Interior Modelling and Object Metamorphosis with Parametric Solids

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Abstract

Mathematical concepts of multivariate *parametric solids*, their description by a B-spline basis functions are discussed. Parametric solids can model both the shape and unisotropic interior. Three types of parametric solids i.e. Coons body 0, 1 and 2 are used to demonstrate the interior modelling often used as initial or boundary conditions in numerical simulation. Multiple parametric solids elements can be joined together to form a complicated shape. Continuity between elements can be defined similarly as in B-spline modelling. Proposed methodology and modelling technique is applied to metamorphosis of two given 3-D shapes.

1. Introduction

Many unambiguous solid representation techniques like primitive instancing, cell decomposition, constructive solid geometry, sweep, boundary or medial axis representation [8] have one limitation, they cannot offer ways of representing internal behavior. The representation is considered to be unambiguous when it corresponds to one and only one object in object space. Unfortunately, more late and modern technique like real function representation [4] does not solve this problem either. Those techniques assume total internal homogenity of the model. Nevertheless, they are adequate for many simulations and design applications. Still more and more complicated physical models where a scalar, vector or tensor-valued physical fields are needed, increase the demand on modelling both the shape and the distributions of fields as initial or boundary conditions for simulation. Some of the applications include areas of structural mechanics and ablation thermo- and aerodynamics [5] and description of inhomogeneous material. The representation that offers the interior representation is a parametric function representation: $F: R^n \rightarrow R^m$, where R^n and R^m are parameter and object space, respectively. In the trivariate case, the natural extension to triangles and rectangles are tetrahedra, pentahedra, and hexahedra, extensively used domains for Lagrange, and Hermite basis functions, both in the finite element methods (FEM) literature [9].

Sederberg and Parry [2] introduced the Free-Form Deformations (FFDs). This method imposes an initial deformation lattice on a parallelepiped, and defines the deformable space as the trivariate Bezier volume defined by lattice points. Griessmain and Purtgathofer [1] modified the technique by utilizing a B-spline volume representation.

This paper first, in Section 2. introduces the mathematical concepts of multivariate *parametric solids*, here the definition of a Coons patch is extended to Coons solids. Three types of parametric solids namely Coons body 0, 1 and 2 are used to demonstrate the interior modelling often used as initial or boundary conditions in numerical simulation. In Section 3., we modify the control points of boundary surfaces and curves to model the interior and the shape of objects. Modelling with multiple Coons solids is also discussed, here. Finally, the applications of the proposed technique and the object metamorphosis is discussed in Section 4..

2. Three Construction Steps of Coons Body

We will restrict ourselves to parametric function representation: $F: R^3 \rightarrow R^3$. The well-known Coons patch is generalized to tensor-product Coon parametric solids noted as body 0, 1, and 2. Each of three representations has different ability to control the interior of parametric solid. The simplest and most limited is *Coons body* 0 where only the shape of a quadrilateral can be changed, while *Coons body* 1 allows us to control the interior by modification the control points along the edge curves of a parametric solid. The most general is the *Coons body* 2, having the ability to modify shape and interior by all control points within the boundary surfaces of a parametric solid.

In general a *tensor-product solid* of degree (l, m, n) is defined to be

$$\mathbf{p}(u, v, w) = \sum_{i=0}^{l} \sum_{j=0}^{m} \sum_{k=0}^{n} \mathbf{b}_{ijk} F_i^l(u) F_j^m(v) F_k^n(w),$$

where u, v, $w \in [0,1]$, $\mathbf{b}_{ijk} \in R^3$, and F_i are the basis functions, respectively. Notice that for a volume of degree $(3,3,3)$, there are $4\times4\times4 = 64$ algebraic vectors and 64 geometric vectors, so there are 192 coefficients, see Fig. 1.

