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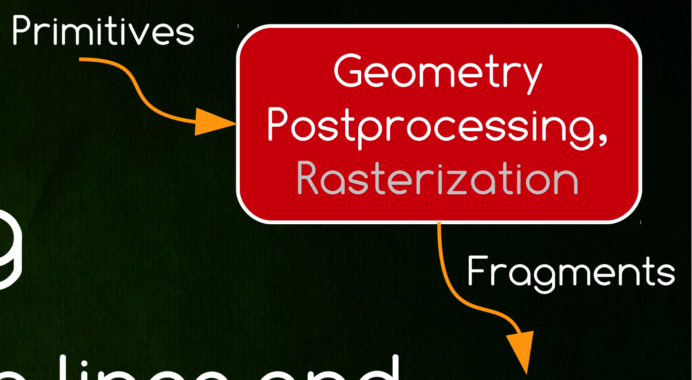
Clipping

Lesson 06

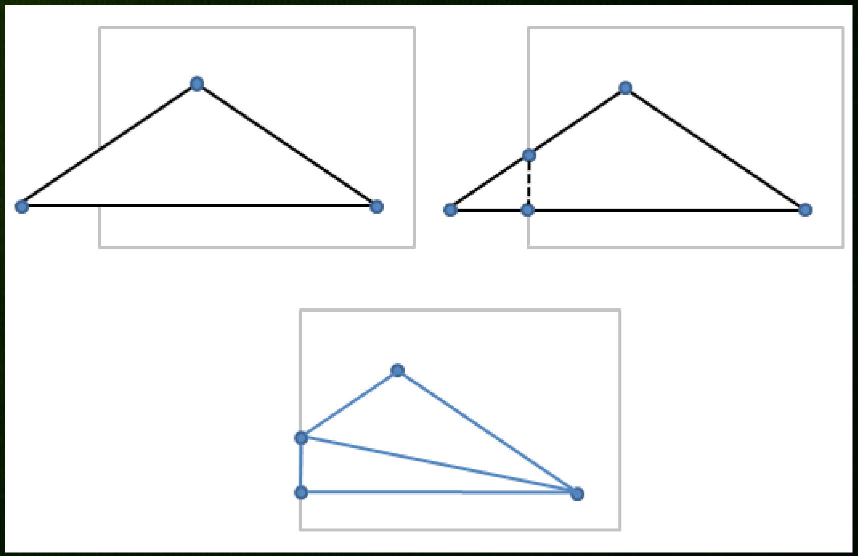
Outline of Lesson 06

- Line clipping algorithms in the CG Pipeline
- Cohen-Sutherland
- Cyrus-Beck
- Nicholl-Lee-Nicholl

The CG Pipeline
Geometry Postprocessing



- During geometry postprocessing lines and triangles are clipped against the window
  - We can not write outside the frame buffer
- Clipping should be
  - Fast for many primitives
  - Implemented on HW (GPU)

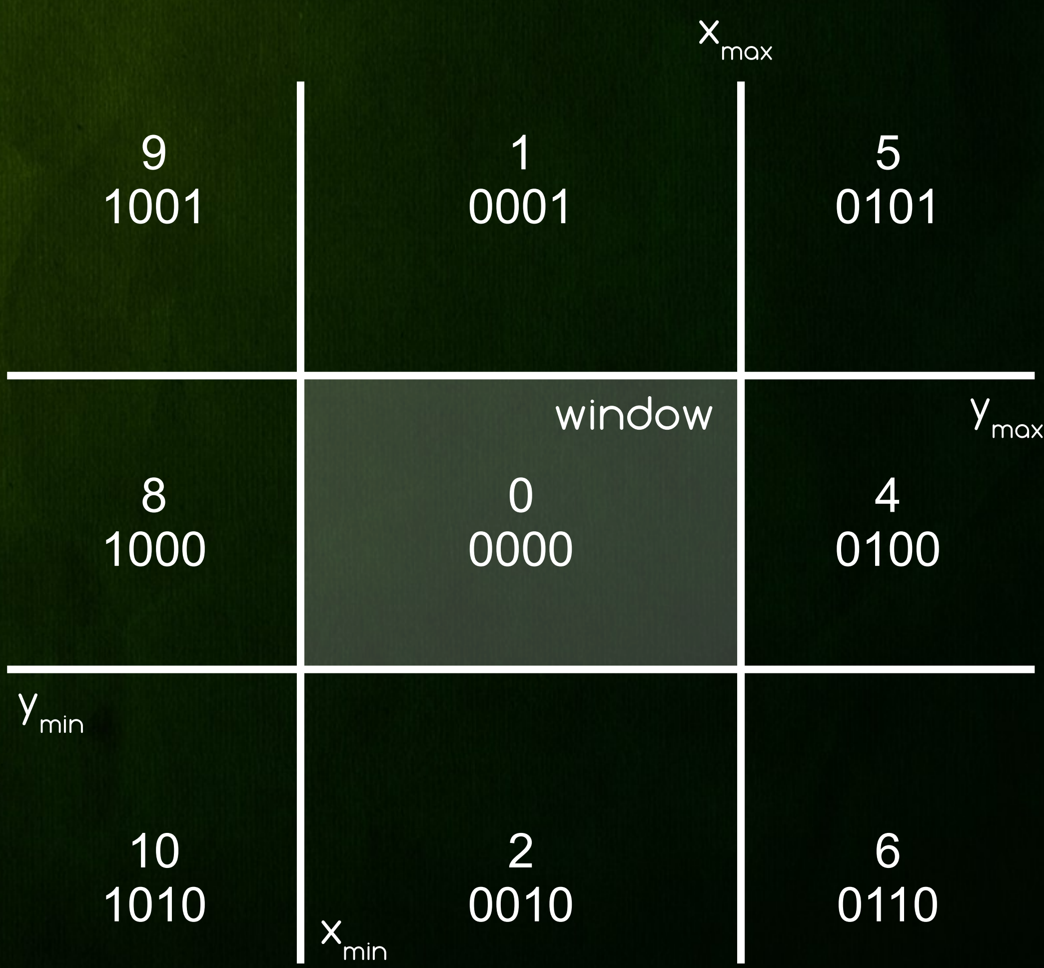


Cohen-Sutherland

- Main Purpose
  - Clipping lines against rectangular (axis aligned) 2D (3D) window
- Algorithm Principle
  - Divides a 2D (3D) space into 9 (27) regions
  - Efficiently determine the (portions of) lines that are visible in the window
  - Clip lines against window edges

