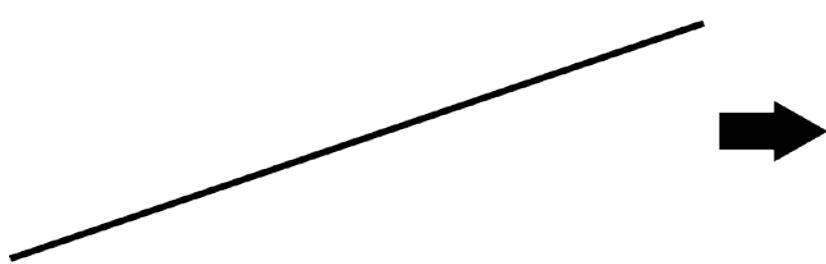
Line Rasterization

DDA, Bresenham's algorithm, midpoint algorithm

Line Rasterization

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- Very frequent
- Basic operation
- Displaying of complex object can be reduced to drawing a lot of lines
- Generate satisfactory set of points



Line Rasterization - Criteria

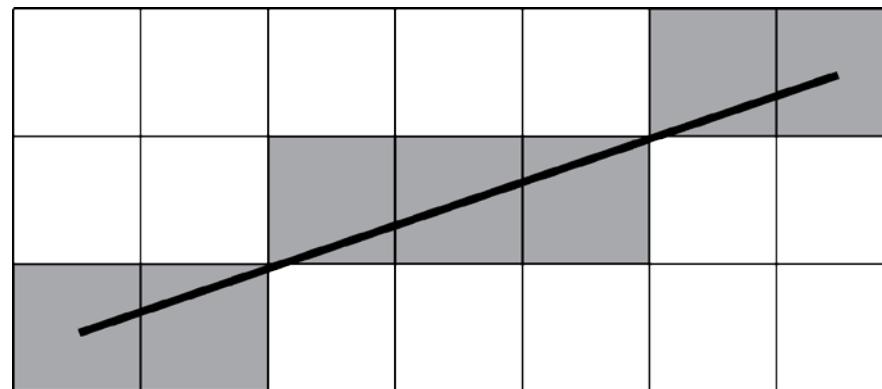
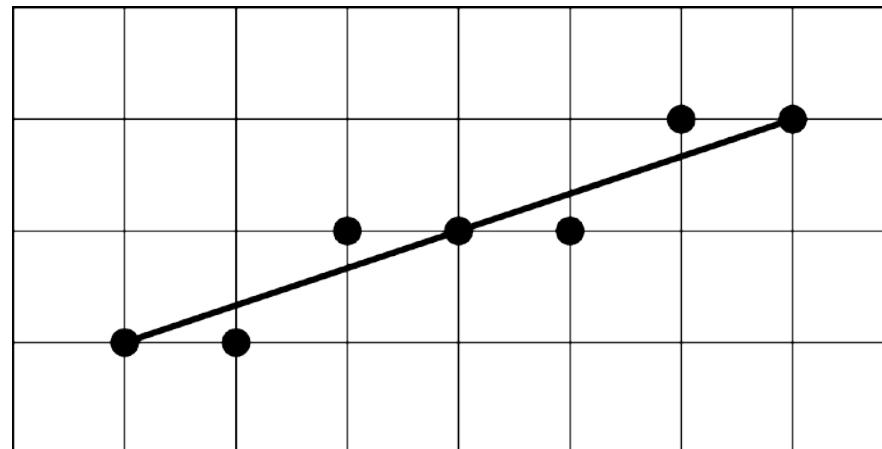
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- Visually as straight as possible
- Accurate end and start
 - Avoid gaps between lines
- Even visual thickness
 - Constant density
 - Thickness independent of slope and length
- Fast rasterization

DDA

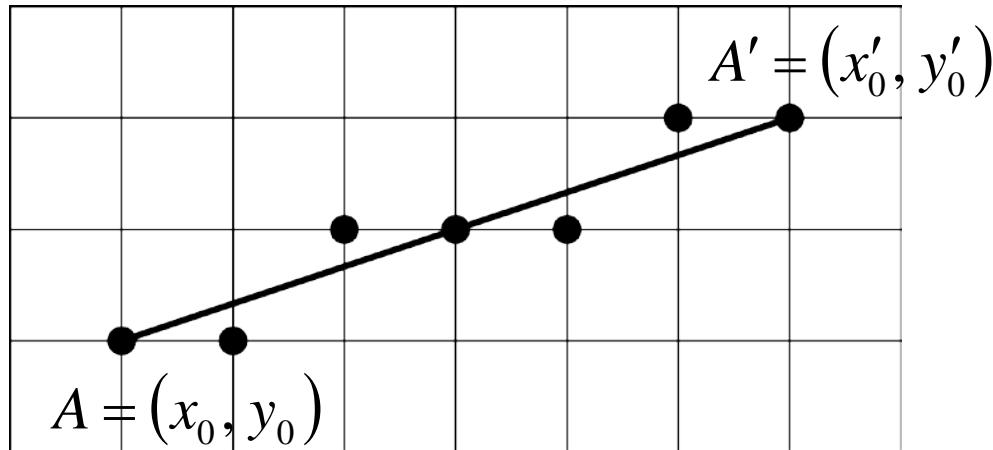
7

- Digital differential analyzer
- Linear interpolation of values between endpoints



DDA

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$$A = (x_0, y_0)$$

$$y = mx + b$$

$$A' = (x'_0, y'_0)$$

$$m = \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

$$\Delta x = x'_0 - x_0$$

$$\Delta y = y'_0 - y_0$$

$$dx = 1 \Rightarrow dy = m dx = m$$

DDA - Algorithm

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- Draw (x_0, y_0)

- Iteration:

$$x_{i+1} = x_i + 1$$

$$y_{i+1} = y_i + m$$

- Draw $(x_{i+1}, \text{round}(y_{i+1}))$

DDA - Algorithm

10

- Draw (x_0, y_0)

- Iteration:

$$x_{i+1} = x_i + 1$$

$$y_{i+1} = y_i + m$$

- Draw $(x_{i+1}, \text{round}(y_{i+1}))$

- What if $\Delta y > \Delta x$?

DDA - Algorithm(2)

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$$\Delta y > \Delta x \Rightarrow m > 1$$

- **Solution:** $dy = 1$

$$dx = \frac{dy}{m} = \frac{1}{m}$$

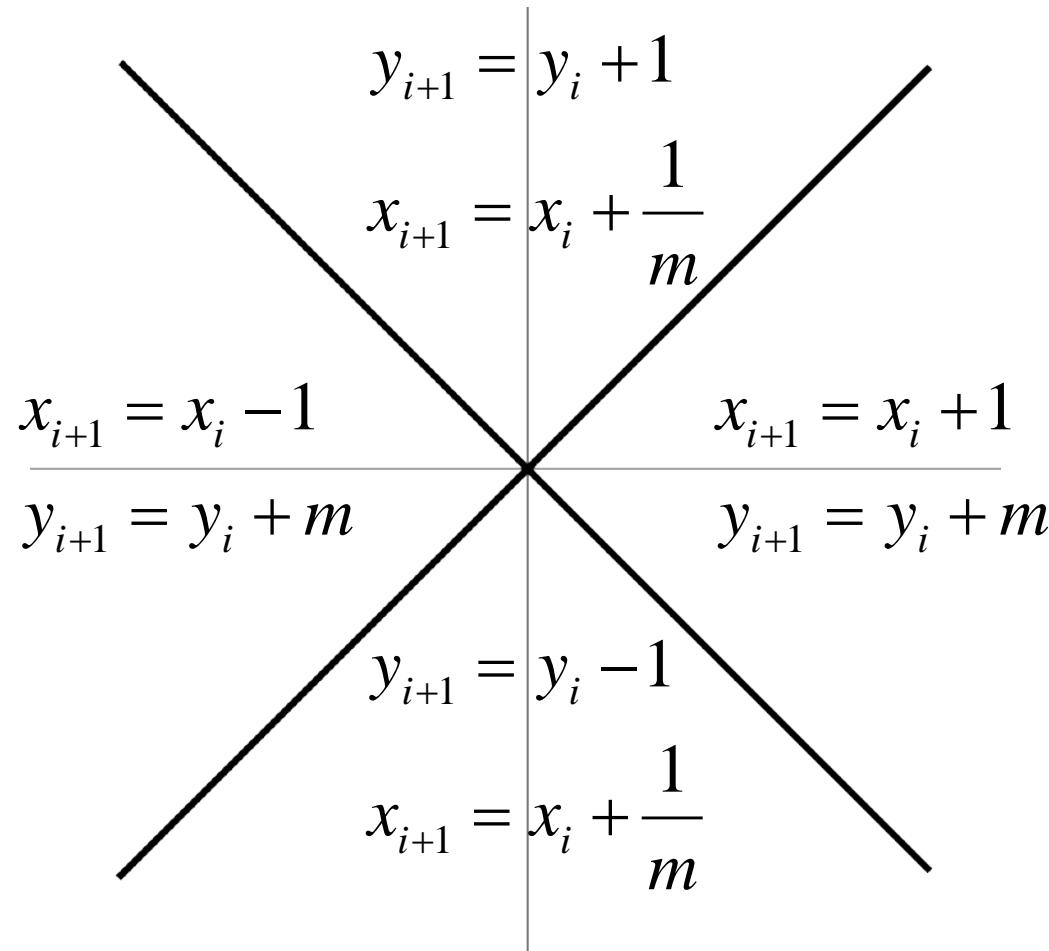
- **Iteration:** $y_{i+1} = y_i + 1$

$$x_{i+1} = x_i + \frac{1}{m}$$

- **Draw** $(\text{round}(x_{i+1}), y_{i+1})$

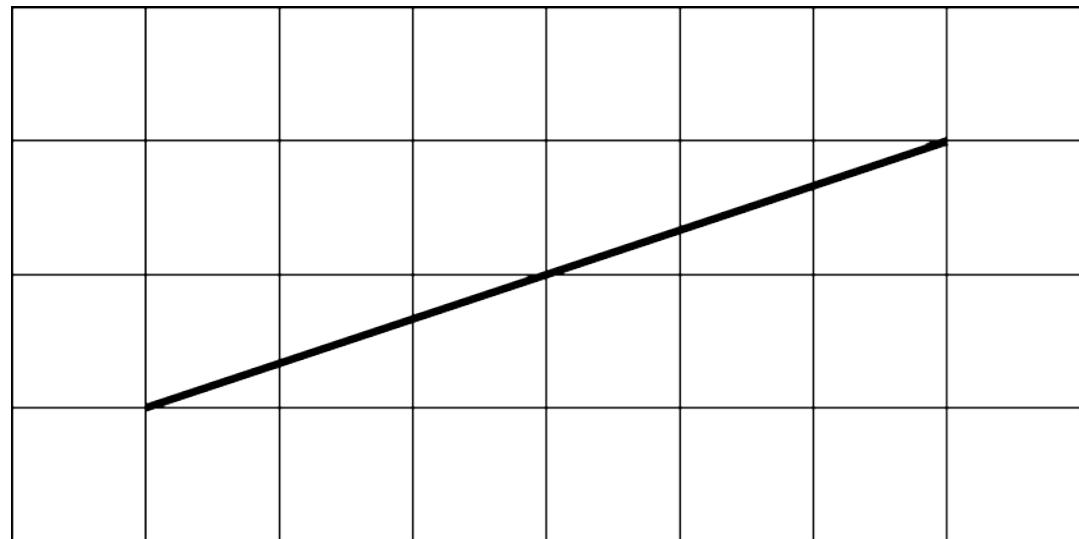
Line Rasterization - Quadrants

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DDA - Example

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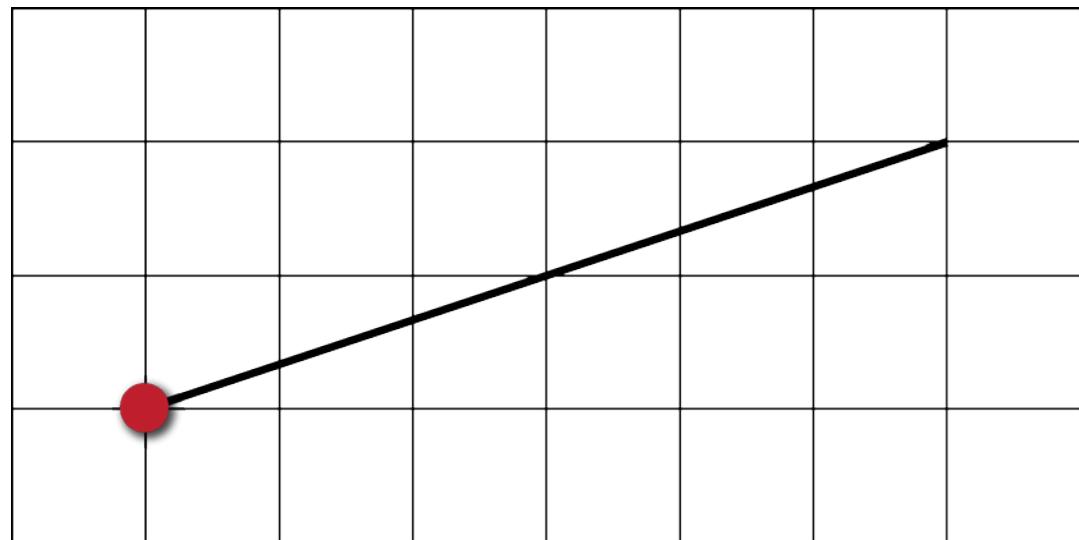
$$A = (1,1) \quad A' = (7,3)$$