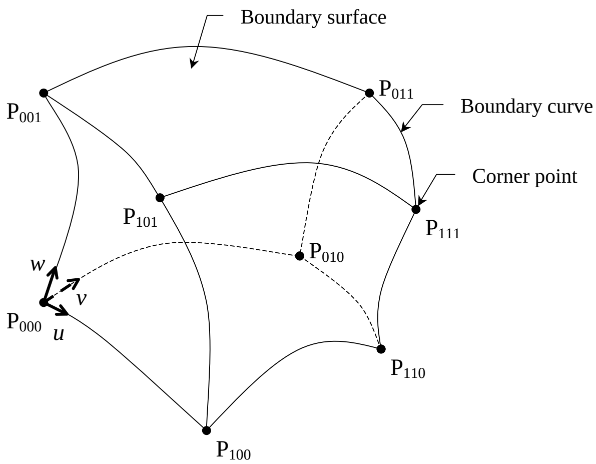

Figure 1. Tensor-product solid of degree (3,3,3).

2.1 Visualization

Simple mappings $F : R^3 \rightarrow R^3$ with small polynomial degrees, can be understood by displaying a set of isoparametric surfaces of the object, see Figure 2. One way to get a good idea of the shape of an object is to display several different images. For example, we can visualize the shaded shape with projected parametric curves and display three parametric surfaces corresponding to constant u, v, and w. To show the shape and interior changes in time, i.e. mappings $F : R^4 \rightarrow R^4$, time animation of isoparametric surfaces and outer shape is one possibility. The visualization techniques just mentioned are used during the course of this paper.

2.2 Coons Body 0

Coons body 0 is a trilinear interpolation of eight points P_{ijk}, i, j, $k \in \{0,1\}$ defined to be

$$T(P_{ijk}; u, v, w) = (1-u, u)\times$$
$$\times \begin{pmatrix} (1-v, v) \begin{pmatrix} P_{000} & P_{001} \\ P_{010} & P_{011} \end{pmatrix} \begin{pmatrix} 1-w \\ w \end{pmatrix} \\ (1-v, v) \begin{pmatrix} P_{100} & P_{101} \\ P_{110} & P_{111} \end{pmatrix} \begin{pmatrix} 1-w \\ w \end{pmatrix} \end{pmatrix}$$

where u, v, $w \in [0,1]$.

Coons body 0 has some obvious properties that can be derived from the tensor-product solid:

- *Derivatives*: Because of the linear independence u, v, w coordinates, the partial derivatives with respect to u, v, w have obvious geometric interpretation and they do coincide with the derivatives along parametric lines.
- *Boundary surfaces*: There are six hyperbolic paraboloids on the boundary, namely $T(P_{ijk}; 0, v, w)$, $T(P_{ijk}; 1, v, w)$, $T(P_{ijk}; u, 0, w)$, $T(P_{ijk}; u, 1, w)$, $T(P_{ijk}; u, v, 0)$, and $T(P_{ijk}; u, v, 1)$.
- *Boundary curves*: The boundary curves of Coons body 0 are linear segments. Putting two parameters of u, v, and w equal to 0, or 1 we get 12 linear sements, $T(P_{ijk}; 0, 0, w)$, $T(P_{ijk}; 0, 1, w)$, ...
- *Control grid*: Control grid points coincide with eight vertices of quadrilateral, P_{ijk}.

An example of Coons body 0 element is shown in Figure 2. Simple cube is deformed by moving four corner points in upward and downward directions, as a result the shape changes but the interior "density" can't be modified. This element is often used in FEM methods when only linear basis functions are used within a cubic element.

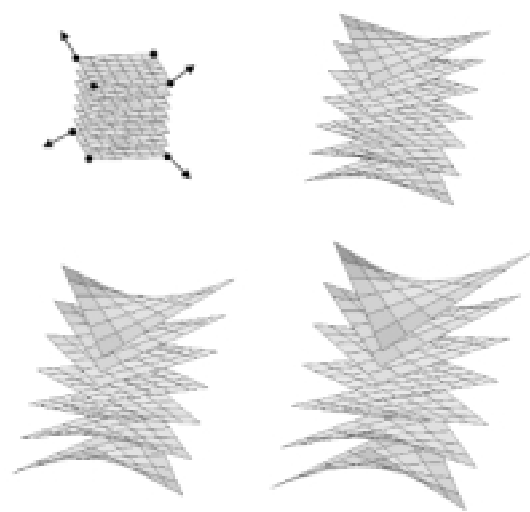

Figure 2. Coons body 0: Shape morphing of original cube by eight control points.

