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Set Theoretic Operations Between Components Defined by Sketching Silhouette Curves

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Abstract

We propose the method for object modeling based on sketched silhouette curves. The 3D shape of the object is calculated from set of silhouette curves using two approaches studied within this paper namely the skeleton-based convolution surfaces and variational implicit surfaces. Both methods are extended to handle smooth set theoretic operations between components defined by sketching silhouette curves on different projection planes. Our approach includes additional extensions to existing sketch-based modeling systems like automatic skeleton generation that can be directly used for object animation, carving operation, creating surfaces with handles and surface texturing.

Mathematics Subject Classification 2000: I.3.6, I.3.7

Additional Key Words and Phrases: Sketch-based Modeling, Variational Surfaces, Convolution Surfaces, F-rep

1. INTRODUCTION

Sketching is a very natural process for most people, it makes the design faster and intuitive. In this work we focus on designing a 3D freeform shape from sketched silhouette curves supporting natural human abilities and offering the rich expression afforded by pen movements.

Inspired by the above approaches like Teddy [1] and ConvMo [2] we have developed a system consisting of two approaches the skeleton based convolution surfaces and variational surfaces to reconstruct a 3D shape from user defined silhouette. By comparing of two approaches we conclude that the mixture of both methods for different parts of the shape should work best of all. In our first approach we reduce unwanted bulging effect by simplifying the skeleton and diminishing of field contribution at skeletal joints. Since this approach is not enough to remove the effect, we proposed the global interpolation with initial solution found by local interpolation step. Moreover, we propose the method for reduction of constraints thus reducing the computation costs of variational implicit surface and unwanted oscillations on surfaces. At the end we propose several operation for shape manipulation that are robust and very simple to implement.

The paper is organized as follows. The proposed methods are discussed in Section 2 We demonstrate the results obtained using our techniques in Section 3and we conclude this paper in Section 4

2. RECONSTRUCTION OF SURFACE FROM SILHOUETTE CURVE

In this section we describe the main steps for creating convolution and variational implicit surfaces from sketched silhouette curves. Both methods have some common steps but they differ in technique used for extracting the data needed for shape reconstruction from the silhouette curve.

2.1 Skeleton based surface

The following method summarizes the main steps of skeleton based convolution surface reconstruction from sketch:

(1) Process the input stroke and extract the skeleton from its simplification.

(2) Convolution surface construction by local fitting of weighted kernel along the skeleton segments to the silhouette curve.

(3) Global fitting of convolution surface to the silhouette by least-square method.

(4) Modify the generated surface components by merging, carving, and other editing operations, refer to Fig. 1.

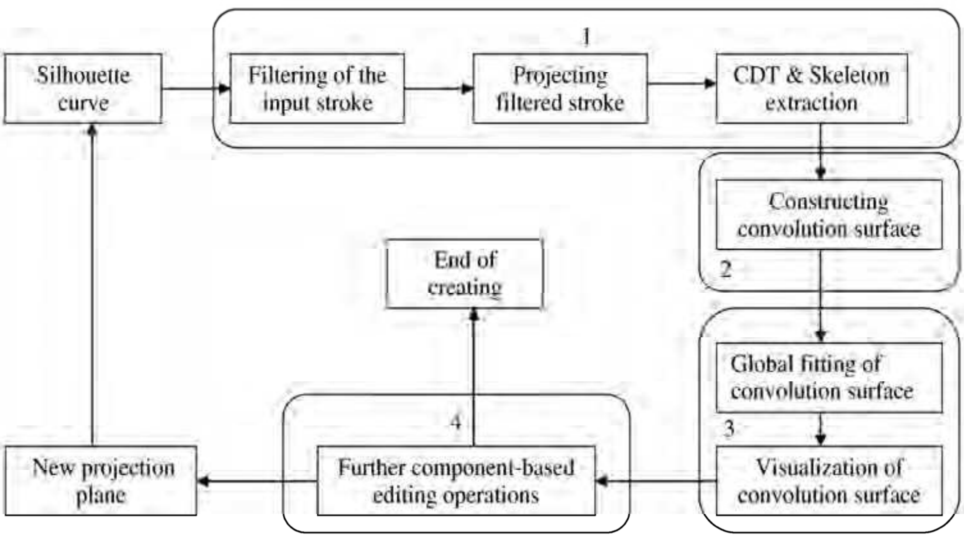

Fig. 1. Basic steps for skeleton based convolution surface generation from sketch.

