Functional Representation of Human Embryo Brain Models

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Abstract

The shape of a human embryo brain is organic and has many folds that are difficult to model or animate with conventional techniques. The function representation is a good choice in modeling such organs because the smooth variations of set theoretic operation can be used. One approach proposed for animating the organ growth uses a tubular skeleton calculated automatically from a 2D object outline. The growth speed varies with the position within the organ and thus the model is divided into multiple geometric primitives that are later glued by a blending union operation. Animation frames of brain growth are shown.

1 Introduction

Human organs during their development can undergo significant changes in shape through a variety of global transformations, such as bending or twisting. Because, it is very difficult to see and to understand growth process of organs the embryologists found the realistic models of human organs and animations of their shape changes during the growth necessary in their studies. Biological organ grows nonlinearly and yet obeys certain rules.

The purpose of this manuscript is to model the outer shape and the shape metamorphosis during the growth of some human embryo organs, particularly brain and stomach. Popular methods like 3D shape reconstruction from Computer Tomography (CT) sections or ultrasound data can not be used for this type of modeling because the resolution of the devices used in those methods are much higher comparing to the size of human embryo. Four weeks old embryo is approximately 3 *mm* tall while the CT resolution is 1 *mm* giving us only three sections for a reconstruction process. Usually, the microscopic cross-sections are used to reconstruct the polygonal representation of an embryo, which is exact but complicated process. In case of such destructive approach often a mouse embryo is used instead of the human embryo [1]. To control the shape metamorphosis between two mesh objects become a problem when they have different topology and geometry. To create the realistically looking human organ models and to generate the animations demonstrating the growth process requires an appropriate methodology. The aim of this paper is to present a methodology based on the functional representation and convolution surfaces [2, 3].

Even though the convolution surfaces provide nice blending between several parts of organs, the control of the blend shape is very limited. The functional representation is a tool that generalize the set theoretic operations and generates full range of shapes from simple object union to smooth blend. The animation of such surfaces follow the changes smoothly, even if the topology changes. Because of this advantage the functional representation become a popular tool where the shapes to be modeled are from the natural world. We explain here our modeling experience that can be useful for others.

2 Function Representation

Let us consider closed subsets of n-dimensional Euclidean space E^n with the definition:

$$f(x_1, x_2, ..., x_n) \geq 0, \tag{1}$$

where f is a real continuous function defined on E^n. The above inequality is called a function representation (F-rep) of a geometric object and function f is called the defining function. In three-dimensional case the boundary of such a geometric object is called implicit surface. The major requirement on the function is to have at least C^0 continuity. The set of points $X_i(x_1, x_2, ..., x_n) \in E^n$, $i = 0, ..., N$ associated with Eq. 1 can be classified as follows:

$$\begin{aligned} f(X_i) &> 0 \text{ if } X_i \text{ is inside the object,} \\ f(X_i) &= 0 \text{ if } X_i \text{ is on the boundary} \\ &\quad \text{of the object ,} \\ f(X_i) &< 0 \text{ if } X_i \text{ is outside the object.} \end{aligned}$$

Let us consider from now on the defining function given by the convolution operator between a kernel and all the points of a skeleton, i.e. function F as defined in the last equation of the previous section.

2.1 Set-theoretic Operations

The binary operations on geometric objects represented by functions can be also defined in the form of function representation by

$$\mathcal{F}(f_1(X), f_2(X)) \geq 0, \tag{2}$$

where $\mathcal{F}$ is a continuous real function of two variables [3]. Such operations are closed on the set of function representations. After set theoretic operation between two subjects defined by functions f_1 and f_2 the resulting object has the defining function as follows:

- For object union

$$\begin{aligned} f_3 &= f_1 | f_2 \\ &\equiv \frac{1}{1+a}(f_1 + f_2 + \sqrt{f_1^2 + f_2^2 - 2af_1f_2}), \end{aligned}$$

- for object intersection

$$\begin{aligned} f_3 &= f_1 \& f_2 \\ &\equiv \frac{1}{1+a}(f_1 + f_2 - \sqrt{f_1^2 + f_2^2 - 2af_1f_2}), \end{aligned}$$