2.3 Coons Body 1

Let us given the following compatible curves $C(u,j,k)$, $C(i,v,k)$, and $C(i,j,w)$; $i,j,k \in \{0,1\}$; $u,v,w \in [0,1]$ holding the following property

$$\begin{aligned} C(u,j,k)_{u=i} &= C(i,v,k)_{v=j} = C(i,j,w)_{w=k} \\ &= C(i,j,k) =: C_{ijk}, \end{aligned}$$

for each $i, j, k \in \{0, 1\}$.

Coons body 1 is a bilinear interpolation between couples of compatible curves defined to be

$$\begin{aligned} C1(u,v,w) &= \\ &= (1-u,u)\begin{pmatrix} C(0,0,w) & C(0,1,w) \\ C(1,0,w) & C(1,1,w) \end{pmatrix}\begin{pmatrix} 1-v \\ v \end{pmatrix} \\ &+(1-u,u)\begin{pmatrix} C(0,v,0) & C(0,v,1) \\ C(1,v,0) & C(1,v,1) \end{pmatrix}\begin{pmatrix} 1-w \\ w \end{pmatrix} \\ &+(1-v,v)\begin{pmatrix} C(u,0,0) & C(u,0,1) \\ C(u,1,0) & C(u,1,1) \end{pmatrix}\begin{pmatrix} 1-w \\ w \end{pmatrix} \\ &-2T(C_{ijk}; u, v, w). \end{aligned}$$

Coons body 1 has the following properties:

- *Derivatives*: The partial derivatives with respect to u, v, w do coincide with the derivatives along parametric lines.
- *Boundary surfaces*: There are six Coons patches on the boundary derived from four-tuples of boundary curves.
- *Boundary curves*: The boundary curves of Coons body 1 are compatible curves $C(u, j, k)$, $C(i, v, k)$, and $C(i, j, w)$ [3]. In a simplest case when the boundary curves are linear the Coons body 1 is actually the Coons body 0.
- *Vertices*: Vertices of $C1(u, v, w)$ are the points C_{ijk}.
- *Control grid*: Control grid points coincide with the control points of boundary curves.

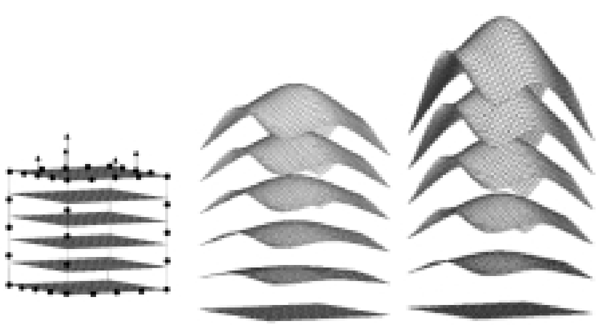

Figure 3. Coons body 1 with B-spline boundary curves. B-spline de Boor points along the edges deform the cube.

An example of Coons body 1 shown in Figure 3 demonstrates the cubic element defined by B-spline boundary curves each having five control points. The B-spline de Boor points are distributed along the edges of a simple cube and only the points with assigned arrows in the image, are used to deform the shape. The boundary curves of this solid can define the influence of boundary surfaces on the interior without changing the outer shape. The effect of density modification is shown in Figure 4, left image shows the original body with few parametric surfaces, the middle image shows the solid with modified interior. It can be observed that the interior in right image deforms much earlier than in left image. The most right image of Figure 4 is the outer shape that holds constant during the interior modelling. The interior modelling is still limited in Coons body 1 since we can not directly modify the center of boundary surfaces.

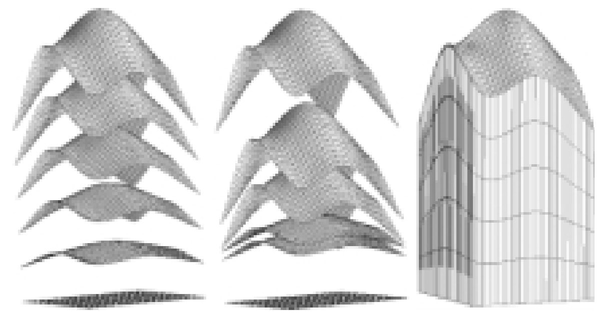

Figure 4. Coons body 1: Preserving the shape while changing the interior density.