$$\Delta x = 6 \quad \Delta y = 2$$

$$m=1/3 \quad x_{i+1} = x_i$$

DDA - Example

14



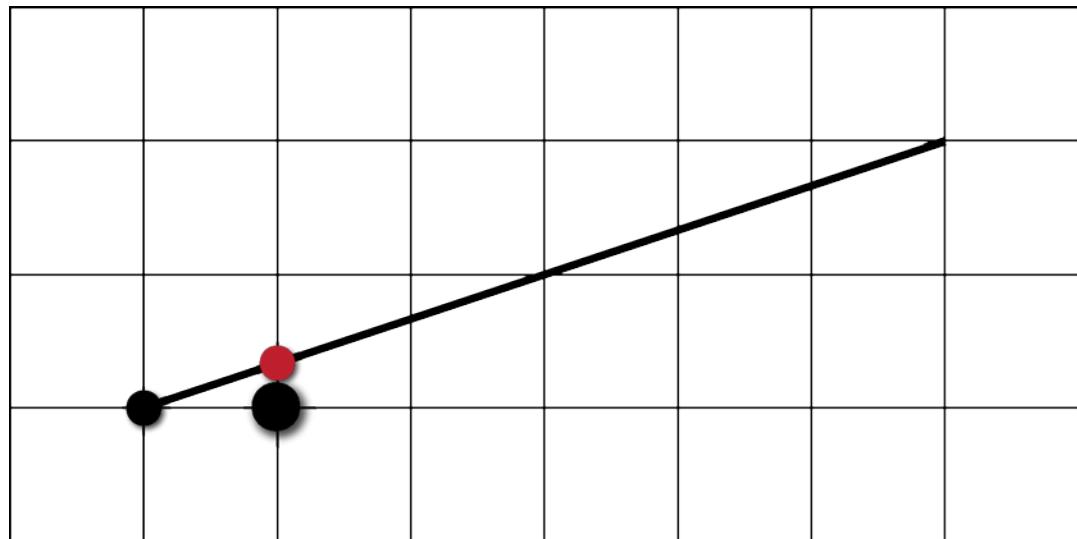
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DDA - Example

15



$$A = (1,1) \quad A' = (7,3)$$

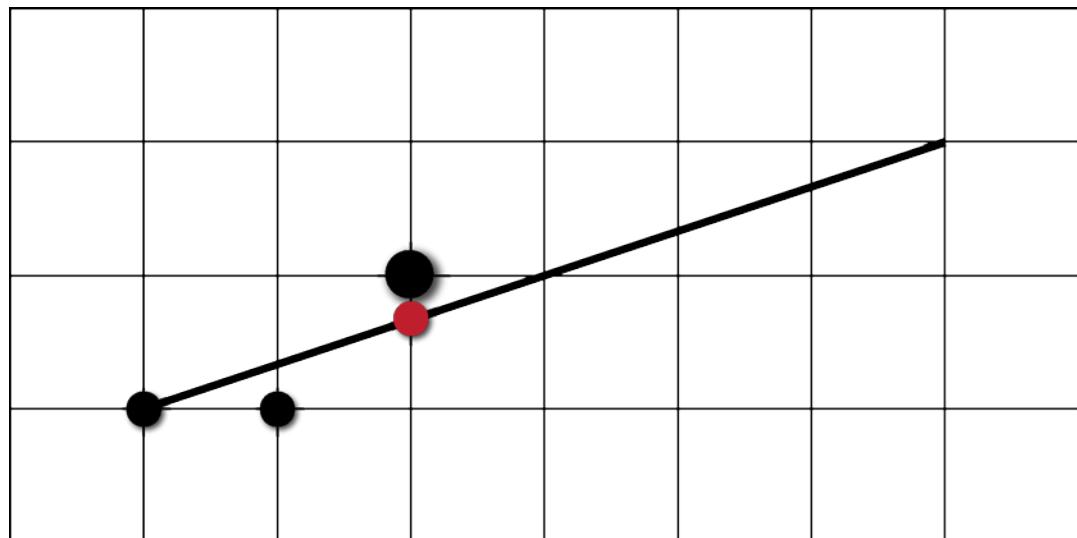
$$\Delta x = 6 \quad \Delta y = 2$$

$$m = 1/3 \quad x_{i+1} = x_i$$

i	(x _i ,y _i)	draw	error
0	(1,1)	(1,1)	0
1	(2,1 1/3)	(2,1)	1/3
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DDA - Example

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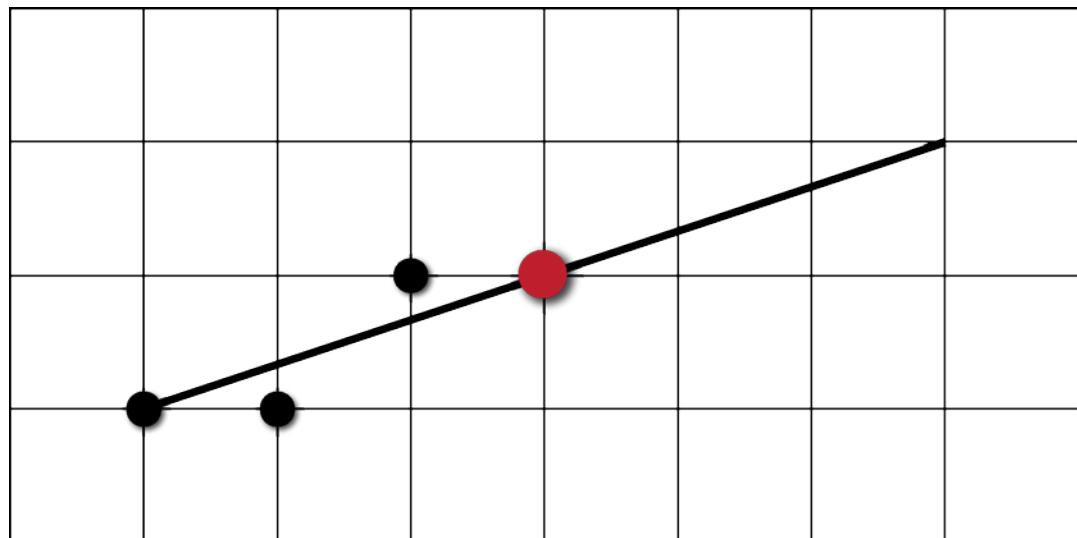
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DDA - Example

17



$$A = (1,1) \quad A' = (7,3)$$

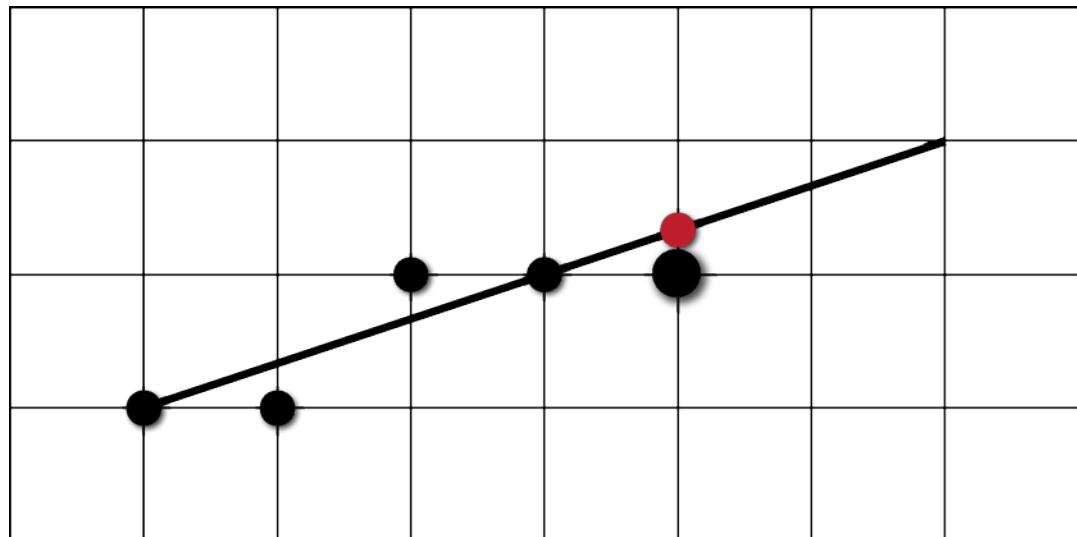
$$\Delta x = 6 \quad \Delta y = 2$$

$$m = 1/3 \quad x_{i+1} = x_i$$

i	(x _i ,y _i)	draw	error
0	(1,1)	(1,1)	0
1	(2,1 1/3)	(2,1)	1/3
2	(3,1 2/3)	(3,2)	-1/3
3	(4,2)	(4,2)	0

DDA - Example

18



$$A = (1,1) \quad A' = (7,3)$$

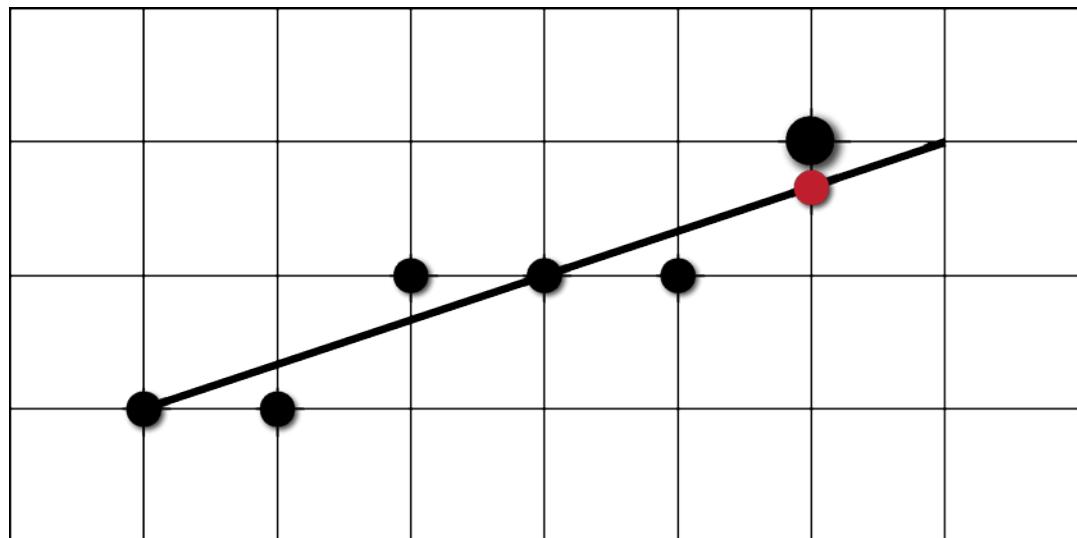
$$\Delta x = 6 \quad \Delta y = 2$$

$$m = 1/3 \quad x_{i+1} = x_i$$

i	(x _i , y _i)	draw	error
0	(1,1)	(1,1)	0
1	(2, 1 1/3)	(2,1)	1/3
2	(3, 1 2/3)	(3,2)	-1/3
3	(4,2)	(4,2)	0
4	(5,2 1/3)	(5,2)	1/3

DDA - Example

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$$A = (1,1) \quad A' = (7,3)$$