- for object subtraction

$$f_3 = f_1 \backslash f_2 \equiv f_1 \& (-f_2),$$

where $|, \&, \backslash$ are notations of so-called R-functions and parameter $a = a(f_1, f_2)$ is the arbitrary continuous function satisfying the conditions

$$\begin{gathered} -1 < a(f_1, f_2) \leq 1 \\ a(f_1, f_2) = a(f_2, f_1) = a(-f_1, f_2) = a(f_1, -f_2). \end{gathered}$$

Please, note that even thought the resulting defining function for set above theoretic operations is continuous, the resulting object is not continuous in general.

2.2 Blending Union Operation

Intuitively the blending union operation between two initial objects from the set of function representations is a gluing operation. It allows us to control the gluing type in the wide range of shapes from pure set union to convolution like summation of terms. Mathematically the blending union operation is defined by

$$\mathcal{F}(f_1, f_2) = f_1 + f_2 + \sqrt{f_1^2 + f_2^2} + \frac{a_0}{1 + (\frac{f_1}{a_1})^2 + (\frac{f_2}{a_2})^2},$$

where f_1 and f_2 are functions representing objects that are blended. The absolute value a_0 defines the total displacement of the bending surface from two initial surfaces. The values $a_0 > 0$ and $a_1 > 0$ are proportional to the distance between blending surface and the original surface defined by f_1 and f_2, respectively. The effect of this operation compared to other possible object connections is demonstrated on two object primitives whose skeleton consists of two line segments one vertical and the other one diagonal, see Figure 1 top-left. Simple plus operation between convolution functions deforms the thickness of vertical convolution cylinders as shown in top-right image. Considering four line segments as a single skeleton of geometric primitive results in the shape shown in top-center image. The sequence of shapes shown on bottom of Figure 1 are the blending union operations between two parallel geometric primitives. The geometric primitives and their skeletons do not change but the blending parameters used to blend them are different for each image. In orderer from left side the used parameters are $a_i = 0.01$, $a_i = 0.07$, $a_i = 0.3$, $a_i = 0.5$, and $a_i = 0.7$, respectively. We can conclude that in the case when the shape and size of geometric primitives must be preserved the blending union operation with different parameters a_0, a_1, and a_2 is a good choice. On the other hand when the blending shape is main concern the convolution plus operation should be used. When both the shape of geometric primitives and that of blending are important the small values of blending union parameters is a choice. The F-rep blending union operation has similar advantages as simple convolution union with respect to minimizing unwanted bulges.

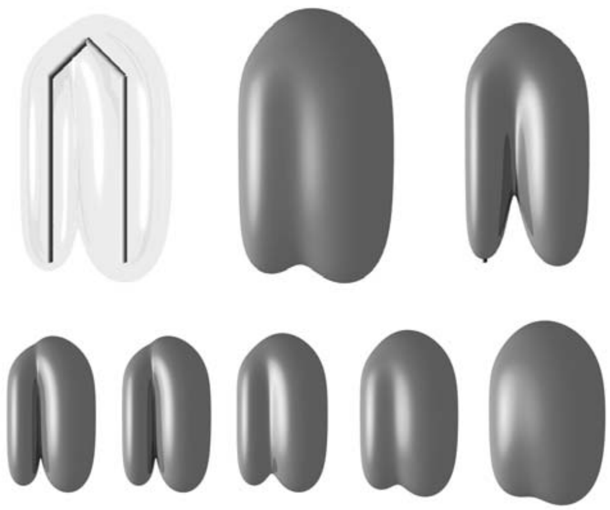

Figure 1: Blending union operation. top: standard and bottom: Blending union operation.

3 Organic Brain Models

This section will discuss a method to model the organic shapes by F-rep, where each of the geometric primitives is defined by

$$\sum_{i=1}^{N} F_i(x_1, x_2, x_3) - T = 0,$$

where F_i are the source potentials of skeleton primitives i.e. points, lines or triangles and T is an iso-potential threshold value. The source potential F_i is usually obtained via convolution operator between a kernel and a skeleton primitive.