2.4 Coons Body 2

Let us given three couples of arbitrary opposite patches $Z_k(u,v) = C(u,v,k)$; $Y_j(u,w) = C(u,j,w)$; $X_i(v,w) = C(i,v,w)$; $i,j,k \in \{0,1\}$; $u,v,w \in [0,1]$ having the common boundary curves as stated bellow:

$$C(u,v,k)_{v=j} = C(u,j,w)_{w=k} =: C(u,j,k);\ j,k = 0,1$$
$$C(u,v,k)_{u=i} = C(i,v,w)_{w=k} =: C(i,v,k);\ i,k = 0,1$$
$$C(i,v,w)_{v=j} = C(u,j,w)_{u=i} =: C(i,j,w);\ i,j = 0,1.$$

To make thinks easier, the connected patches should have no other intersections, e.g. $C(u,j,k) = Y_j(u,w) \cap Z_k(u,v)$, $C(i,v,k) = X_i(v,w) \cap Z_k(u,v)$ and $C(i,j,w) = X_i(v,w) \cap Y_j(u,w)$, $i,j,k \in \{0,1\}$.

Coons body 2 is defined to be

$$C2(u,v,w) = D(u,v,w) - C1(u,v,w) - T(C_{ijk}; u,v,w)$$

where $T(C_{ijk}; u,v,w)$ and $C1(u,v,w)$ are Coons body 0 and 1, respectively; $D(u,v,w)$ is a linear interpolation between couples of opposite patches given by

$$\begin{aligned} D(u,v,w) &= (1-u,u)\begin{pmatrix} C(0,v,w) \\ C(1,v,w) \end{pmatrix} \\ &+ (1-v,v)\begin{pmatrix} C(u,0,w) \\ C(u,1,w) \end{pmatrix} \\ &+ (1-w,w)\begin{pmatrix} C(u,v,0) \\ C(u,v,1) \end{pmatrix}. \end{aligned}$$

Coons body 2 has the following properties:

- *Derivatives*: The partial derivatives with respect to u, v, w do coincide with the derivatives along parametric lines.
- *Boundary surfaces*: There are six boundary faces $X_i(v,w)$, $Y_j(u,w)$, and $Z_k(u,v)$ used in definition [3]. Note, the case when patches are the Coons patches defined by four-tuples of boundary curves then the Coons body 2 is actually the Coons body 1. In the simplest case when patches have linear boundary curves the Coons body 0 is obtained.
- *Boundary curves*: The boundary curves of Coons body 2 are common boundary curves $C(u,j,k)$, $C(i,v,k)$, and $C(i,j,w)$.
- *Vertices*: Vertices of $C2(u,v,w)$ are the points C_{ijk}.
- *Control grid*: Control grid points coincide with control grid points of boundary surfaces.

An example of Coons body 2 using B-spline boundary surfaces is shown in Figure 5. The de Boor control points of this body are distributed at corner, on edges, and within the interior of boundary surfaces. De Boor control vertices distributed over the B-spline surfaces create the shape folding and deform a simple cube in Figure 5. Additional freedom in modelling is gained from control points distributed within the center of B-spline patches. It can be shown that the Coons body 2 defined by the B-spline boundary surfaces is a B-spline body [3]. A nail represented as a single Coons body 2, illustrated on Figure 6, has two different interior densities while preserving the same outer shape. The most right image shows the shaded shape and surface parametric curves of a nail. Moving control points belonging to edges along the nail body modifies its interior density.

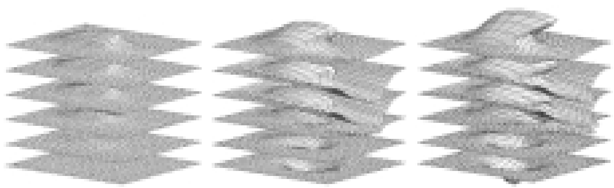

Figure 5. Coons body 2 with B-spline boundary surfaces. The de Boor points of boundary surfaces can deform the shape and interior of original cube.