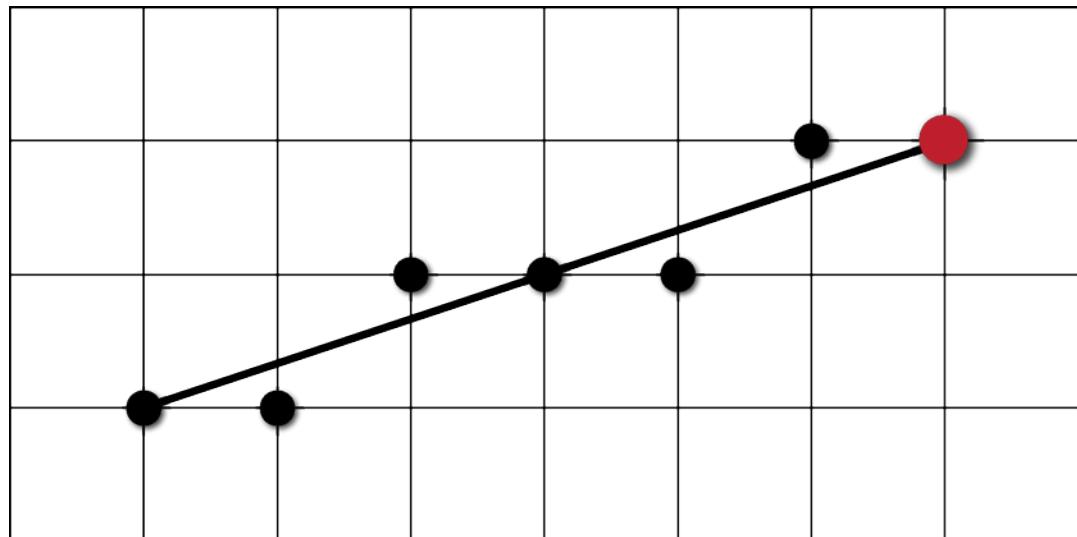
$$\Delta x = 6 \quad \Delta y = 2$$

$$m = 1/3 \quad x_{i+1} = x_i$$

i	(x _i ,y _i)	draw	error
0	(1,1)	(1,1)	0
1	(2,1 1/3)	(2,1)	1/3
2	(3,1 2/3)	(3,2)	-1/3
3	(4,2)	(4,2)	0
4	(5,2 1/3)	(5,2)	1/3
5	(6,2 2/3)	(6,3)	-1/3

DDA - Example

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$$A = (1,1) \quad A' = (7,3)$$

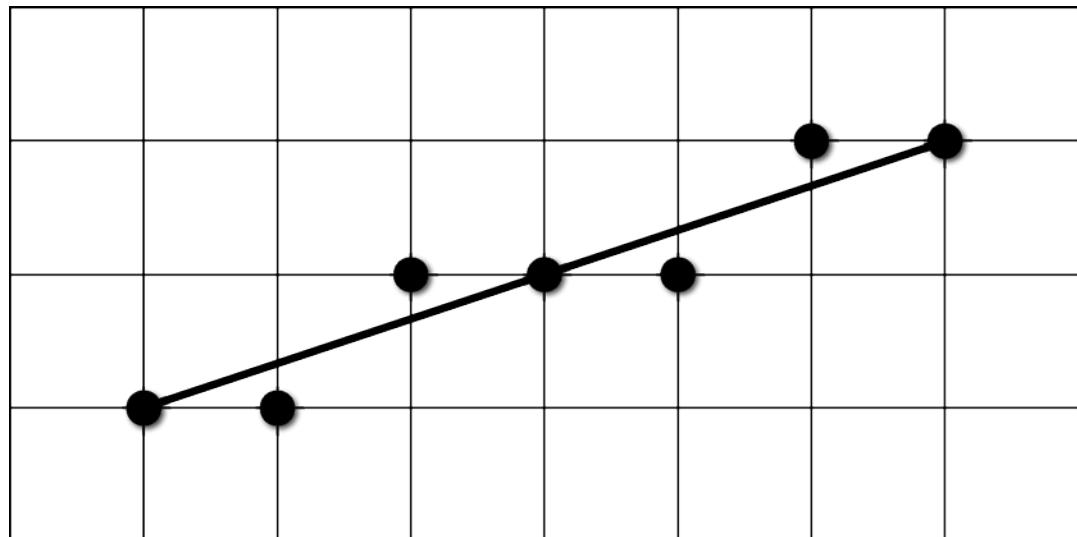
$$\Delta x = 6 \quad \Delta y = 2$$

$$m = 1/3 \quad x_{i+1} = x_i$$

i	(x _i , y _i)	draw	error
0	(1,1)	(1,1)	0
1	(2, 1 1/3)	(2,1)	1/3
2	(3, 1 2/3)	(3,2)	-1/3
3	(4,2)	(4,2)	0
4	(5, 2 1/3)	(5,2)	1/3
5	(6, 2 2/3)	(6,3)	-1/3
6	(7,3)	(7,3)	0

DDA - Example

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$$A = (1,1) \quad A' = (7,3)$$

$$\Delta x = 6 \quad \Delta y = 2$$

$$m = 1/3 \quad x_{i+1} = x_i$$

i	(x _i ,y _i)	draw	error
0	(1,1)	(1,1)	0
1	(2,1 1/3)	(2,1)	1/3
2	(3,1 2/3)	(3,2)	-1/3
3	(4,2)	(4,2)	0
4	(5,2 1/3)	(5,2)	1/3
5	(6,2 2/3)	(6,3)	-1/3
6	(7,3)	(7,3)	0

Problem with DDA

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- Floating point arithmetic
 - Computationally expensive (problem of old computers)
 - Inaccuracy
- Decimal
0.1
- Binary
 $e = -4$
 $s = 1100110011001100110011001100110011...$
- Decimal
0.10000001490116119384765625

Bresenham's Algorithm

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- Fast drawing algorithm
- Integer arithmetic
- Following the drawn point (x_i, y_i) while storing the error $\frac{p_i}{q_i}$
- q_i is constant ($q_{i+1} = q_i$)
- Iteration: $(x_i, y_i), p_i \rightarrow (x_{i+1}, y_{i+1}), p_{i+1}$

Bresenham -Iteration

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$$(x_i, y_i), p_i \rightarrow (x_{i+1}, y_{i+1}), p_{i+1}$$

$$\begin{aligned}x_{i+1} &= x_i + 1 & p_{i+1} &= \frac{p_i}{q_{i+1}} + \frac{\Delta y}{\Delta x} = \frac{p_i + \Delta y}{q_i} \\p_{i+1} &= p_i + \Delta y & q_{i+1} &= q_i\end{aligned}$$

□ **If** $p_i > \frac{q_i}{2} = \frac{\Delta x}{2}$ **then** $y_{i+1} = y_i + 1$ $p_{i+1} = p_i - \Delta x$
else $y_{i+1} = y_i$

Bresenham -Iteration

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$$(x_i, y_i), p_i \rightarrow (x_{i+1}, y_{i+1}), p_{i+1}$$

$$\begin{aligned}x_{i+1} &= x_i + 1 & p_{i+1} &= \frac{p_i}{q_{i+1}} + \frac{\Delta y}{\Delta x} = \frac{p_i + \Delta y}{q_i} \\p_{i+1} &= p_i + \Delta y & q_{i+1} &= q_i\end{aligned}$$

- If $p_i > \frac{q_i}{2} = \frac{\Delta x}{2}$ then $y_{i+1} = y_i + 1$ $p_{i+1} = p_i - \Delta x$
else $y_{i+1} = y_i$
- Problem: $\frac{\Delta x}{2}$ (integer arithmetic)

Bresenham -Iteration

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$$(x_i, y_i), p_i \rightarrow (x_{i+1}, y_{i+1}), p_{i+1}$$

$$\begin{aligned}x_{i+1} &= x_i + 1 & p_{i+1} &= \frac{p_i}{q_{i+1}} + \frac{\Delta y}{\Delta x} = \frac{p_i + \Delta y}{q_i} \\p_{i+1} &= p_i + \Delta y & q_{i+1} &= q_i + \Delta x\end{aligned}$$

- If $p_i > \frac{q_i}{2} = \frac{\Delta x}{2}$ then $y_{i+1} = y_i + 1$ $p_{i+1} = p_i - \Delta x$
else $y_{i+1} = y_i$
- Problem: $\frac{\Delta x}{2}$ (integer arithmetic)
- Solution: multiply the whole computation by 2

Bresenham – Correction (1)

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- Initialization: $(x_0, y_0) \quad p_0 = 0$

$$x_{i+1} = x_i + 1$$

$$p_{i+1} = p_i + 2\Delta y$$

- If $p_i > \Delta x$ then $y_{i+1} = y_i + 1 \quad p_{i+1} = p_i - 2\Delta x$
else $y_{i+1} = y_i$

Bresenham – Correction (1)

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- Initialization: $(x_0, y_0) \quad p_0 = 0$

$$x_{i+1} = x_i + 1$$

$$p_{i+1} = p_i + 2\Delta y$$

- If $p_i > \Delta x$ then $y_{i+1} = y_i + 1 \quad p_{i+1} = p_i - 2\Delta x$
else $y_{i+1} = y_i$

- Problem: double modification of p_{i+1}

Bresenham – Correction (1)

29

- Initialization: $(x_0, y_0) \quad p_0 = 0$

$$x_{i+1} = x_i + 1$$

$$p_{i+1} = p_i + 2\Delta y$$

- If $p_i > \Delta x$ then $y_{i+1} = y_i + 1 \quad p_{i+1} = p_i - 2\Delta x$
else $y_{i+1} = y_i$

- Problem: double modification of p_{i+1}
- Solution: move the modification to the previous step

Bresenham – Correction (2)

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□ Initialization: (x_0, y_0) $p_0 = \boxed{2\Delta y}$

$$\boxed{x_{i+1} = x_i + 1}$$

□ If $p_i > \Delta x$ then $y_{i+1} = y_i + 1$ $\boxed{p_{i+1} = p_i + 2\Delta y - 2\Delta x}$
else $y_{i+1} = y_i$ $\boxed{p_{i+1} = p_i + 2\Delta y}$

Bresenham – Correction (2)

31

- Initialization: $(x_0, y_0) \quad p_0 = 2\Delta y$

$$x_{i+1} = x_i + 1$$

- If $p_i > \Delta x$ then $y_{i+1} = y_i + 1 \quad p_{i+1} = p_i + 2\Delta y - 2\Delta x$
else $y_{i+1} = y_i \quad p_{i+1} = p_i + 2\Delta y$
- Final correction: compare p_{i+1} with 0

Bresenham – Final Corrections

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- Initialization: (x_0, y_0) $p_0 = 2\Delta y - \Delta x$

$$x_{i+1} = x_i + 1$$

- If $p_i > 0$ then $y_{i+1} = y_i + 1$ $p_{i+1} = p_i + 2\Delta y - 2\Delta x$
 else $y_{i+1} = y_i$ $p_{i+1} = p_i + 2\Delta y$