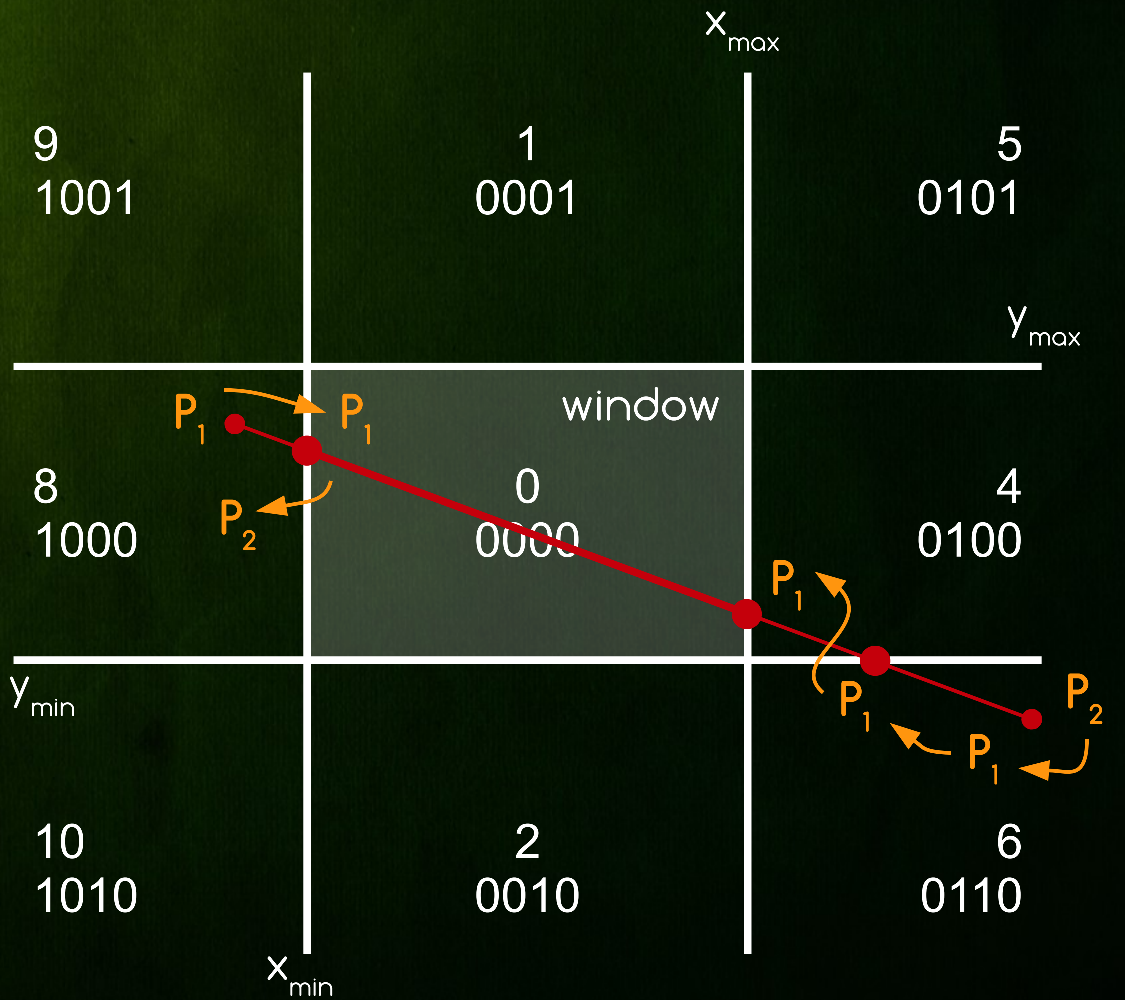
Cohen-Sutherland

- 9 codes (4bit) for each region: code = $b_3b_2b_1b_0$
- X cases
  - $b3 = (x < x_{min})\ ?\ 1 : 0$
  - $b2 = (x > x_{max})\ ?\ 1 : 0$
- Y Cases
  - $b1 = (y < y_{min})\ ?\ 1 : 0$
  - $b0 = (y > y_{max})\ ?\ 1 : 0$



Cohen-Sutherland

- Execution example
  - Clip P_1 against x_{min}
  - Swap P_1 and P_2
  - Clip P_1 against y_{min}
  - Clip P_1 against x_{max}
  - Done with P1P2



Cohen-Sutherland

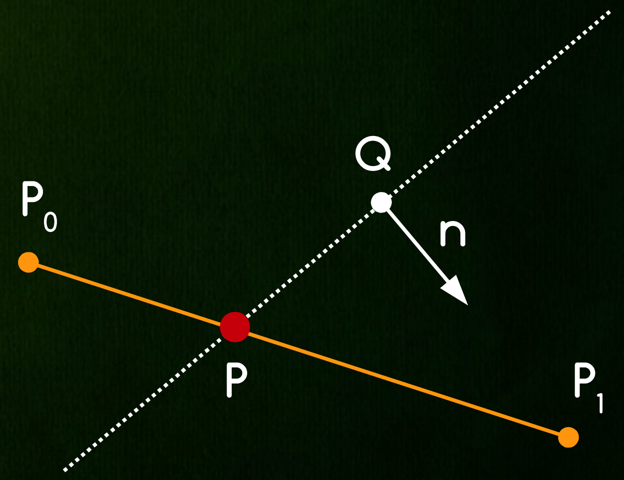
```
c2 = code(x2, y2);
while (false) {
    c1 = code(x1, y1);
    if (c1 & c2 != 0) return false;
    else if (c1 | c2 == 0) return true;
    else {
        if (c1 == 0) { swap(x1, x2); swap(y1, y2); swap(c1, c2); }
        else if (c1 ∈ {1, 5, 9} ) { x1 = x1 + (x2-x1) * (y_max-y1) / (y2-y1); y1 = y_max; }
        else if (c1 ∈ {2, 6, 10} ) { x1 = x1 + (x2-x1) * (y_min-y1) / (y2-y1); y1 = y_min; }
        else if (c1 ∈ {4, 5, 6} ) { y1 = y1 + (y2-y1) * (x_max-x1) / (x2-x1); x1 = x_max; }
        else if (c1 ∈ {8, 9, 10} ) { y1 = y1 + (y2-y1) * (x_min-x1) / (x2-x1); x1 = x_min; }
    }
}
```