3. Modelling with Coons Bodies

A lattice is defined to be a set of points that generates a volume. A control lattice has vertices, edges, faces, and cells. In the case of Coons bodies, each cell of the control lattice is defined by six faces and each face by four vertices. Each vertex has connectivity six. Each cell defines a Coons body (volume). Because, any Coons body 2 is a B-spline volume, the shape continuity is simply controlled in a similar way as it is done for B-splines. Similarly, multiple control points can also be used in Coons body 2 to create sharp edges and corners.

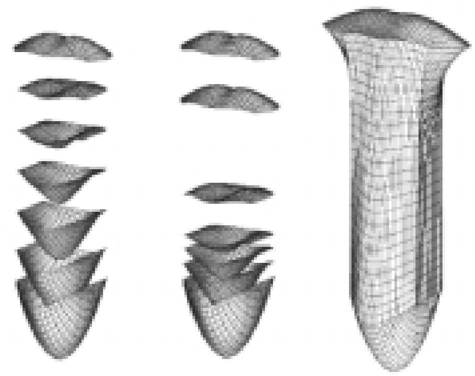

Figure 6. Nail as Coons body 2 with two different interior density distributions.

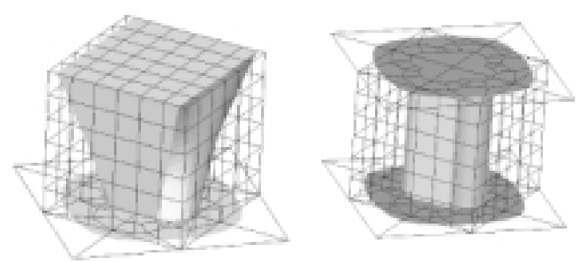

Figure 7. Modelling a spool as Coons body 2 starting from a cube.

In an example shown in Figure 7 the modelling of a spool starts from a cube. First, the bottom of a cube is rounded with control points on the bottom side of a cube, next the top is created by modifying the control points on top side. Top and bottom points also define the thickness of the spool. Other control points are not modified. Control grid and the shaded surfaces for two steps of spool modelling are shown in Figure 7.

Multiple Coons bodies 2 combined with set theoretic operations can create complicated shapes. In Figure 8, three bodies with union set operation are used to form the final shape. In the case when the blending between several parts is required, the single Coons body 2 with extended number of control points

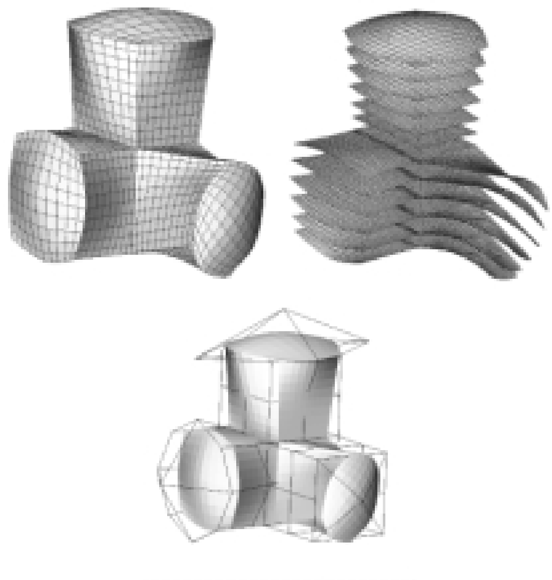

Figure 8. Union set operation between three Coons bodies 2.

can be used. Five Coons bodies 2 were used to design an ergonomic chair in Figure 9. The control grid used for modelling is shown in right image.

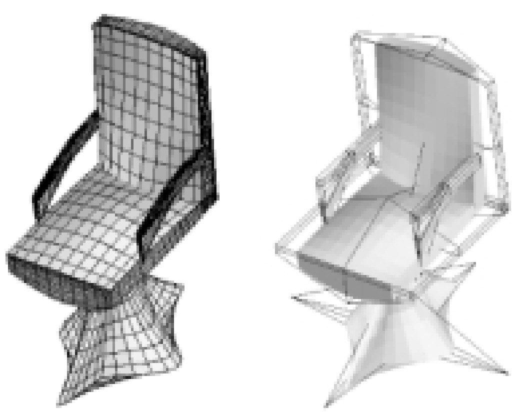

Figure 9. Chair represented with five Coons bodies 2 and its control grid.