Bresenham – Final Algorithm

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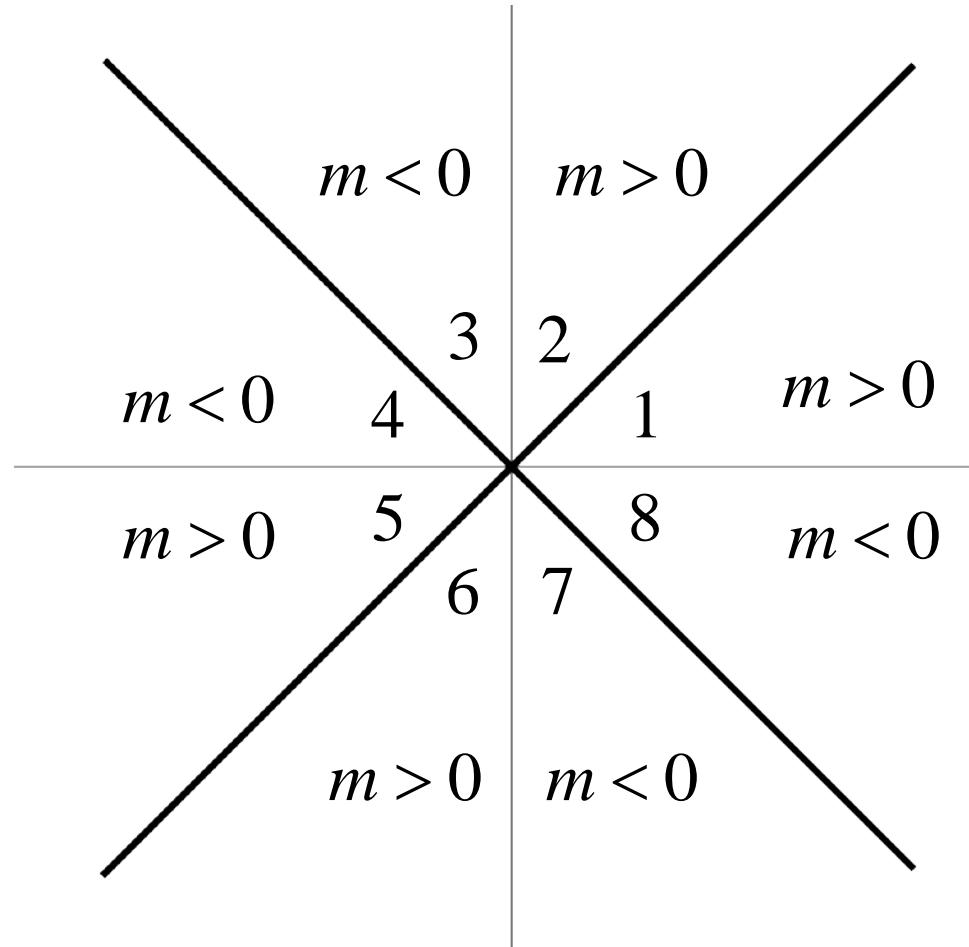
- **Initialization:** (x_0, y_0) $p_0 = 2\Delta y - \Delta x$

$$x_{i+1} = x_i + 1$$

- If $p_i > 0$ then $y_{i+1} = y_i + 1$ $p_{i+1} = p_i + 2\Delta y - 2\Delta x$
 else $y_{i+1} = y_i$ $p_{i+1} = p_i + 2\Delta y$
- **Precompute:** $2\Delta y - \Delta x = p_0$
 $2\Delta y$
 $2\Delta y - 2\Delta x$

Bresenham - Octants

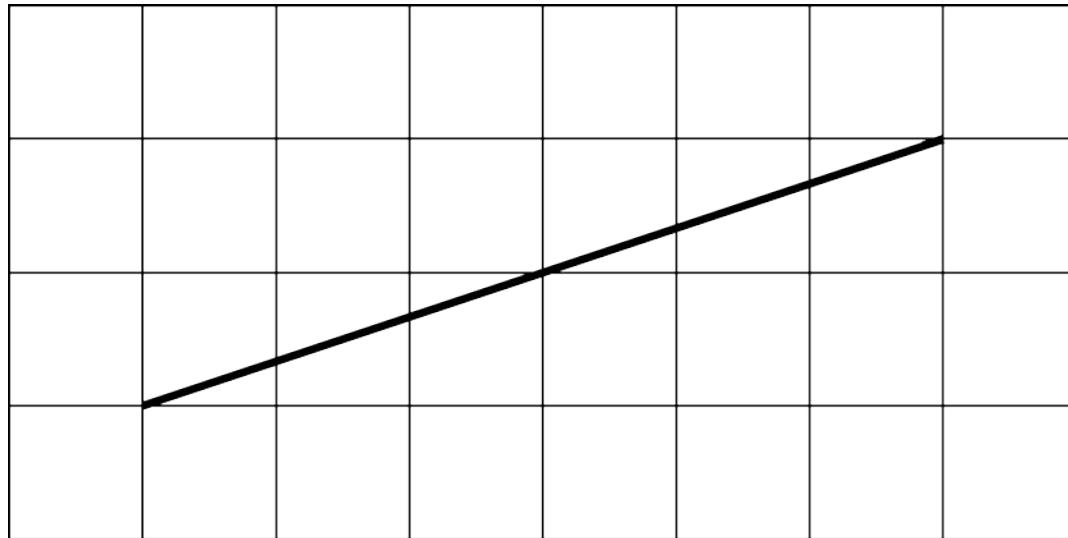
34



- Previous algorithm works only for the first octant

Bresenham - Example

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i	draw	p _i

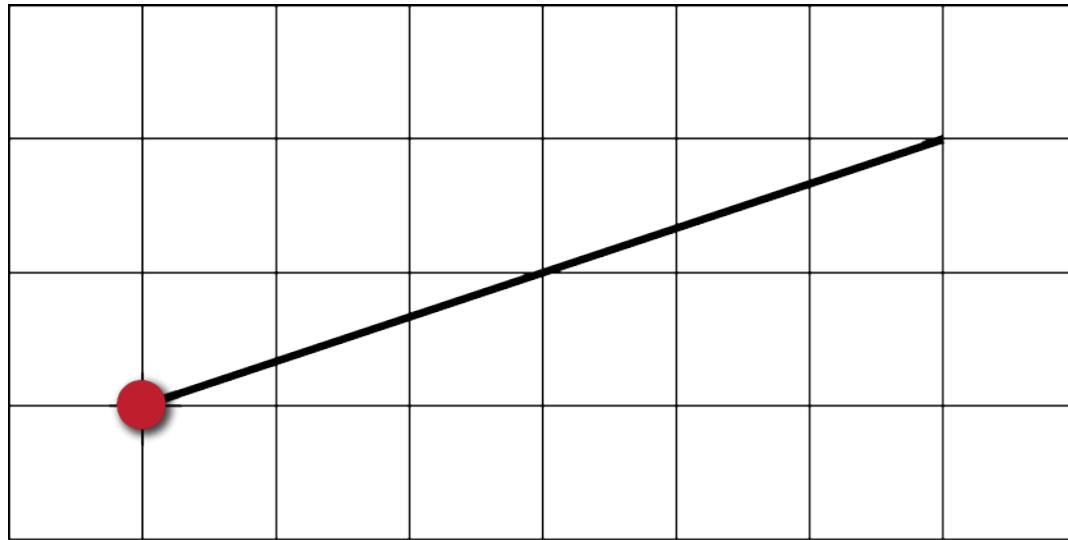
$$A = (1,1) \quad A' = (7,3)$$

$$\Delta x = 6 \quad \Delta y = 2$$

$$2\Delta y = 4 \quad 2\Delta y - 2\Delta x = -8 \quad 2\Delta y - \Delta x = -2$$

Bresenham - Example

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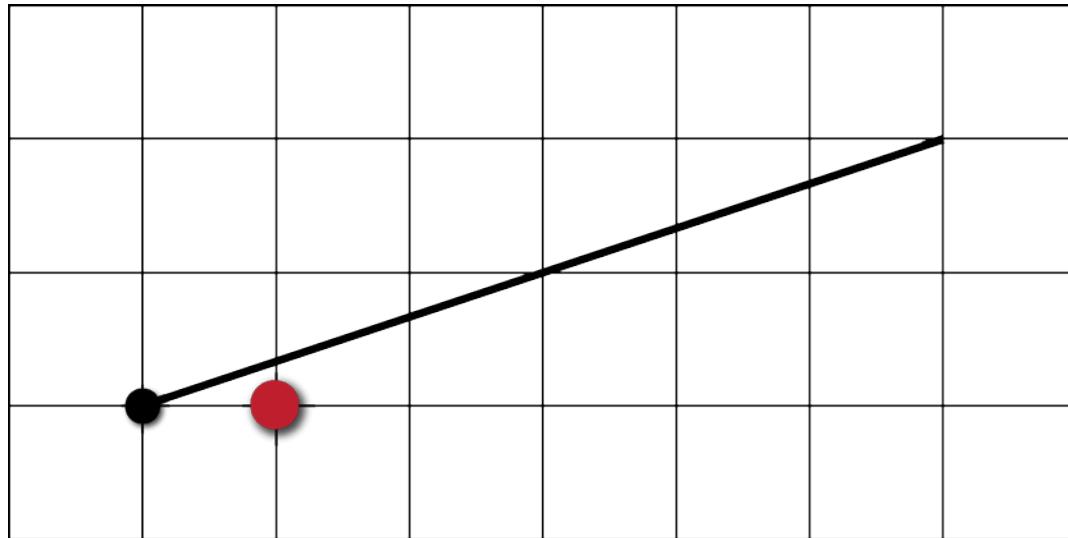
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Bresenham - Example

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$$A = (1,1) \quad A' = (7,3)$$

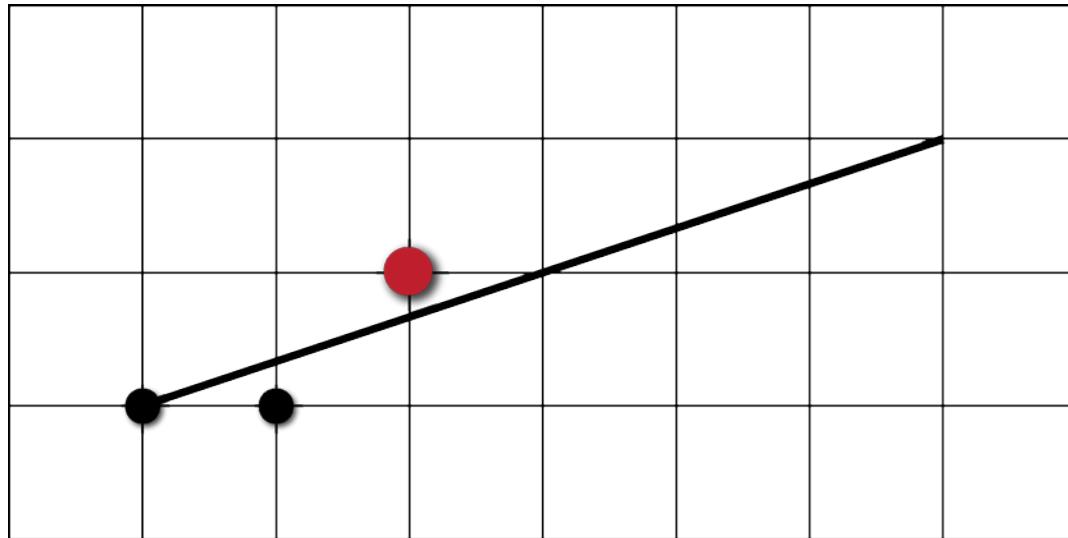
$$\Delta x = 6 \quad \Delta y = 2$$

$$2\Delta y = 4 \quad 2\Delta y - 2\Delta x = -8 \quad 2\Delta y - \Delta x = -2$$

i	draw	p _i
0	(1,1)	-2
1	(2,1)	2

Bresenham - Example

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$$A = (1,1) \quad A' = (7,3)$$

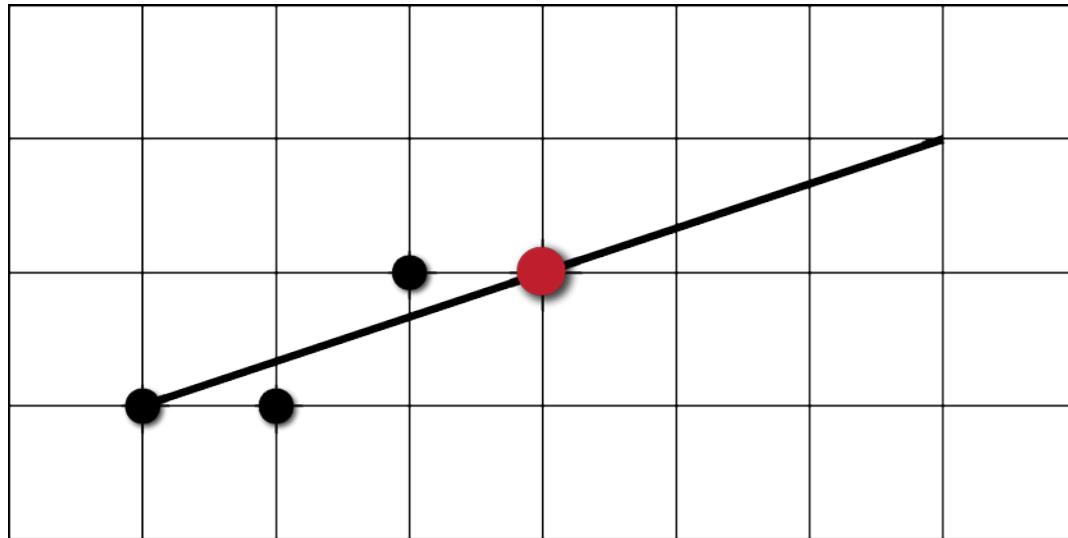
$$\Delta x = 6 \quad \Delta y = 2$$

$$2\Delta y = 4 \quad 2\Delta y - 2\Delta x = -8 \quad 2\Delta y - \Delta x = -2$$

i	draw	p _i
0	(1,1)	-2
1	(2,1)	2
2	(3,2)	-6

Bresenham - Example

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$$A = (1,1) \quad A' = (7,3)$$