3.1 Data acquisition

The real size measurements of embryo brain for different stages of development are obtained from atlas of embryology. The atlas contains hand-drawing pictures ordered by age. In the proposed growth simulation the models from day 28 - 56 days old brain are used. In our case the brain shown on the pictures was first divided into several physiological parts and the outlines were measured by a ruler (see Figure 2).

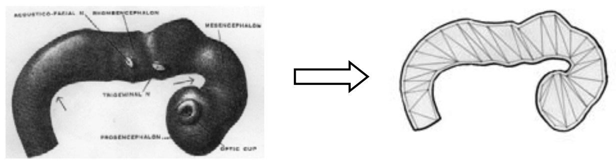

Figure 2: Triangular strip creates the central skeleton.

3.2 Central Skeleton

The result of the measurements is a 2D planar contour, call *the central skeleton*, nearly outlining the outer contour of the shape. Interior of central skeleton is triangulated such that it crates a triangular strip. One can observe different growth speed for different pars of embryo brain. It is therefore natural to divide the central skeleton into those parts. Additional tarts could be necessary to model the folds and control the unwanted blending problem near the folding areas. Figure 3, shows namely the part I corresponding to the part of brain called rhombencephalon, part II will develop to mesencephalon and part III is a prosencephalon. The next step is to calculate the central line that will be used as a base to define the thickness of the model along the line forming the tubular object. Central line passes through the center of central skeleton, connecting the mid points of vertical edges of a triangular strip.

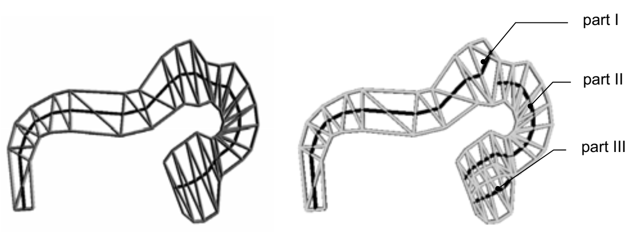

Figure 3: Dividing the central skeleton to 3 parts. The line in the middle of the central skeleton is called central line.

4 Tubular Skeleton

By adding the thickness to 2D central skeleton the 3D skeleton of the model is obtained. Multiple number of copies of central skeleton are slightly scaled and shifted to left and right sides of central skeleton. By this way the cross sections are produced which are then connected to form the tubular skeleton, see Figure 4:

- Each of side skeletons is scaled to fit the ellipses whose center is on the central line. Radius a of the ellipse is a distance to the central line from the border of the central skeleton. Radius b follows the equation, $b = \alpha a$, where α is a ratio parameter.
- As next step, for a given θ the side skeletons are translated by distance $t = c \cos \theta$, where c is known from parametric equation of ellipse shown in Figure 5.
- Finally, side skeletons are connected with a central skeleton or with other side skeletons by a triangular mesh.
- After erasing all interior triangular patches we obtain multiple tubular shapes forming together the entire skeleton of the brain.

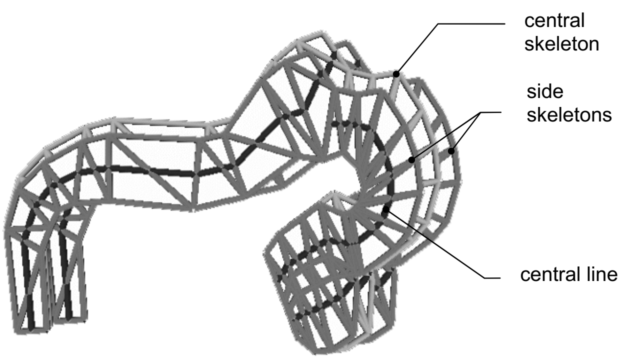

Figure 4: Adding the thickness by scaling and shifting the central skeleton.