4. Metamorphosis with Coons Bodies

In the case of Coons bodies, we are dealing, for example, with the deformation of an unit cube in the sense that we operate on the material forming the cube to alter its shape by pressing, drawing, twisting, or turning. This deformation process has local character within the body. In general, the position of every point in the interior as well as those on the surface is altered when grid point is moved. In the case when two different sets of control points define the same boundary surface (except the parametrization) the shape of the body does not change but the interior will change, see Figures 4, 6. In order to carry out the metamorphosis of two objects represented by Coons body, for every grid point of the first object the associated grid point from other object must be find. Then the grid deformation by moving grid point along the path defines the metamorphosis, similar to motion path animation techniques. Metamorphosis between the cube and the spool, shown in Figure 10, uses the grid deformation technique.

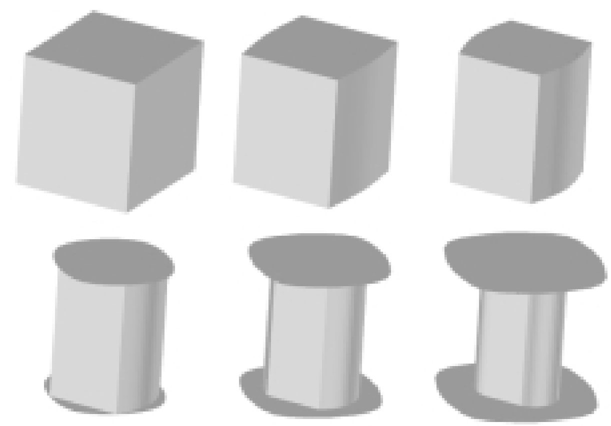

Figure 10. Metamorphosis from cube to spool using the Coons body2.

5. Conclusion

Proposed system implemented under the Linux Red Hat 6.2 operating system uses the parametric Coons bodies to model the interior and outer shape of a body. Presented examples show how easy is to define and change the interior. Natural extension to interior modelling is the shape deformation, and metamorphosis discussed in this paper.

The authors see the advantage of this technique in ability to define the boundary and internal initial conditions prior to numerical simulations. The method works well for symmetrical shapes used in engineering simulation. Parametric solids can be effectively applied on finite elements with irregular or *trimmed boundaries* to avoid difficult problems in model design for FEM.

Proposed method was actively tested in shape deformation examples using both engineering and organic shapes. It was quite difficult to apply the parametric solids for representation and deformation of organic shapes. Coons bodies can be easily utilized in FFD applications. Better results of shape deformation could be obtained using the fifth dimensional parametric solids under current investigation.

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References

Proceedings Papers:

[1] J. Griessmair and W. Purtgathofer. Deformation of solids with trivariate B-splines. *Proceedings of the Eurographics '98 conference*, September, 137-148, 1989.

[2] T. W. Sederberg and S. R. Parry. Free-form deformation of solid geometric models. *Computer Graphics (SIGGRAPH '86)*, 20:151–160, 1986.

[3] V. Zatko and Z. Mederlyová. On the coons bodies. *Proceedings of the 15th Spring Conference on Computer Graphics - SCCG 1999*, Budmerice, Slovakia, May, 247–253, 1999.

Journal Papers:

[4] A. Pasko, V. Adzhiev, A. Sourin and V. Savcenko. Function representation in geometric modeling: concepts, implementation and applications. *The Visual Computer*, 11:429–446, 1995.

[5] E.L. Stanton, L.M. Crain and T.F. New. A parametric cubic modeling system for general solids of composite material. *International Journal of Numerical Methods in Engineering*, 11:653–670, 1977.

Books:

[6] J. Hoschek and D. Lassers. Fundamentals of computer aided geometric design. *A K Peters*, Wellesley, 462–504, 1993.

[7] M. E. Mortenson. Geometric modeling. *John Wiley and Sons, Inc.*, New York, 237–260, 1997.

[8] A.A.G. Requicha. Representation of rigid solids: theory, methods, and systems. *Computing Surveys*, 12:437–465, 1980.

[9] J.C. Sabonnadiere and J.L. Coulomb. Finite element methods in CAD: electrical and magnetic fields. *Springer*, Berlin Heidelberg, 1987.