2.1.1 *Processing the input stroke. Filtering the input stroke.* At this step the silhouette curve drawn by the user is converted into a simple polygon by sampling the input device movement. Several different algorithms can be found for point reduction in a polygon. In general they can be calcified into two groups those dependent on screen resolution using the edges with uniform length [1] and those using adaptive edge length [2]. Original nonuniform distribution of points along the silhouette is re-sampled to space the points uniformly on simplified polygon, see Fig. 2. All simplified strokes with self-intersections are rejected at this stage and user is requested to draw a new stroke.

Projecting filtered stroke. At this point we transform the stroke from screen coordinates to 3D coordinates by projecting it to a projection plane. Projection plane is parallel to screen plane passing through 3D position of a first silhouette point obtained from camera depth.

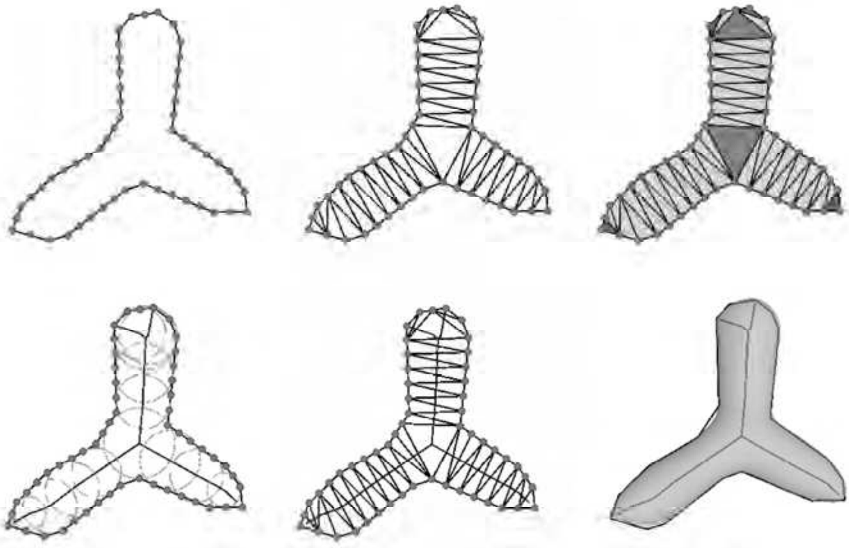

Fig. 2. Process of generating convolution surface from silhouette. Top from left: Filtered silhouette curve; result of CDT; terminal (blue), sleeve (green), and junction (red) triangles. Bottom: Influence circles; simplified skeleton; reconstructed convolution surface.

Skeleton extraction. At this step a good skeleton must be found to get the best guess of surface close to the silhouette. First, the constrained Delaunay triangulation (CDT) is applied, where constrains are the edges of filtered polygon. Second, triangles are claciffied into three categories, see Fig. 2 top raw:

—Terminal triangle (T-triangle) is a triangle with two external edges (marked in blue),

—Sleeve triangle (S-triangle) is a triangle with one external edge (marked in green),

—Junction triangle (J-triangle) is a triangle that has no external edges (marked in red).

To find a good initial guess an algorithm based on medial axis comprising of line segments should be used as a skeleton. Unfortunately this approach will produce unstable results for so called "folding" strokes.

We propose the skeleton extraction algorithm which unlike [2] uses not only circumscribed circles of triangles but rather for T-triangle it uses inscribed circles with their incenters; for J-triangles it uses circumscribed with circumcenters; and for S-triangles it uses either circumscribed circles or circles with external edge of polygon as tangent and third vertex of triangle as point of circle. Which one to choose is decided by the largest overlapping area with the polygon, see Fig. 2 bottom raw.