Cyrus-Beck

- Main Purpose
  - Clipping lines against any convex polygon
- Algorithm Principle
  - Find line parameter of intersection with each edge of polygon
  - Update min and max line parameter to be inside the halfspace of each edge
  - If min < max calculate clipped line segment points

Cyrus-Beck

- Intersection of hyperplane and line segment
  - Hyperplane (origin O, normal n)
  - Line segment (start point P0, end point P1)
- P lies on line segment
  - P = P0 + t (P1 - P0) | 0 <= t <= 1
- P lies on hyperplane
  - (P - Q) *n = 0
- Solve t = (Q-P0) *n / (P1-P0) *n
  - dq = (Q-P0) *n | d1 = (P1-P0) *n → t = dq / d1

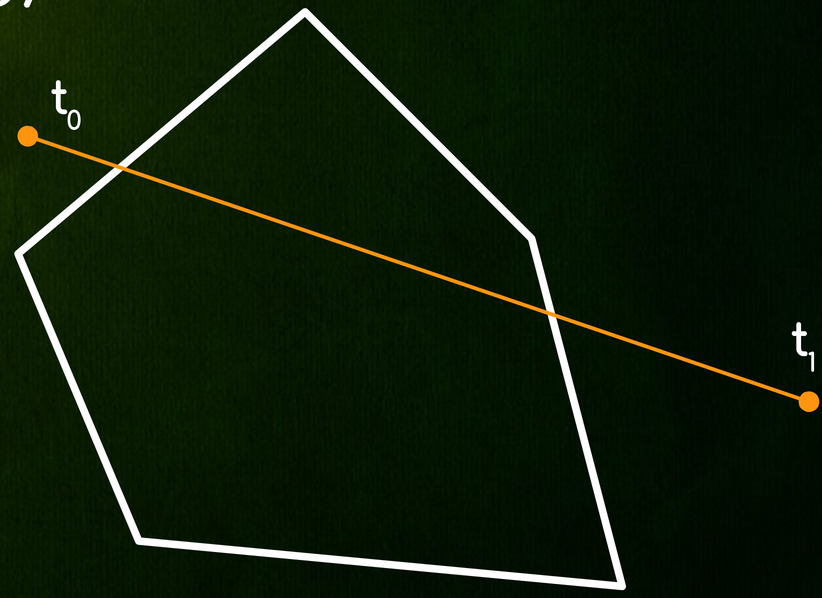


Cyrus-Beck

- Instead of calculating new intersected points Cyrus-Beck operates only on line parameters t0 and t1 - this is faster
- First set t0 = 0 and t1 = 1 (original line segment)
- For each edge find intersection parameter t and set
  - If (d1 > 0) t0 = max(t, t0) (out-to-in case)
  - If (d1 < 0) t1 = min(t,t1) (in-to-out case)
- This will find the smallest intersection interval
- At the end find new P0 and P1 for t0 and t1

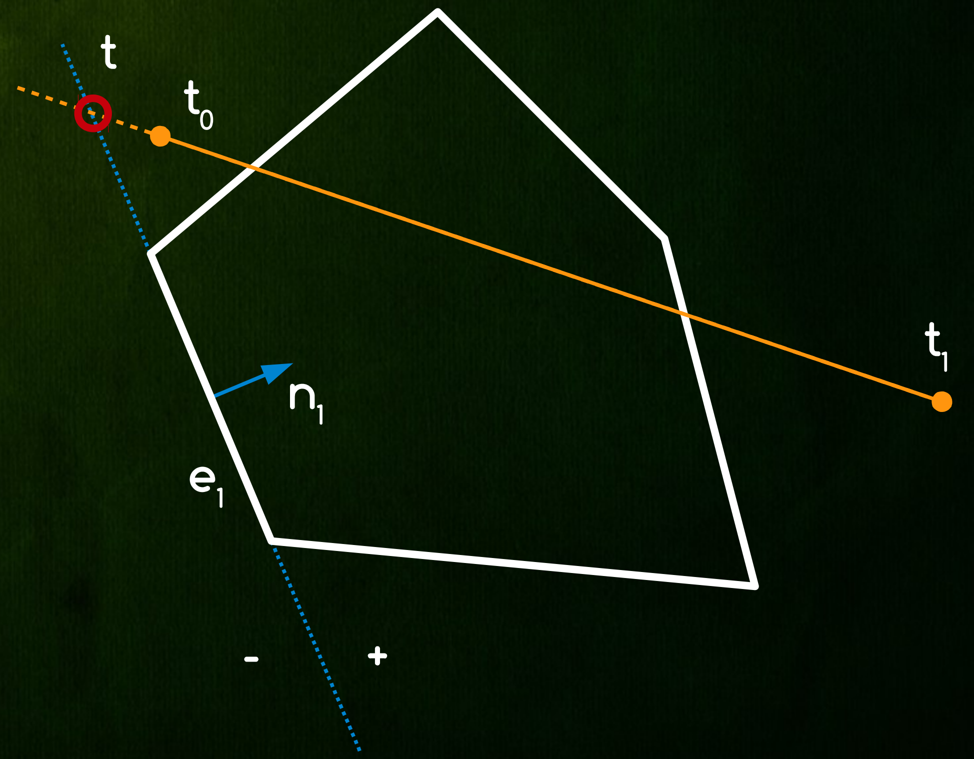
Cyrus-Beck

- Input: Convex polygon and line segment
- Output: Clipped line segment being fully inside given polygon (or nothing)
- Set clipping parameters
  - → t0 = 0, t1 = 1