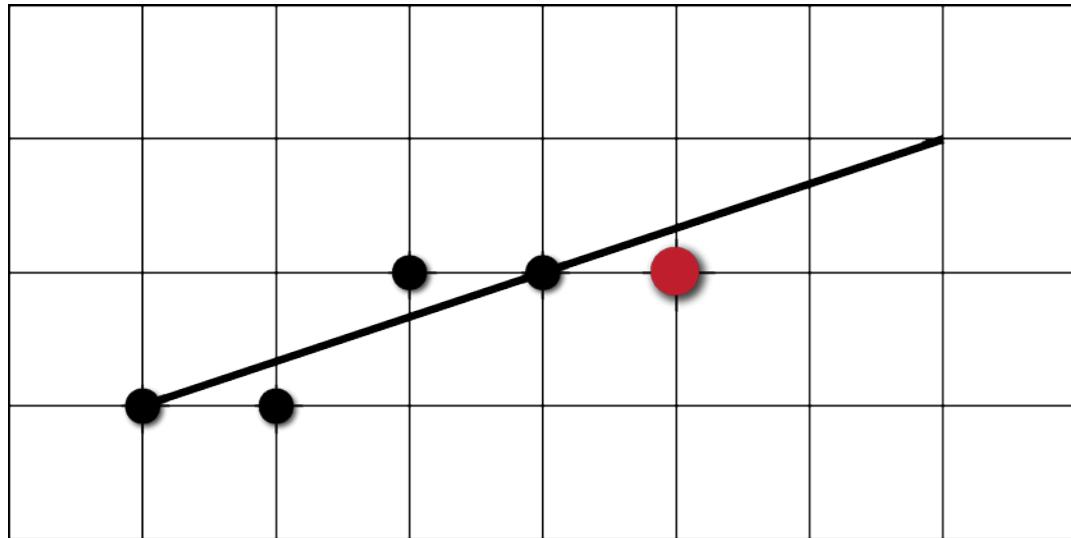
$$\Delta x = 6 \quad \Delta y = 2$$

$$2\Delta y = 4 \quad 2\Delta y - 2\Delta x = -8 \quad 2\Delta y - \Delta x = -2$$

i	draw	p _i
0	(1,1)	-2
1	(2,1)	2
2	(3,2)	-6
3	(4,2)	-2

Bresenham - Example

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$$A = (1,1) \quad A' = (7,3)$$

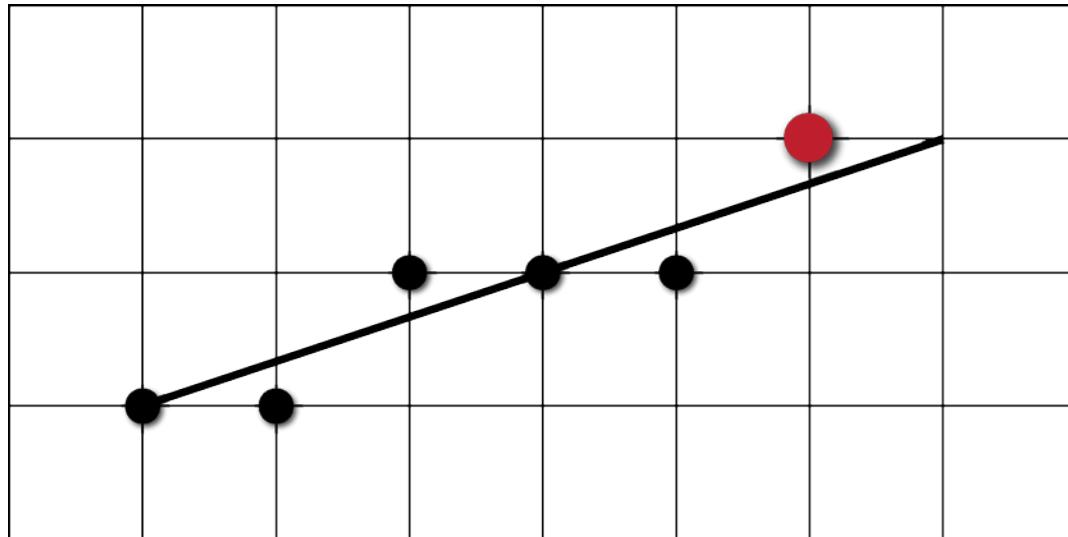
$$\Delta x = 6 \quad \Delta y = 2$$

$$2\Delta y = 4 \quad 2\Delta y - 2\Delta x = -8 \quad 2\Delta y - \Delta x = -2$$

i	draw	p _i
0	(1,1)	-2
1	(2,1)	2
2	(3,2)	-6
3	(4,2)	-2
4	(5,2)	2

Bresenham - Example

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$$A = (1,1) \quad A' = (7,3)$$

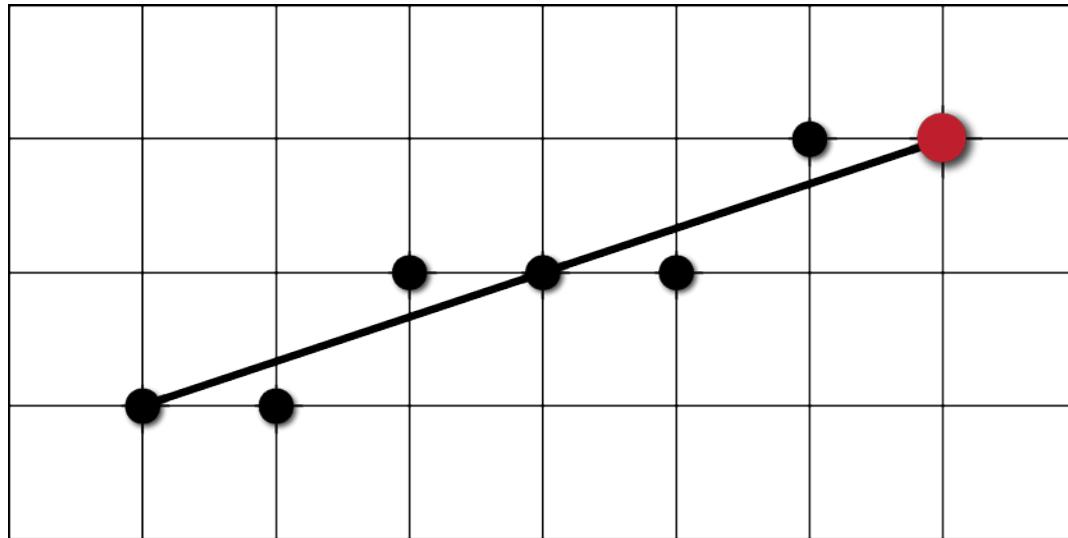
$$\Delta x = 6 \quad \Delta y = 2$$

$$2\Delta y = 4 \quad 2\Delta y - 2\Delta x = -8 \quad 2\Delta y - \Delta x = -2$$

i	draw	p _i
0	(1,1)	-2
1	(2,1)	2
2	(3,2)	-6
3	(4,2)	-2
4	(5,2)	2
5	(6,3)	-8

Bresenham - Example

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$$A = (1,1) \quad A' = (7,3)$$

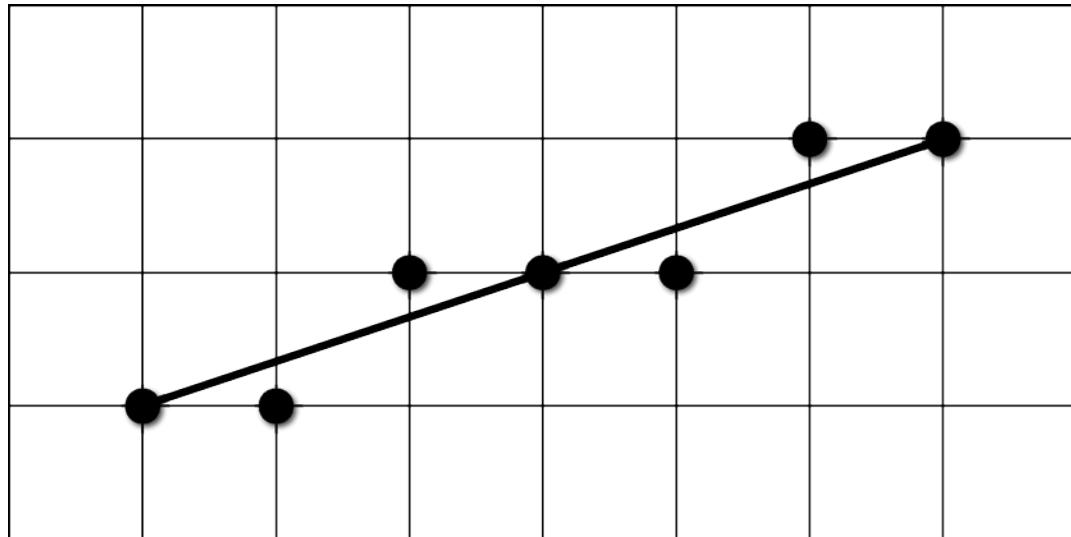
$$\Delta x = 6 \quad \Delta y = 2$$

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i	draw	p _i
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2	(3,2)	-6
3	(4,2)	-2
4	(5,2)	2
5	(6,3)	-8
6	(7,3)	-2

Bresenham - Example

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$$A = (1,1) \quad A' = (7,3)$$

$$\Delta x = 6 \quad \Delta y = 2$$

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i	draw	p _i
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3	(4,2)	-2
4	(5,2)	2
5	(6,3)	-8
6	(7,3)	-2

Midpoint Algorithm

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- Alternative construction of Bresenham's algorithm
 - Produces the same results
- Sometimes preferred to previous approach
 - Better applicable for curve drawing algorithms
- Assume nonvertical line (first quadrant)
$$f(x, y) = ax + by + c = 0 \quad -b \geq a \geq 0$$
- Positive for points above the line
- Negative for lines below the line

Midpoint Algorithm

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- Increase the x-coordinate : $x_{i+1} = x_i + 1$
- Determine only the y coordinate based on the sign:

$$d_i = f(x_i + 1, y_i + 0.5)$$

- If $d_i \leq 0$ then

$$\begin{aligned} d_{i+1} &= f(x_i + 2, y_i + 0.5) = a(x_i + 2) + b(y_i + 0.5) + c = \\ &= d_i + a \end{aligned}$$

- If $d_i > 0$ then

$$\begin{aligned} d_{i+1} &= f(x_i + 2, y_i + 1.5) = a(x_i + 2) + b(y_i + 1.5) + c = \\ &= d_i + a + b \end{aligned}$$