In order to remove unwanted bulging effect we delete redundant center if its circle lies mostly inside the next obtained circle in process order from starting T-triangle till a J-triangle or another T-triangle is reached.

2.1.2 *Constructing a convolution surface..* To find the convolution that fits given silhouette, we need to determine parameters of its field function $F(\mathbf{r})$. The first approximation to $F(\mathbf{r})$ is computed by considering each skeleton segment as isolated, i.e. neglecting the influence on other elements. In such a case for a single line segment with end points **p**, **q** and length l we assign weights w_0 and w_1 at the end points by setting the field contribution over **p** at distance $R_{\mathbf{p}}$ to $\mathbf{r}_{\mathbf{p}}$ and over **q** at distance $R_{\mathbf{q}}$ to $\mathbf{r}_{\mathbf{q}}$. Finally, we can estimate the weights w_0, and w_1. Finally, the field function of resulting convolution surface consisting of several linear skeletal elements can be obtained by summing the field function for all line segments L_i, $i = 1, 2, \dots, n$: $F(\mathbf{r}) = \sum_{i=1}^{n} \lambda_i F(\mathbf{r}, L_i)$, where $lambda_i$ is control scale factor. In the first approximation step $lambda_i = 1$.

2.1.3 *Global fitting of the convolution surface.*. With $lambda_i$ and weights w_0, w_1 computed as described above, one could notice that the surface does not interpolate the silhouette curve. This is caused by the fact that when field contribution of all segments are summed the resulting surface begins to inflate. This inflating effect is fully suppressed in global interpolation step. We solve the constrained least-squares problem for unknowns $lambda_i$ while minimizing the distance between the convolution surface and the silhouette polygon points.

2.1.4 *Operations.* *Merging* of two components is for free because two field functions are summed up to form a smooth surface. In order to have larger control over the blending of two components the F-rep blending union operation is implemented.

Carving operation is also implemented as F-rep blending substraction operation.

Surfaces with handles can be created by substraction operation between two components. Hoverer, there is a way create such surfaces directly form two silhouettes one inside the other one. Our algorithm can handle such cases and the result will be a torus, see Fig. 3.

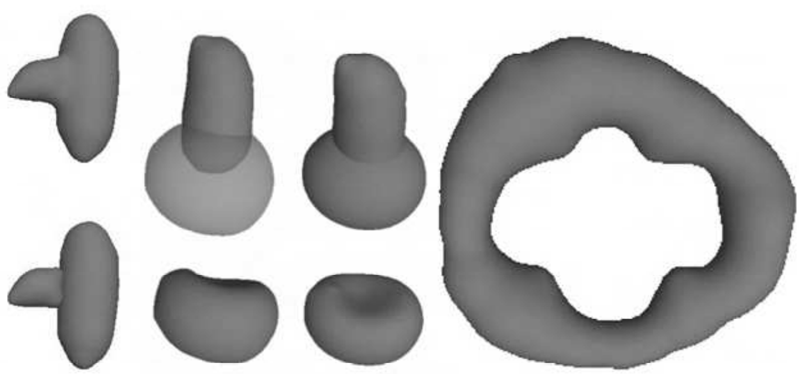

Fig. 3. Operations with skeleton convolution surfaces. From left: blending, carving and handle.

2.2 Variational implicit surface

The main steps of a method for reconstruction of variational implicit surfaces (VIS) are as follows:

(1) Process the input stroke and extract the skeleton from its simplification.

(2) Construction if variational implicit surface by definition of boundary and normal constraints. Final VIS then matches the silhouette curve.

(3) Global fitting of convolution surface to the silhouette by least-square method.

(4) Modify the generated surface components by merging, carving, and other editing operations, refer to Fig. 4.