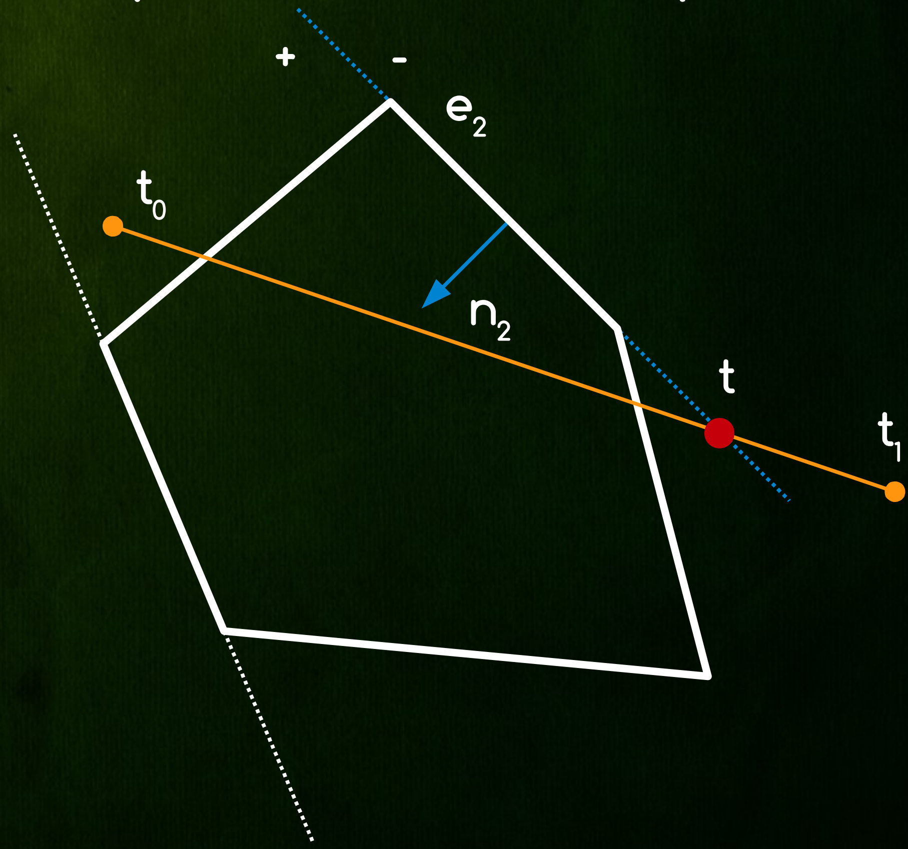
Cyrus-Beck

- Find intersection parameter t with edge e1
- d1 = (P1-P0) *n1 > 0 → clip t0 (out-to-in case)
- t0 = max(t,t0)
  - Since t < t0
  - No update is done



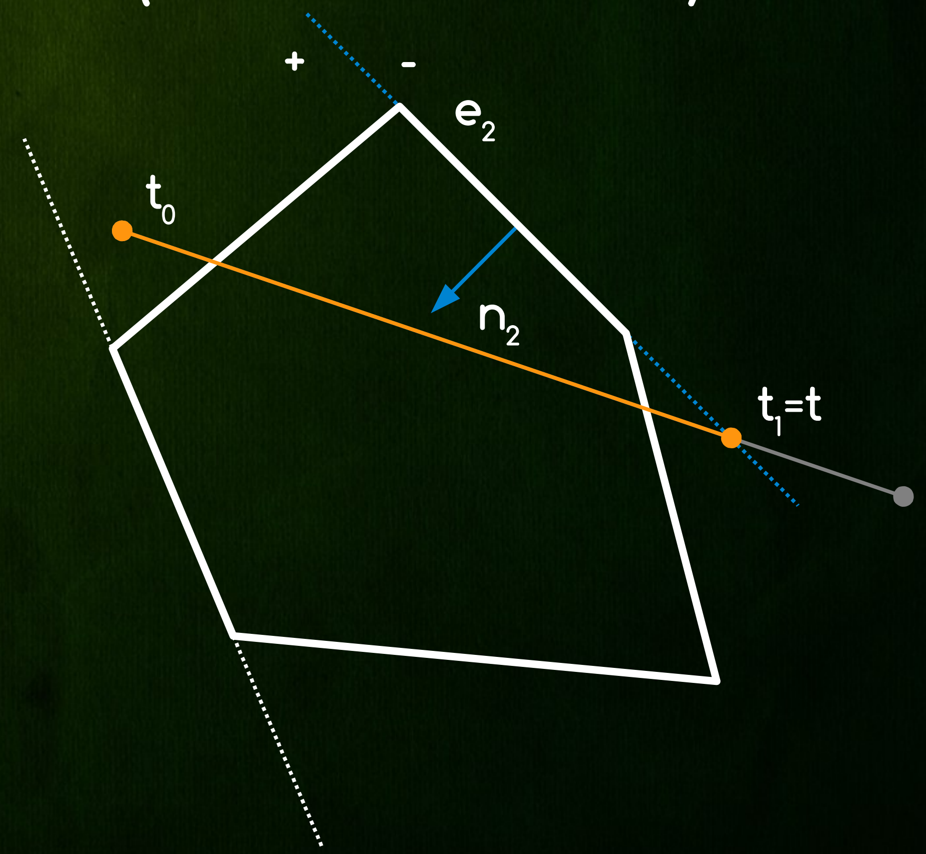
Cyrus-Beck

- Find intersection parameter t with edge e2
- d1 = (P1-P0) *n2 < 0 → clip t1 (in-to-out case)