Midpoint Algorithm

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- Next value of d_i can be computed by simple additions
- Initial value:

$$d_0 = f(x_0 + 1, y_0 + 0.5) = f(x_0, y_0) + a + b/2$$

- Integer computation:

$$F(x, y) = 2f(x, y) = 2(ax + by + c) = 0$$

Midpoint algorithm

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- Equation of the line:

$$f(x, y) = 2(dy)x - 2(dx)y + 2c = 0$$

$$d_0 = f(x_0, y_0) + a + b/2 = \boxed{2dy - dx}$$

$$d_i \leq 0 : d_{i+1} = d_i + a = d_i + \boxed{2dy}$$

$$d_i > 0 : d_{i+1} = d_i + a + b = d_i + \boxed{2dy - 2dx}$$

- d_i is equivalent to the p_i in Bresenham's algorithm

Curve Rasterization

Midpoint circle, Bresenham circle

Midpoint Circle Algorithm

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- **Circle:**

$$f(x, y) = x^2 + y^2 - r^2 = 0$$

$f(x, y) > 0$ outside

$f(x, y) < 0$ inside

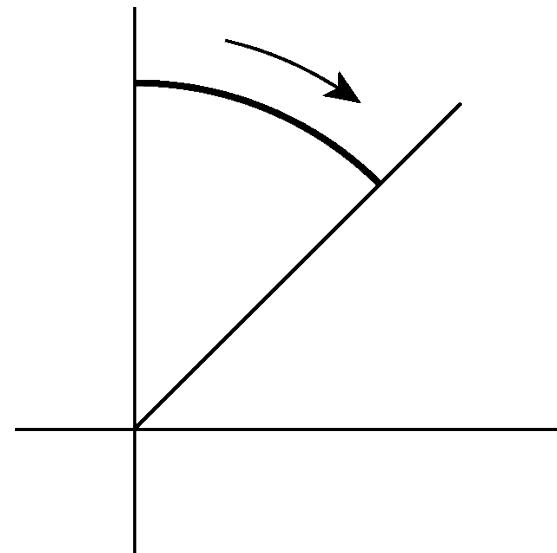
- **Draw 2. octant of the circle**

$$x_{i+1} = x_i + 1$$

$$d_i \geq 0 : y_{i+1} = y_i - 1$$

$$d_i < 0 : y_{i+1} = y_i$$

- **Test :** $d_i = f\left(x_i + 1, y_i - \frac{1}{2}\right)$



Midpoint Circle Algorithm - Step

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- Determine d_i

$$\begin{aligned}d_{i+1} - d_i &= f(x_i + 2, y_{i+1} - \frac{1}{2}) - f(x_i + 1, y_i - \frac{1}{2}) = \\&= (x_i + 2)^2 + (y_{i+1} - \frac{1}{2})^2 - r^2 - (x_i + 1)^2 - (y_i - \frac{1}{2})^2 + r^2 = \\&= 2x_i + 3 + (y_{i+1}^2 - y_{i+1}) - (y_i^2 - y_i)\end{aligned}$$

$$y_{i+1} = y_i \Rightarrow d_{i+1} - d_i = 2x_i + 3$$

$$\begin{aligned}y_{i+1} = y_i - 1 \Rightarrow d_{i+1} - d_i &= 2x_i + 3 + (y_i^2 - 2y_i + 1 - y_i + 1) - (y_i^2 - y_i) = \\&= 2x_i - 2y_i + 5\end{aligned}$$

Midpoint Circle Algorithm- Initialization

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- Compute d_0

$$x_0 = 0, \quad y_0 = r$$

$$d_0 = \left(1, r - \frac{1}{2}\right) = 1 + \left(r - \frac{1}{2}\right)^2 - r^2 = 1 + r^2 - r + \frac{1}{4} - r^2 = \frac{5}{4} - r$$

- Integer computation

$$d_0 = \text{round}\left(\frac{5}{4} - r\right)$$

- If r is integer value

$$d_0 = 1 - r$$

Midpoint Circle Algorithm - Final

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□ Initialization: $(x_0, y_0) = (0, r)$ $d_0 = \text{round}\left(\frac{5}{4} - r\right)$

$$x_{i+1} = x_i + 1$$

□ If $d_i \geq 0$ then $y_{i+1} = y_i - 1$ $d_{i+1} = d_i + 2x_i - 2y_i + 5$
else $y_{i+1} = y_i$ $d_{i+1} = d_i + 2x_i + 3$

Bresenham Circle Algorithm

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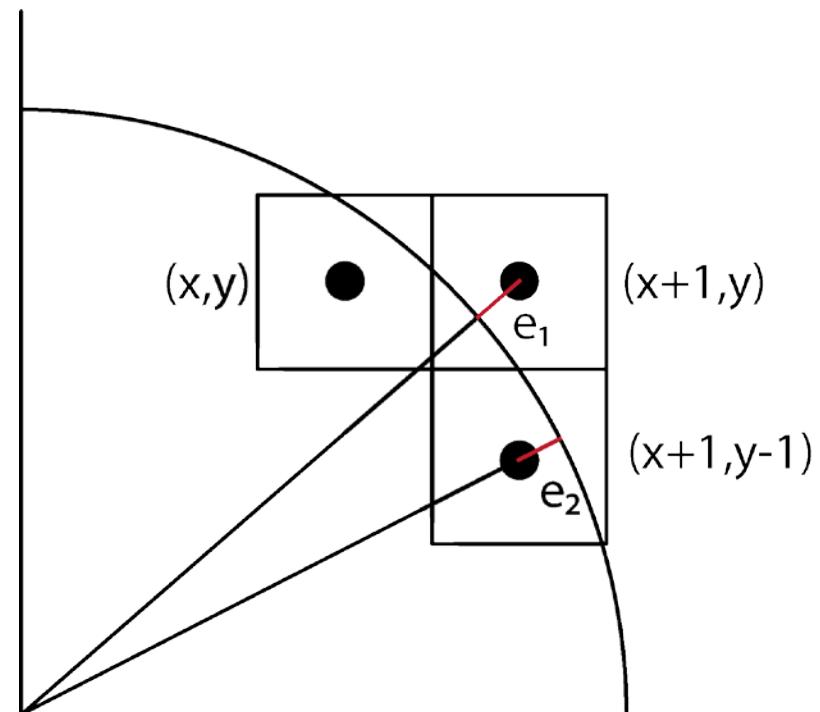
- Generate 2. octant
- Determine if $y_{i+1} = y_i$ or $y_{i+1} = y_i - 1$

- Based on sign of error:

$$p_i = e_{i1} + e_{i2}$$

$$e_{i1} = (x_i + 1)^2 + y_i^2 - r^2$$

$$e_{i2} = (x_i + 1)^2 + (y_i - 1)^2 - r^2$$



Bresenham Circle Algorithm - Step

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- Determine change in p

$$p_i = e_{i1} + e_{i2} = 2(x_i + 1)^2 + y_i^2 + (y_i - 1)^2 - 2r^2$$

$$p_{i+1} = 2(x_i + 2)^2 + y_{i+1}^2 + (y_{i+1} - 1)^2 - 2r^2$$

$$p_{i+1} - p_i = 2((x_i + 2)^2 - (x_i + 1)^2) + (y_{i+1}^2 - y_i^2) + ((y_{i+1} - 1)^2 - (y_i - 1)^2)$$

$$p_{i+1} = p_i + 4x_i + 6 + (y_{i+1}^2 - y_i^2) + ((y_{i+1} - 1)^2 - (y_i - 1)^2)$$

Bresenham Circle Algorithm - Step

55

- Determine change in p

$$p_i = e_{i1} + e_{i2} = 2(x_i + 1)^2 + y_i^2 + (y_i - 1)^2 - 2r^2$$

$$p_{i+1} = 2(x_i + 2)^2 + y_{i+1}^2 + (y_{i+1} - 1)^2 - 2r^2$$

$$p_{i+1} - p_i = 2((x_i + 2)^2 - (x_i + 1)^2) + (y_{i+1}^2 - y_i^2) + ((y_{i+1} - 1)^2 - (y_i - 1)^2)$$

$$p_{i+1} = p_i + 4x_i + 6 + (y_{i+1}^2 - y_i^2) + ((y_{i+1} - 1)^2 - (y_i - 1)^2)$$

- If $p_i \leq 0$ then $y_{i+1} = y_i$

$$p_{i+1} = p_i + 4x_i + 6$$

- If $p_i > 0$ then $y_{i+1} = y_i - 1$

$$p_{i+1} = p_i + 4(x_i - y_i) + 10$$

Bresenham Circle Algorithm - Initialization

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- Compute p_0

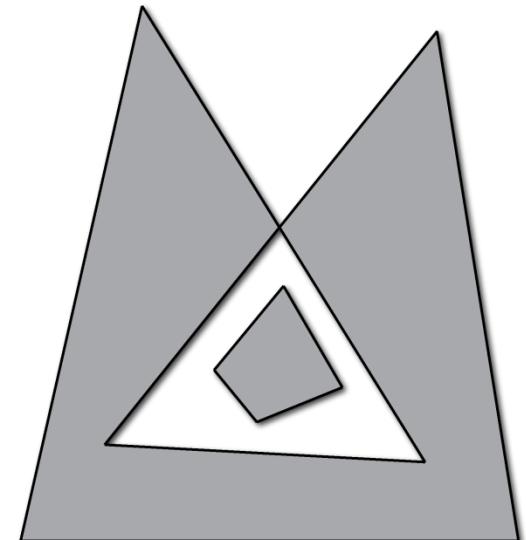
$$p_0 = 2 + r^2 + (r-1)^2 - 2r^2 = 2 + r^2 + r^2 + 1 - 2r - 2r^2 = 3 - 2r$$

Polygon Rasterization

Polygon Rasterization

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- Draw boundary
 - Draw polygon edges
- Draw filled polygon
 - Draw edges and fill the inside
- Problem
 - Need to determine which areas to fill
 - Slow
- Solution
 - Specialized algorithm



Polygon Rasterization

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- Using scan line
 - Scanning from top to bottom (or bottom to top)
- Computing intersections
 - Using coherence
 - Computing intersections incrementally
- Compute intersections only with lines that intersect the scan line
 - Store active edge list (AEL)

Polygon Rasterization: Algorithm - Initialization

60

- Associate an empty bucket to each scan line
- Find the largest y value for each edge and put it into corresponding scan line's bucket
 - Sort according to x
- For each edge store
 - x – x-coordinate of the highest point
 - y – the lowest y-coordinate of the edge
 - dx – change in x between scan lines (slope)
- Initialize the AEL to empty
- Set y to the height of the top scan line

Polygon Rasterization: Algorithm - Iteration

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- Add all edges in the bucket for y to the AEL
 - AEL is always sorted according to x
- Compute intersection for each edge in AEL
 - Intersections are sorted because AEL is sorted
- Fill areas between intersections
- Remove any edge from AEL for which y is equal to the y -coordinate of the scan line
- Process next scan line
 - Exit if all scan lines are processed

Polygon Rasterization: Intersections

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- Compute intersection based on the intersection with previous scan line

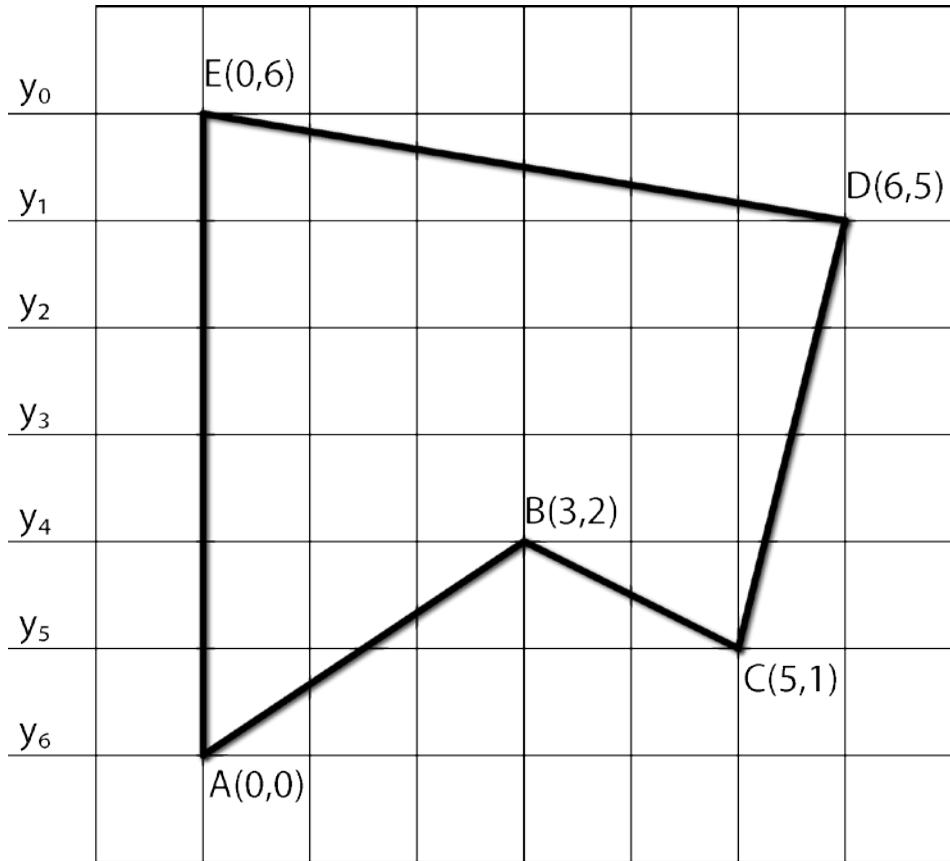
$$x_{k+1} = x_k + dx = x_k + \frac{\Delta x}{\Delta y}$$