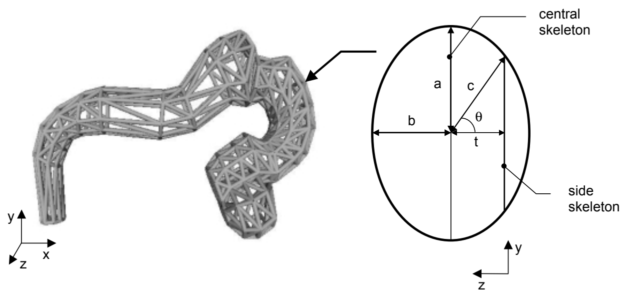

Figure 5: A 3D tubular skeleton for 36 days old human embryo brain.

5 HyperFun Modeling

A smooth convolution surface defined over the triangular mesh of tubular skeleton creates the model of embryo brain. In order to create brain model with convolution surfaces, we use HyperFun [2, 4] as modeling library and POV-Ray [5] as rendering software. HyperFun command hfConvTriangle generates convolution surface over the triangles which suites our problem. Let us discuss all parameter settings for one particular example, the *stage3* human embryo brain shown in Figure 6. The convolution kernel width is set to $s = 0.5$ and iso-potential threshold value is $T = 0.6$. The ration parameters of brain thickness have been set to $\alpha = 1.0$ at parts I and II and to $\alpha = 1.2$ at part III. Nice blending during the animation can be guaranteed by blend-union operation between three parts of this model using the HyperFun command hfBlendUni. The blending parameters $a_1 = a_2 = a_3 = 0.2$ are used for both gluing parts I, II and parts II and III, respectively.

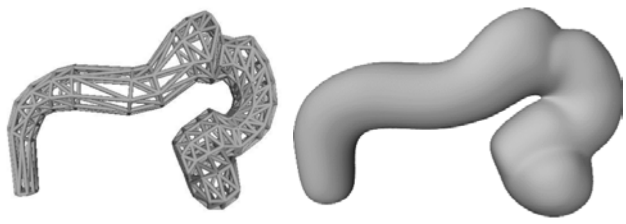

Figure 6: Stage3 human embryo brain. Left: 3D tubular skeleton, right: entire brain model, defined by function representation.

6 Animation

The good key-frame models and correct morphing interpolation between them are necessary for implicit surface animations. The embryological atlas consists of 8 artistic drawings of the human embryo brain development. We have used first

Stages	Age	Size (mm)	Number of	
	[days]	[mm]	skeleton triangles	parts
Stage1	28	3.5	296	1
Stage2	32	5	476	1
Stage3	36	9	588	3
Stage4	42	11	548	3
Stage5	49	15	724	4
Stage6	56	27	704	4
Stage6	72	56	710	4

Table 1: Model data.

7 images as key-frame models for the animation process. The key-frame models named Stage1,..., Stage7 have their basic measurements and statistical information collected in Table 1.

6.1 Morphing

There does not exist a general morphing technique we could use in our case because the available techniques use strictly the shape and topology information but they neglect the known growth processes, movements and knowledge of embryologists. A work described in [6] proposed a skeleton feature vectors but they still use a blending technique to hide the topology errors that occur during their 3D morphing step. We decided to propose the featured-based 3D morphing technique with a simple user interaction to control the complicated growth process. Our technique uses both the global deformations to roughly match the global movements of the brain during the growth and the local morphing technique to correct the shape details. To generate the models between key-frame brain stages we used various types of interpolation. Metamorphosis from Stage1 to Stage2, Stage2 to Stage4 and from Stage4 to Stage6 uses the tricubic interpolation based on Catmull-Rom interpolating curves. The Catmull-Rom splines for one-dimensional case, can be expressed by the following matrix formula [7]:

$$C(u) = UMP^T, \tag{3}$$

where $U = [u^3, u^2, u, 1], P = [p_{i-1}, p_i, p_{i+1}, p_{i+2}]$, and

$$M = \begin{bmatrix} -0.5 & 1.5 & -1.5 & 0.5 \\ 1.0 & -2.5 & 2.0 & -0.5 \\ -0.5 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

$C(u)$ is the interpolated value, $p_{i-1}, p_i, p_{i+1}, p_{i+2}$ are four consecutive data points and $u \in [0, 1]$ is a parameter that defines the fractional position between p_i and p_{i+1}.