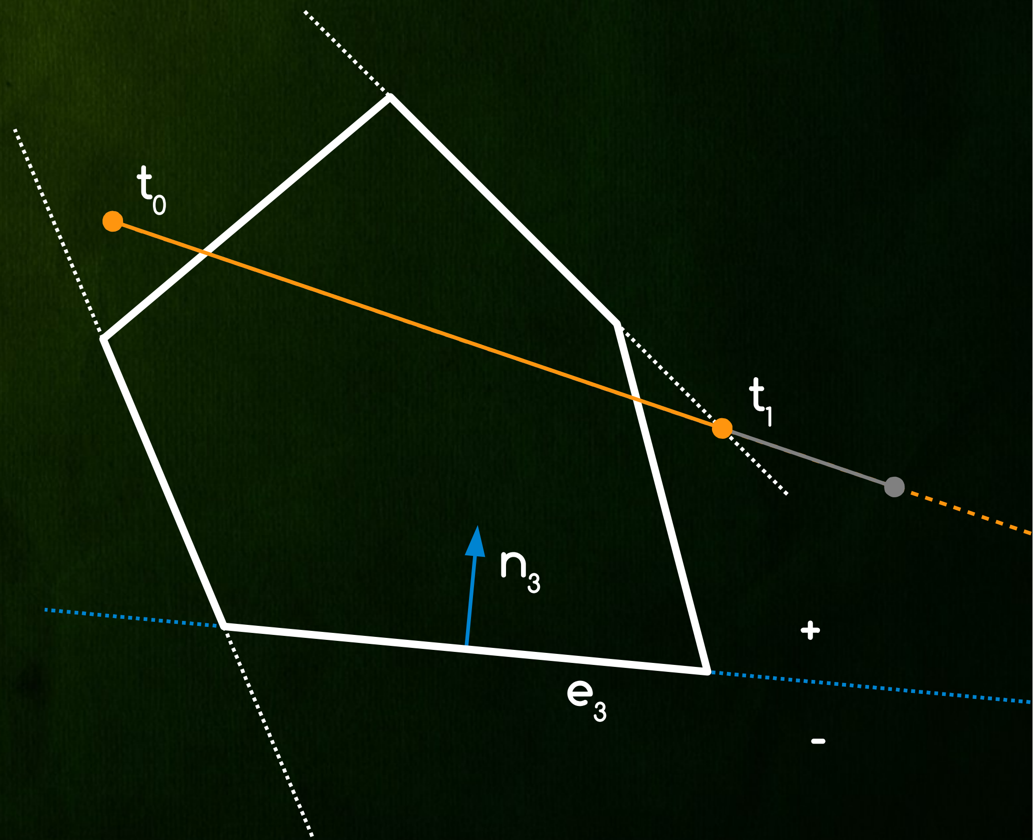
Cyrus-Beck

- Find intersection parameter t with edge e2
- d1 = (P1-P0) *n2 < 0 → clip t1 (in-to-out case)
- t1 = min(t,t1)
  - Since t < t1
  - We update t1 = t



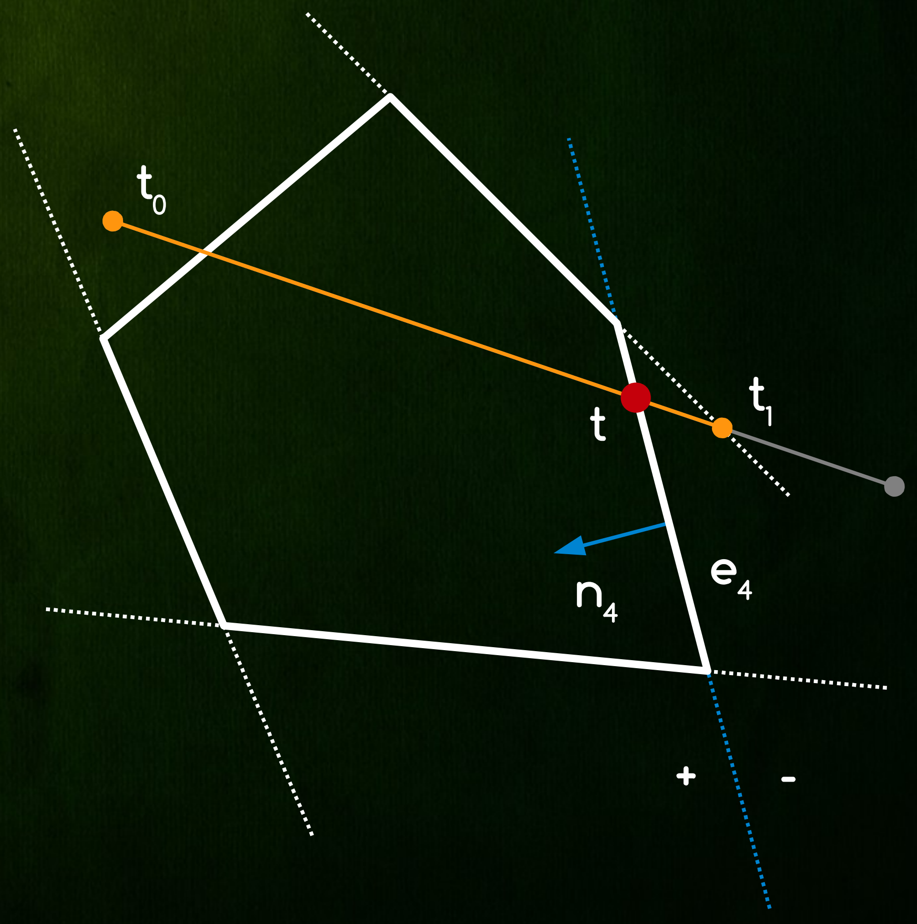
Liang-Barsky

- Find intersection parameter t with edge e3
- d1 = (P1-P0) *n3 < 0 → clip t1 (in-to-out case)
- t1 = min(t,t1)
  - Since t > t1
  - No update is done



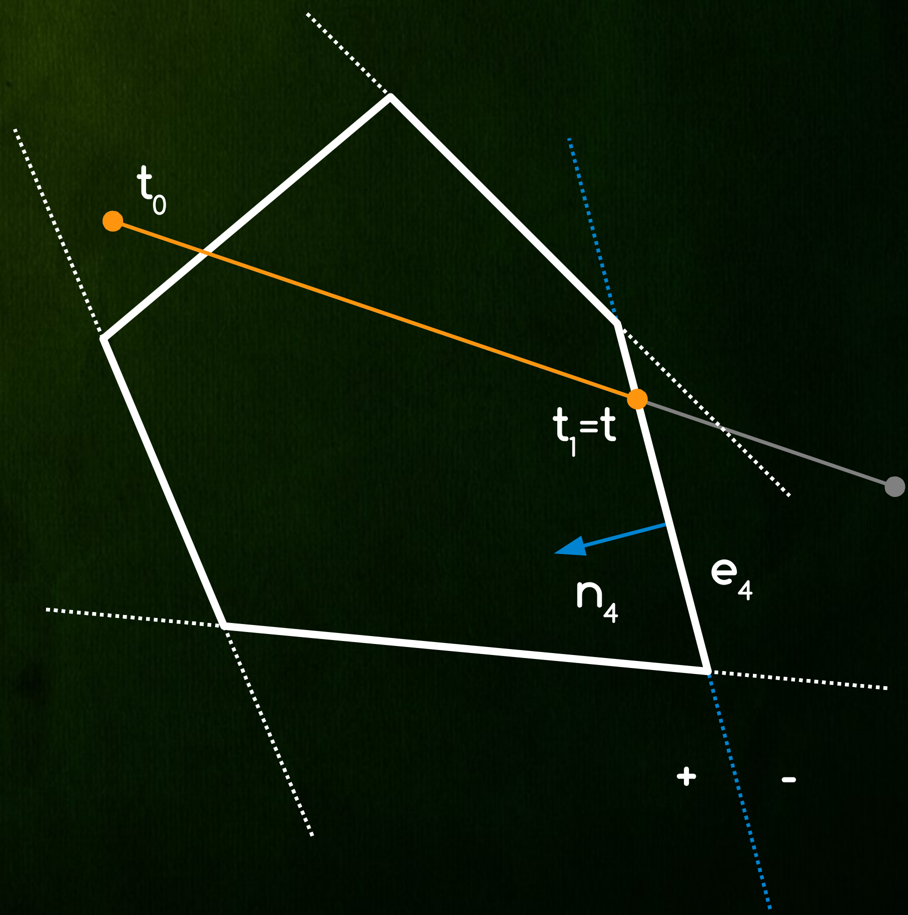
Cyrus-Beck

- Find intersection parameter t with edge e4
- d1 = (P1-P0) *n4 < 0 → clip t1 (in-to-out case)