- Can be altered to use only integer arithmetic
- Parallel implementation
 - Compute intersection for each scan line separately

$$x_{k+1} = x_0 + kdx$$

Polygon Rasterization: Edge List

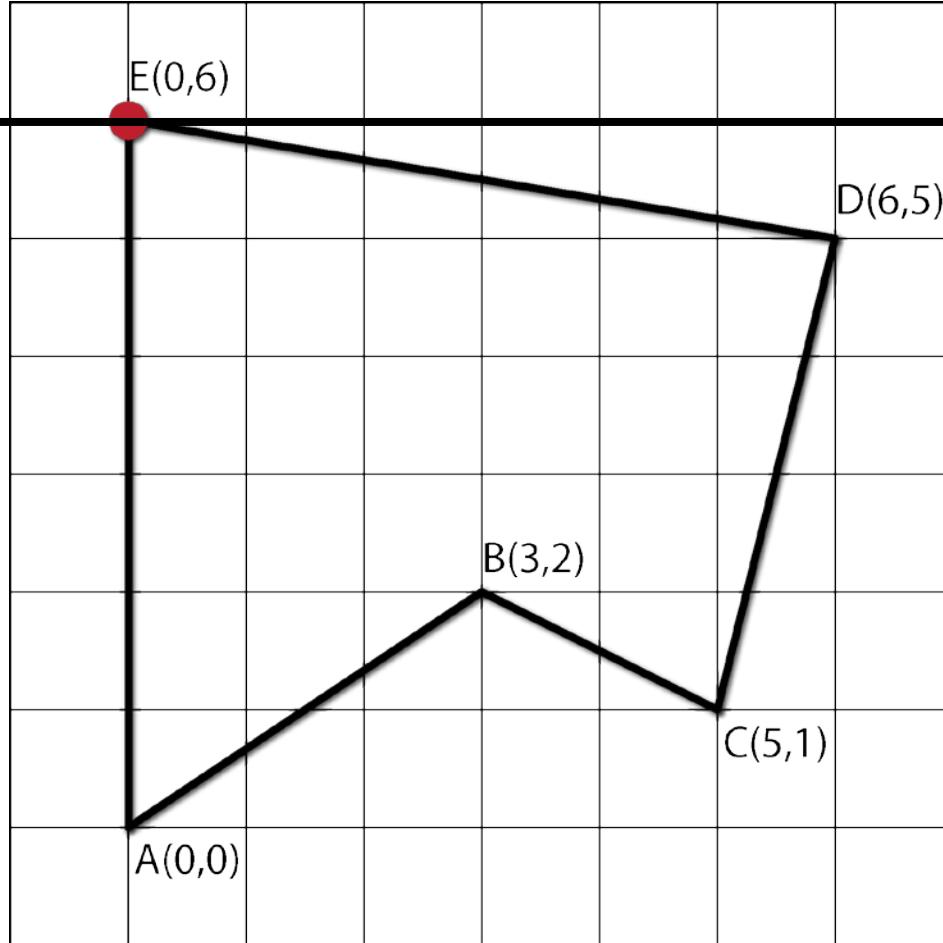
63



EA ED
0 → |0|0|0| → |0|5|6|
1 → |6|1|-1/4|
 DC
2
3
BA BC
4 → |3|0|-3/2| → |3|1|2|
5
6

Polygon Rasterization: Example

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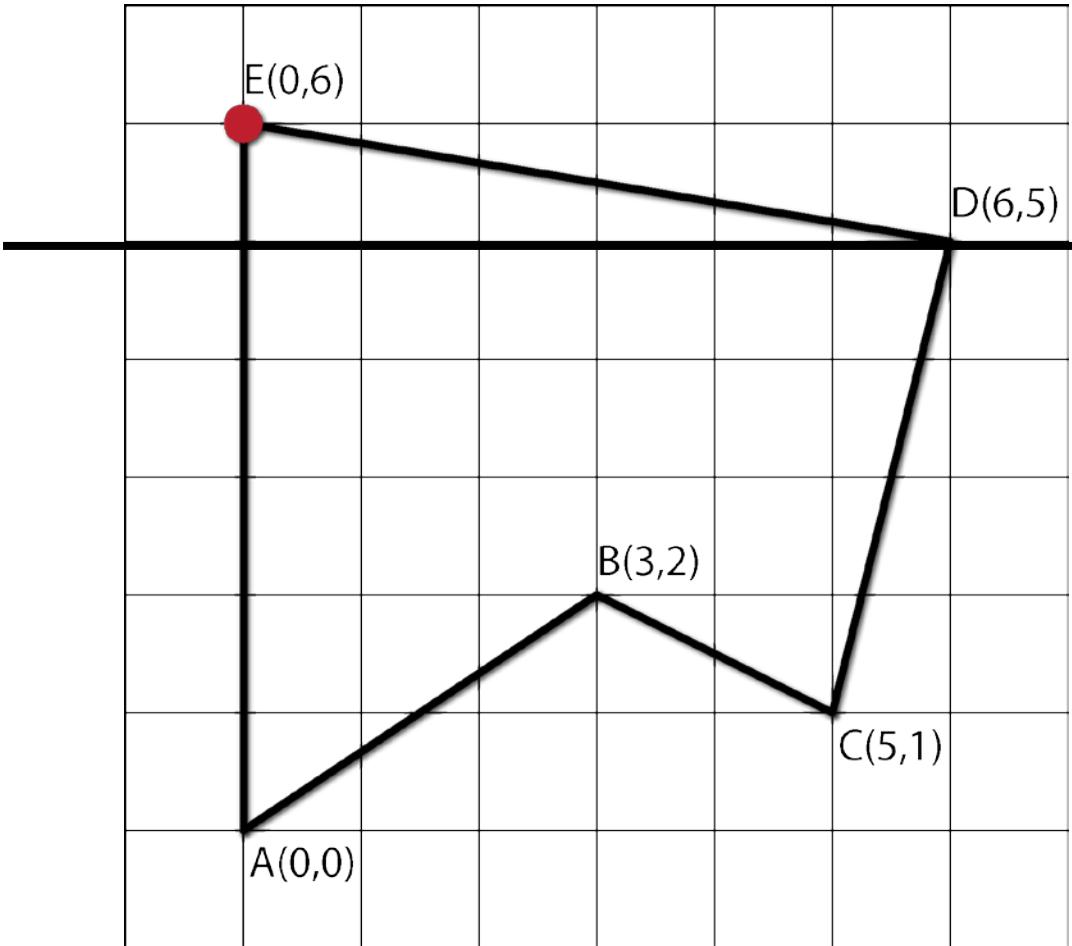
Add EA and ED to AEL

AEL: EA, ED

intersections: (0,6), (0,6)

Polygon Rasterization: Example

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Add DC to AEL

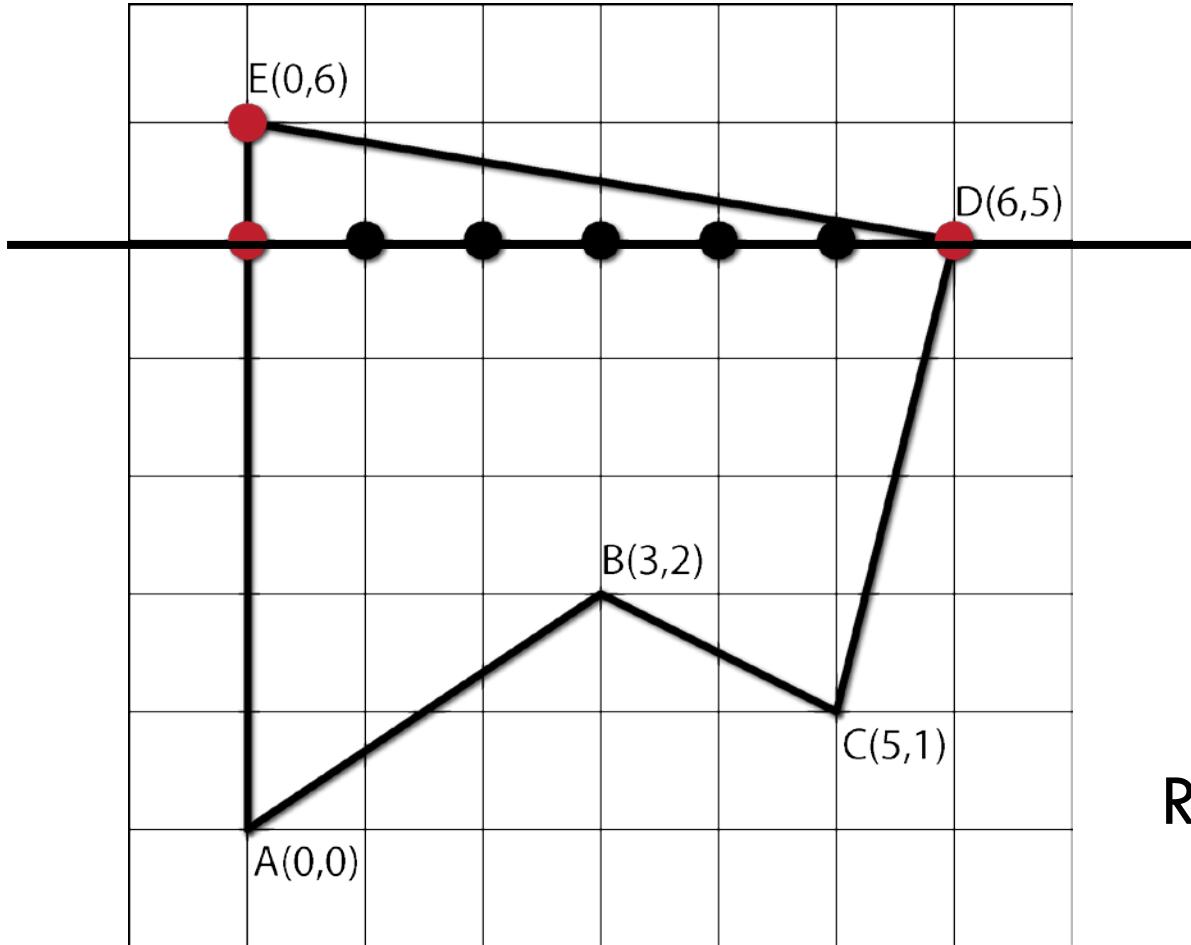
AEL: EA, ED, DC

intersections:
 $(0,5)$, $(6,5)$, $(6,5)$

Remove ED from AEL

Polygon Rasterization: Example

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Add DC to AEL

AEL: EA, ED, DC

intersections:

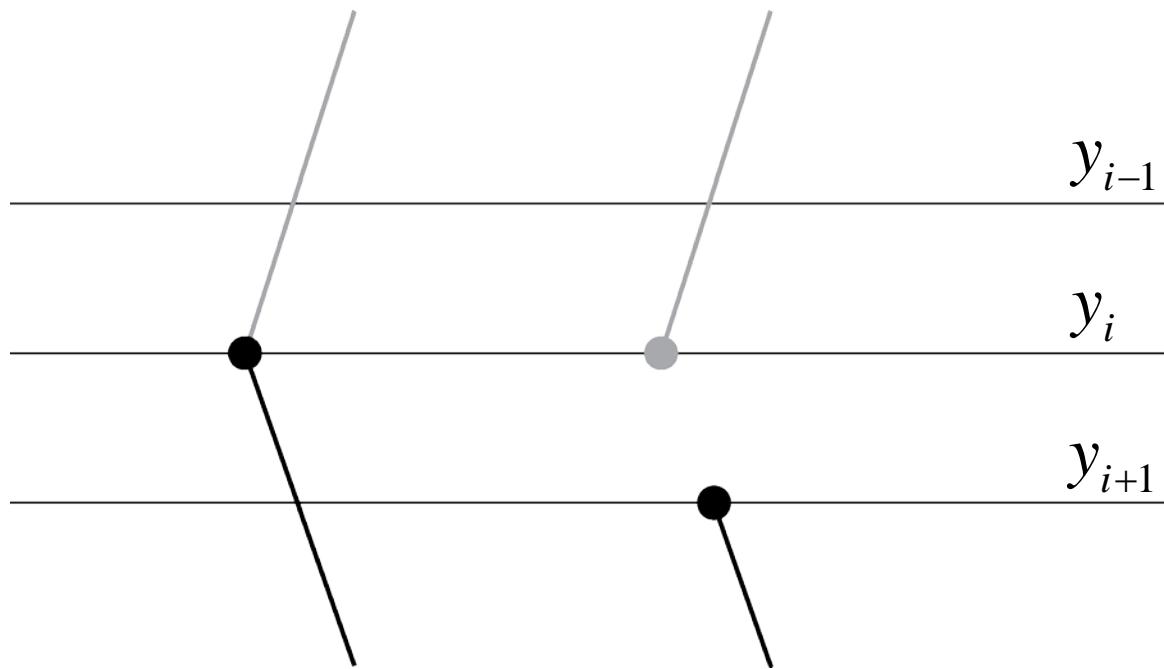
(0,5), (6,5), (6,5)