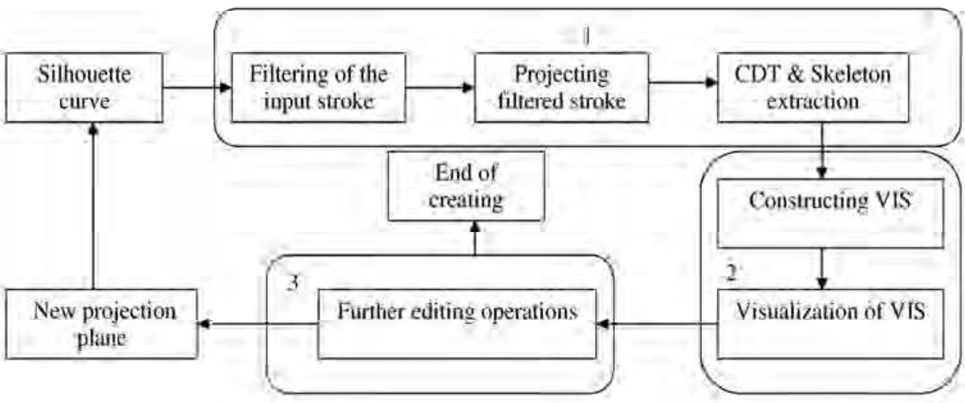

Fig. 4. Basic steps for VIS construction from sketch.

2.2.1 *Processing the input stroke.* The first step of VIS reconstruction method from sketched silhouette curve, processing of the input stroke, is similar to the steps from Section 2.1.1. First two step *Filtering the input stroke* and *Projecting filtered stroke* are exactly the same steps as in Section 2.1.1.

Skeleton extraction. At this step a good skeleton must be found with the height value for each skeleton vertex. The third step is a bit different. To find the skeleton we use our proposed method from above, this guaranties us non-redundant constraints for skeleton vertices. This step is different from commonly used approaches based on VIS where they had many constraints and big matrices. Finally, we assign the height value to each skeleton vertex equal to the average distance from the sketched polygon vertices in the neighborhood. By this approach we have eliminated unwanted surface oscillations.

2.2.2 *Constructing a variational surface.* Construction of variational implicit surface requires the specification of constraints. For each silhouette point we compute the outside normal to the silhouette polygon. Then a normal constraint is defined by shifting the boundary constraint placed at the silhouette point by ε in normal direction. Such normal constraints all lie in the silhouette plane. For each skeleton vertex we generate two normal constraints, one above and one below the silhouette plane. After defining these constraints we have all necessary information to create the linear system. The resulting function interpolates the constraints and no additional step are needed to correct the solution.

2.2.3 *Operations. Merging* of two components is easy. The merging algorithm eliminates boundary constraints and their respective normal constraints which lies in the intersection of these two components.

Carving operation wasn't proposed in previous sketch-based systems based on VIS, even though the mathematical concept is very easy. Similar, to merging algorithm, we eliminate the constraints of the carved component which lie in the intersection of two input components and of eliminating the constraints of carving component which lie outside the carved component. The constraints of carving object that remained are reversed, see Fig. 5.

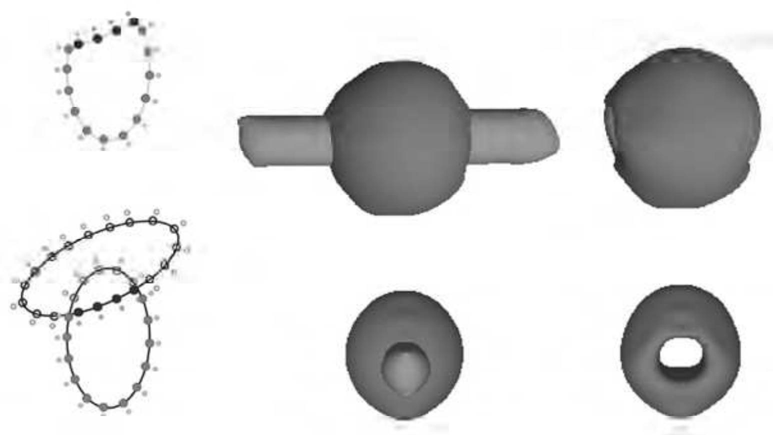

Fig. 5. VIS carving. Left: The constraints of carving object that remained in use are reversed. Right: Carving example.