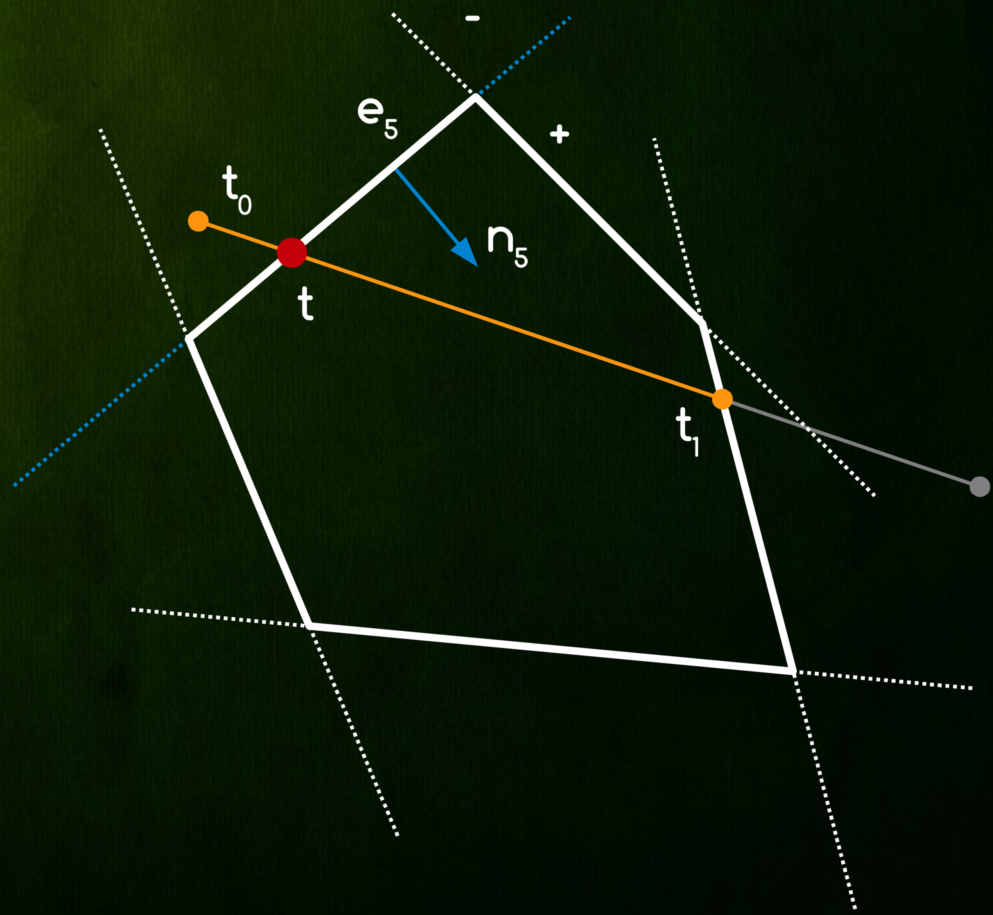
Cyrus-Beck

- Find intersection parameter t with edge e4
- d1 = (P1-P0) *n4 < 0 → clip t1 (in-to-out case)
- t1 = min(t,t1)
  - Since t < t1
  - We update t1 = t



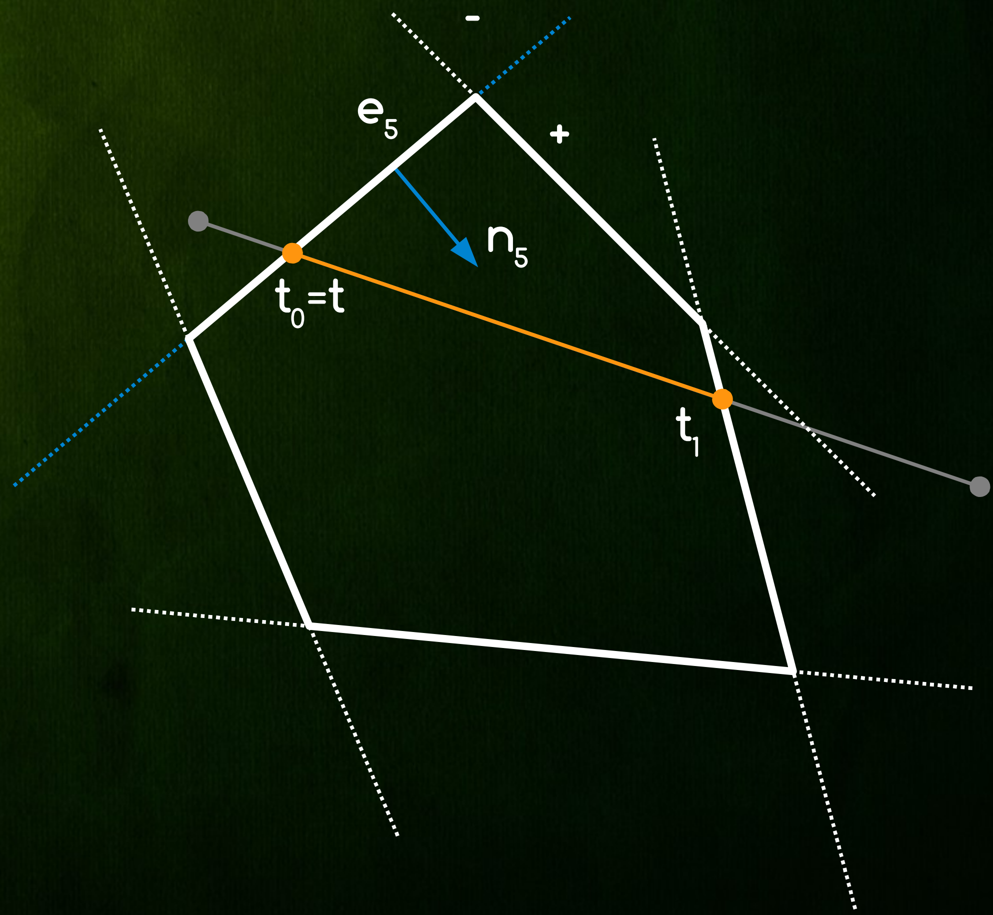
Cyrus-Beck

- Find intersection parameter t with edge e5
- d1 = (P1-P0) *n5 > 0 → clip t0 (out-to-in case)