Catmull-Rom interpolation is used to interpolate position between skeleton points, values of convolution filter parameters s and T, thickness parameters α, θ and the blending parameters a_0, a_1 and a_2. The spline technique can move vertices of existing skeletal triangles, but it can also generate intermediate skeletons by splitting or merging the vertices of existing triangles. Even thought the skeleton change is discrete the convolution surface changes smoothly because of linear property of convolution operator. Similarly when the number of skeleton parts is different a new part is created from a single vertex on the tubular skeleton. As a result we have no visually objective jumps during skeleton based growth animation.

The visual result of modeling human embryo brain using function representation is an animation. The number of in-between key-frame models depended on the age of key-frame models. We generated 10 in-between frames for one day of development. One second in the final animation corresponds to 3 days of human embryo development.

Direct ray-tracing of our convolution surface models took about three weeks on PIII 700MHz, much faster approach is to polygonize the convolution surfaces and then ray-trace them in the complex lighting scene, taking approximately 4 hours. Few frames from animation of *Organ growth* show the embryo stomach and brain described by embryo age and the real size scale bar, see Figures 7, 8.

7 Conclusions

We succeeded to model virtual human embryo brain using convolution surfaces and functional representation. The growth animation of a brain was generated for first 4 months of embryo development. The advantage of skeleton based ap-

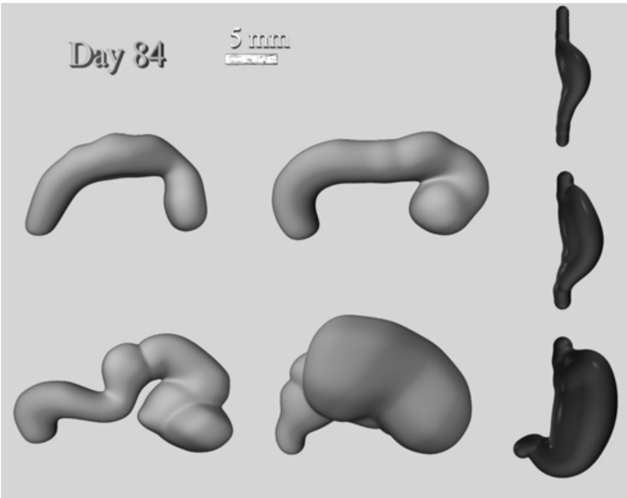

Figure 7: A single frame from the human embryo stomach animation.

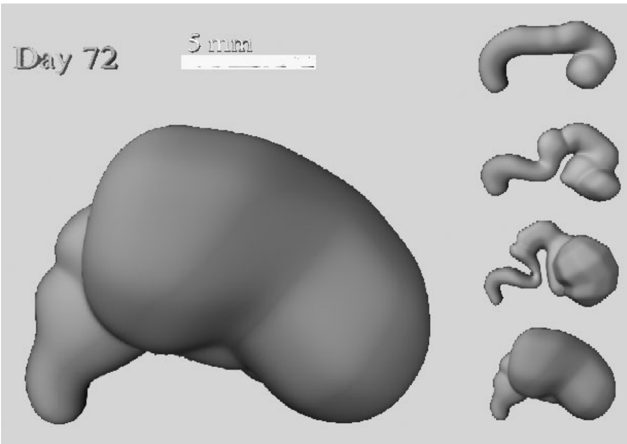

Figure 8: A single frame from the human embryo brain animation.

proach is that it avoids the the topology artifacts that can occur when using the nonlinear interpolation between two defining functions of F-rep models. Variable speed of growth and shape thickness is successfully modeled by convolution plus or blending union between model parts. This is also a solution to unwanted blending problems. The method produces smooth shape changes although the changes of tubular skeleton geometry are discontinuous when new vertices or triangles are created.

In the future work we implement branches and details for brain models and create grown-up brain models and growth animation between older human embryos. The same approaches can be used for other human organs. The future plan is to show the growth of intestinal system combined with other large organs.

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