In order to have better control over the blending of two components the F-rep blending substraction operation was also implemented.

Surfaces with handles can be created directly form two silhouettes one inside the other one. Inner silhouettes have opposite orientation than outer silhouette, and for outer normal constraints we regard the orientation of silhouette. Extracted skeleton lies between two silhouettes and defines the thickness of a model.

3. COMPARISON OF PROPOSED METHODS

The possibilities and limits of skeleton based convolution surface and variational implicit surface for sketch-based modeling. Skeleton contains the topological information needed for assigning the height formation as an additional constraint for convolution surface. The associated height often dependents on the surface representation used in algorithm or it can be defined by the user defined sketched curve from other view direction. Objects defined in this way are directly suitable for animation.

In the case of VIS, the resulting 3D freeform surface can be generated directly from silhouette points because VIS are specified by a set of boundary or normal constraints. Therefore, it is sufficient to sample input stroke and sample its points for constrain definition. See Fig. 6 for several shapes defined by our system. The main problem with VIS is that the thickness is unknown from initial stroke, therefore, new constraint points have to be created in both sides of the silhouette curve. In this way the total number of constraints increases. Problem is even more pronounced by repeated application of editing operations that also increases total number of constraints. Moreover, robustness of resulting linear system strongly depends on the set of constraints, i.e. the change of a single constraint can change the result significantly. Considering the shape manipulation, VIS allows very easy to implement basic operations, however, when more control is needed over the operation (such as shape of blending) skeleton based representation suites better the purpose.

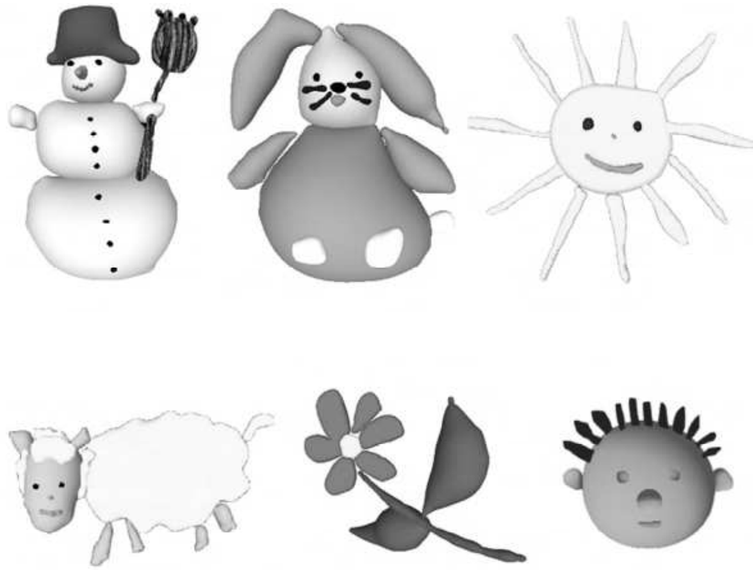

Fig. 6. Examples of various objects created in SketchCo.

4. CONCLUSION

Proposed methods were implemented as SketchCo modeling system. Models in Figure 6 were created in 6 minutes by children from 6 to 15 years. In our first approach with skeleton based convolution surfaces we reduced unwanted bulging effect that appears around junctions of blend line segments by simplifying the skeleton and diminishing of field contribution at skeletal joints. In our approach using VIS we proposed the method for reduction of constraints thus reducing the computation costs and unwanted oscillations on surfaces.

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REFERENCES

[1] Takeo Igarashi and Satoshi Matsuoka and Hidehiko Tanaka: *Teddy: A Sketching Interface for 3D Freeform Design*, Computer Graphics Proceedings, Annual Conference Series,Proceedings of SIGGRAPH August 1999.

[2] C. L. Tai and H. Zhang and C. K. Fong: *Prototype modeling from sketched silhouettes based on convolution surfaces*, Computer Graphics vol. 23 num. 1 , 2004, pp. 71 - 83.

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