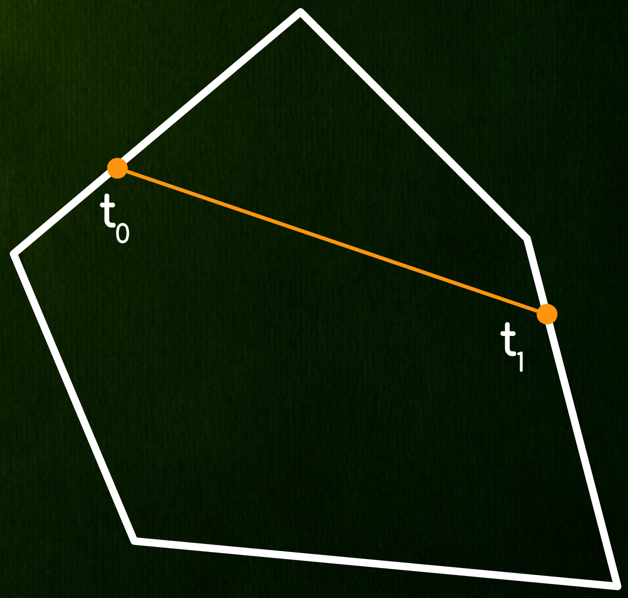
Cyrus-Beck

- Find intersection parameter t with edge e5
- d1 = (P1-P0) *n5 > 0 → clip t0 (out-to-in case)
- t0 = max(t,t0)
  - Since t > t0
  - We update t0 = t



Cyrus-Beck

- No more edges to update with
- If t0 > t1 whole line segment is outside of polygon
- If t0 <= t1 clip line
  - P0' = P0 + t0(P1-P0)
  - P1' = P0 + t1(P1-P0)



Cyrus-Beck

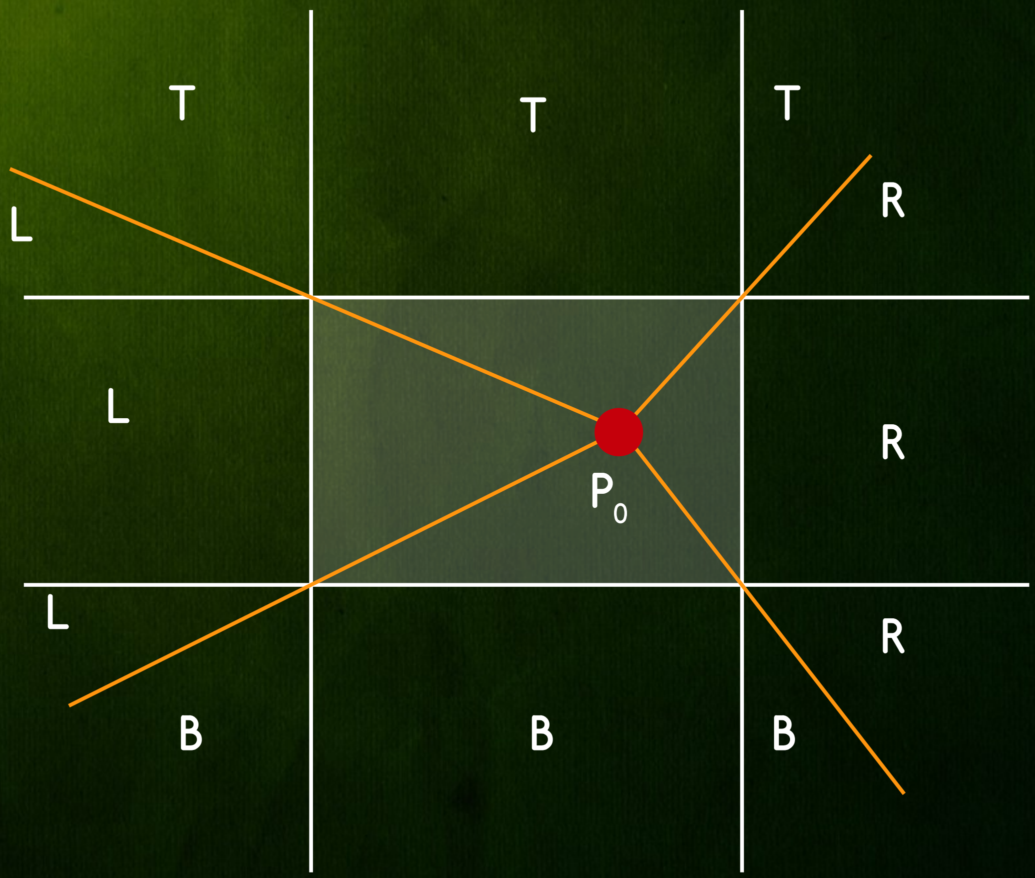
- $t_0 = 0; t_1 = 1;$
- foreach edge $e_i = (q_i, n_i)$ {
  - $d_1 = (p_1 - p_0) * n_i$; $d_q = (q_i - p_0) * n_i$;
  - if $(d_1 > 0)$ { $t = d_q / d_1$; $t_0 = \max(t, t_0)$; } else
  - if $(d_1 < 0)$ { $t = d_q / d_1$; $t_1 = \min(t, t_1)$; } else
  - if $((p_0 - q_i) * n_i < 0)$ return false;   *// line is outside of poly*
- }
- if $(t_0 < t_1)$ return true; else return false;

Nicholl-Lee-Nicholl

- Main Purpose
  - Clipping lines against rectangular (axis aligned) 2D only window
- Algorithm Principle
  - Categorize first point of line segment similarly to Cohen-Sutherland
  - Virtual cast 4 rays from P0 through 4 corners of window and categorize all regions between rays. In each segment we know which window edges we have to clip with
  - Clip line segment with selected edges