Remove ED from AEL

Polygon Rasterization - Correction

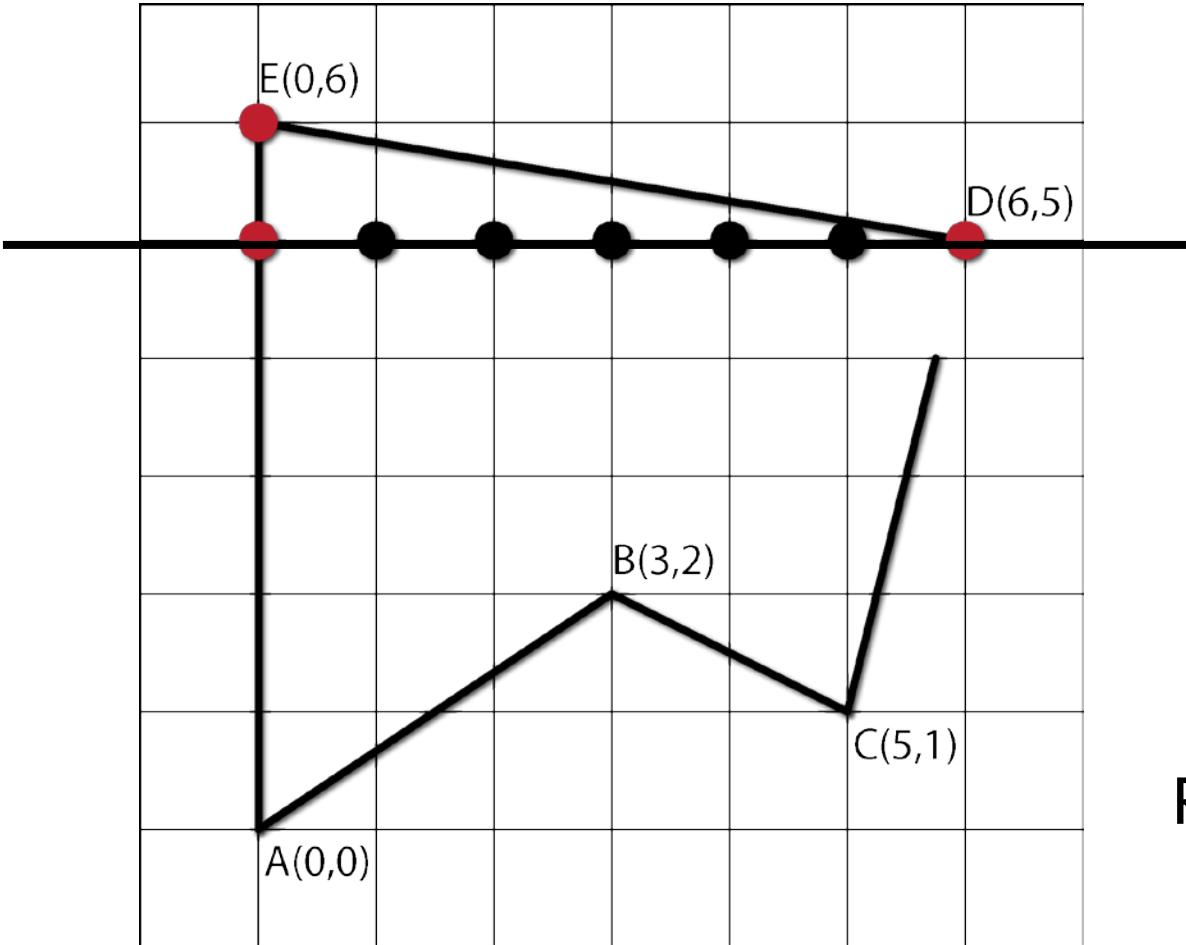
67

- Shorten the by moving the top endpoint of the edge
 - Do in preprocessing



Polygon Rasterization: Example

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Add DC to AEL

AEL: EA, ED, DC

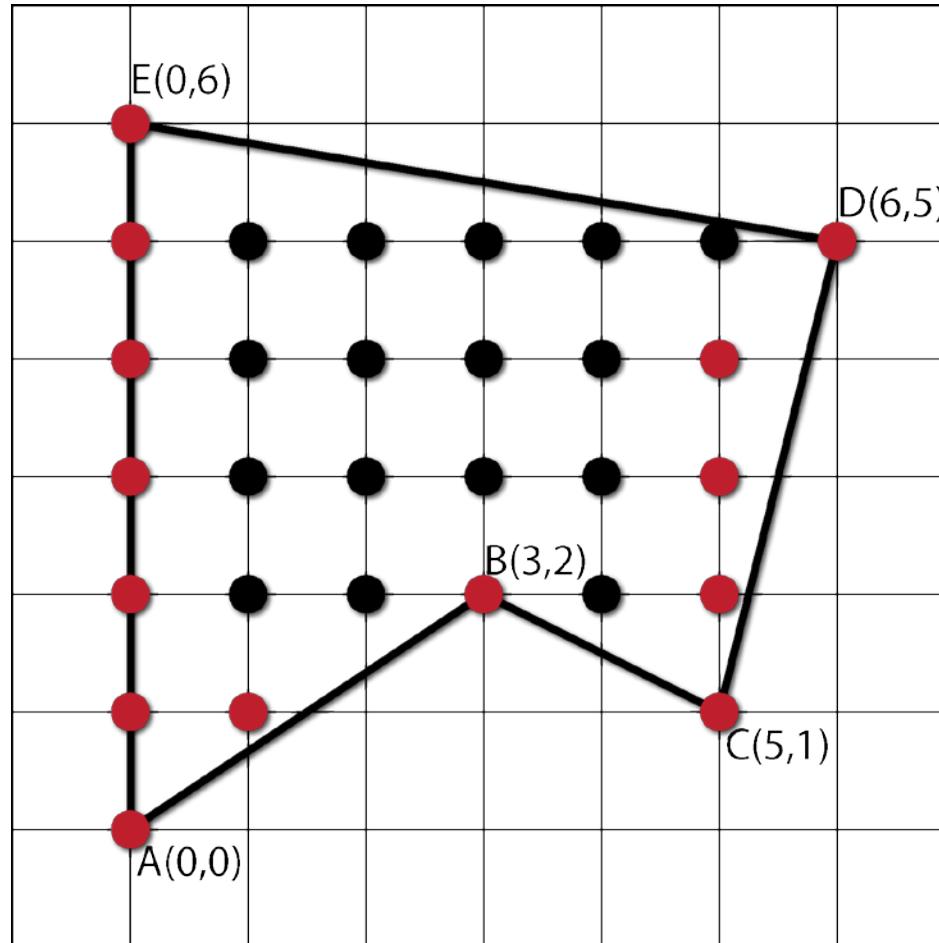
intersections:

(0,5), (6,5)

Remove ED from AEL

Polygon Rasterization: Example

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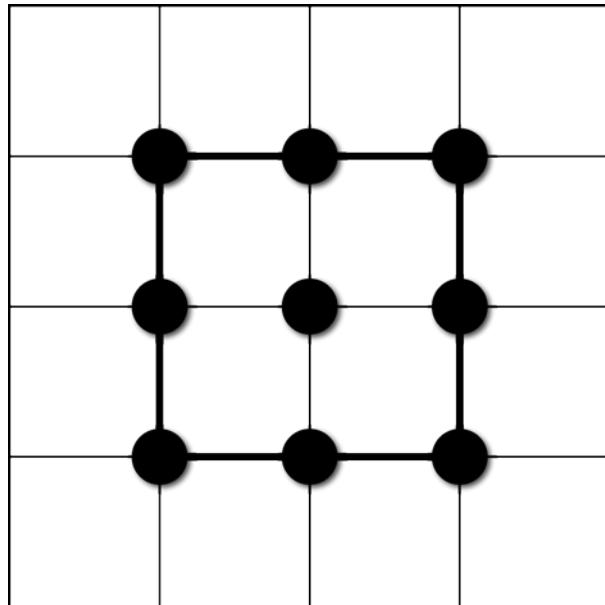


Polygon Rasterization: Problem

70

- Area = 4
- Highlighted points = 9

- Improvement
 - ▣ Filling: omit the right point
 - ▣ Shorten the edges from top by 1



Fill Algorithms

Pixel-Based Fill Algorithms

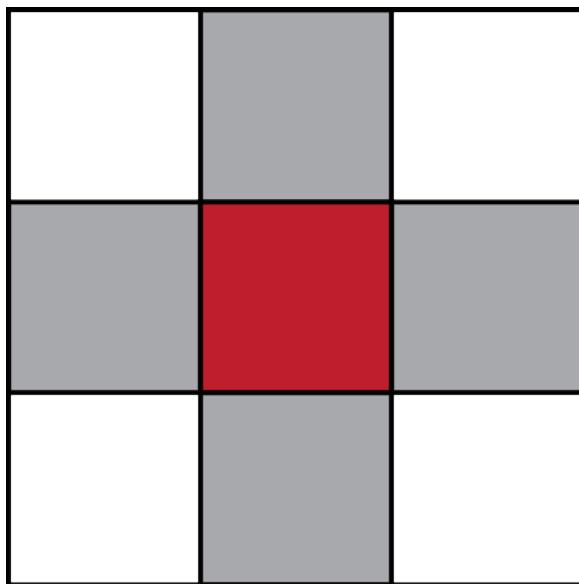
72

- Operate only with pixel
- Filling a specified area with a color
 - Arbitrary boundary (not only polygons)
- Specify starting point (seed)
 - Automatic or user specified
- Specify area
 - Color – flood fill
 - Boundary – boarder fill
- Dependent from adjacency

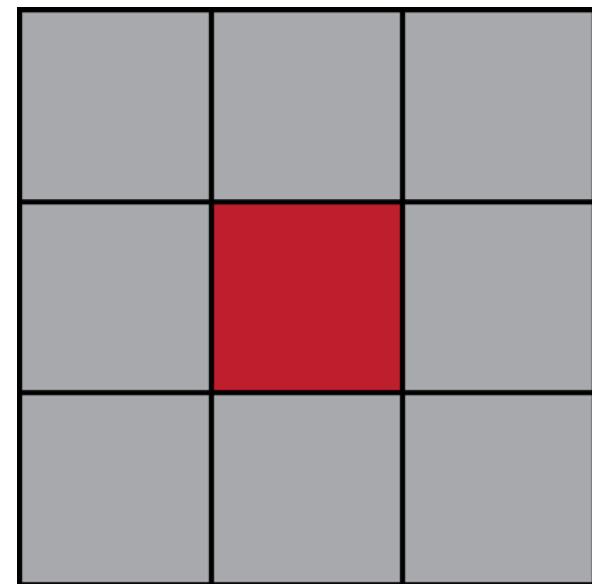
Adjacency

73

4-adjacency



8-adjacency



Flood Fill

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```
FloodFill(x, y, new_color, old_color)
```

```
    if color(x,y) = old_color then
```

```
        color(x,y) = new_color
```

```
        FloodFill(x+1, y, new_color, old_color)
```

```
        FloodFill(x-1, y, new_color, old_color)
```

```
        FloodFill(x, y+1, new_color, old_color)
```

```
        FloodFill(x, y-1, new_color, old_color)
```

- For 8-adjacency add

- $(x+1, y+1), (x-1, y+1), (x+1, y-1), (x-1, y-1)$

Boarder Fill

75

```
FloodFill(x, y, new_color, border_color)
  if (color(x,y) != border_color) and
    (color(x,y) != new_color) then
      color(x,y) = new_color
      FloodFill(x+1, y, new_color, border_color)
      FloodFill(x-1, y, new_color, border_color)
      FloodFill(x, y+1, new_color, border_color)
      FloodFill(x, y-1, new_color, border_color)
```

- Preprocessing
 - Delete new_color from area

Questions ???