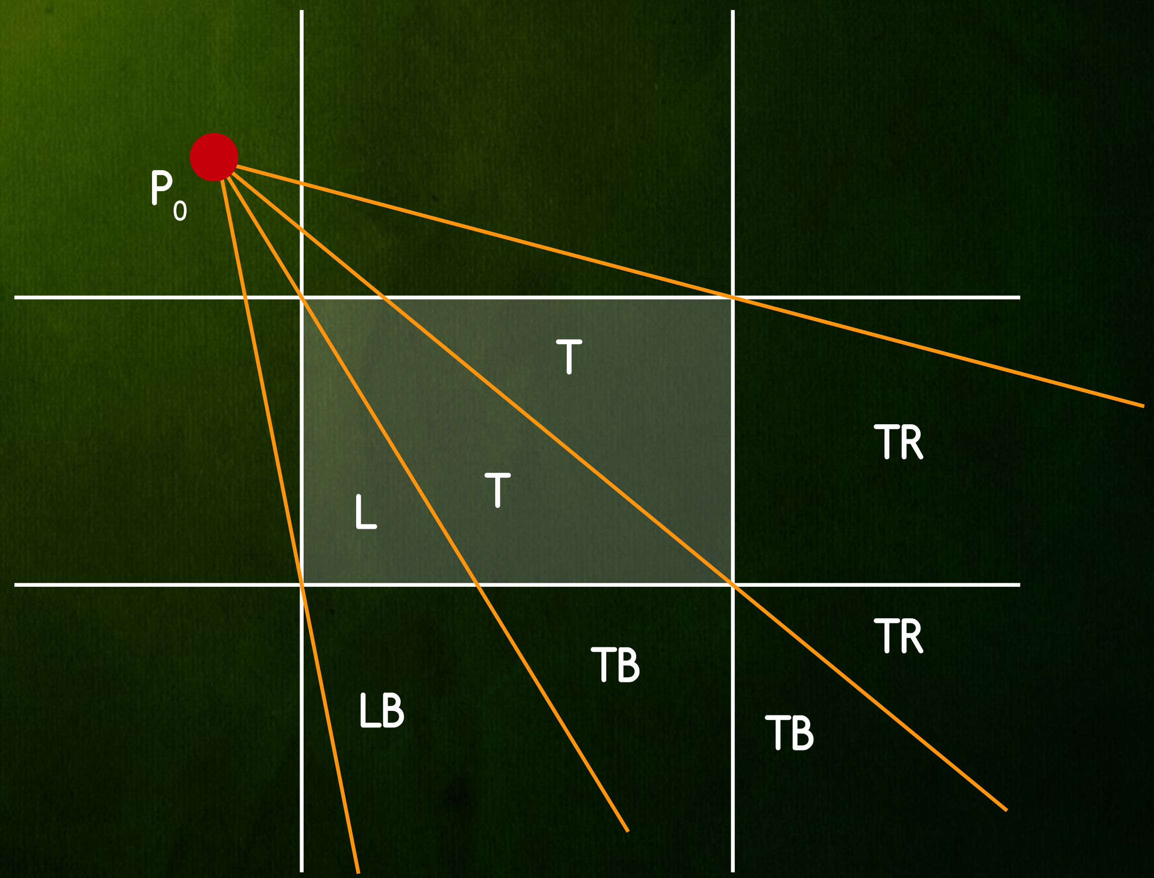
Nicholl-Lee-Nicholl

- Window region



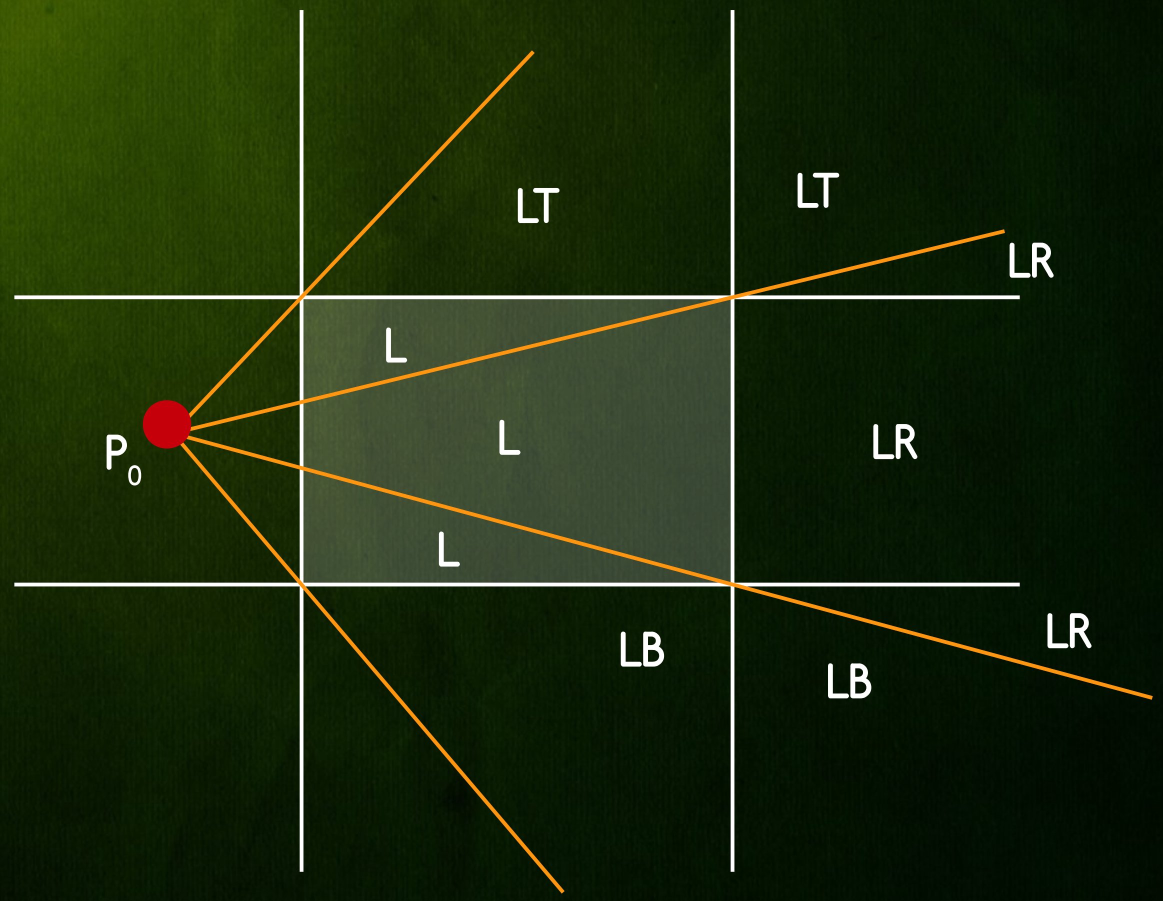
Nicholl-Lee-Nicholl

- Corner region



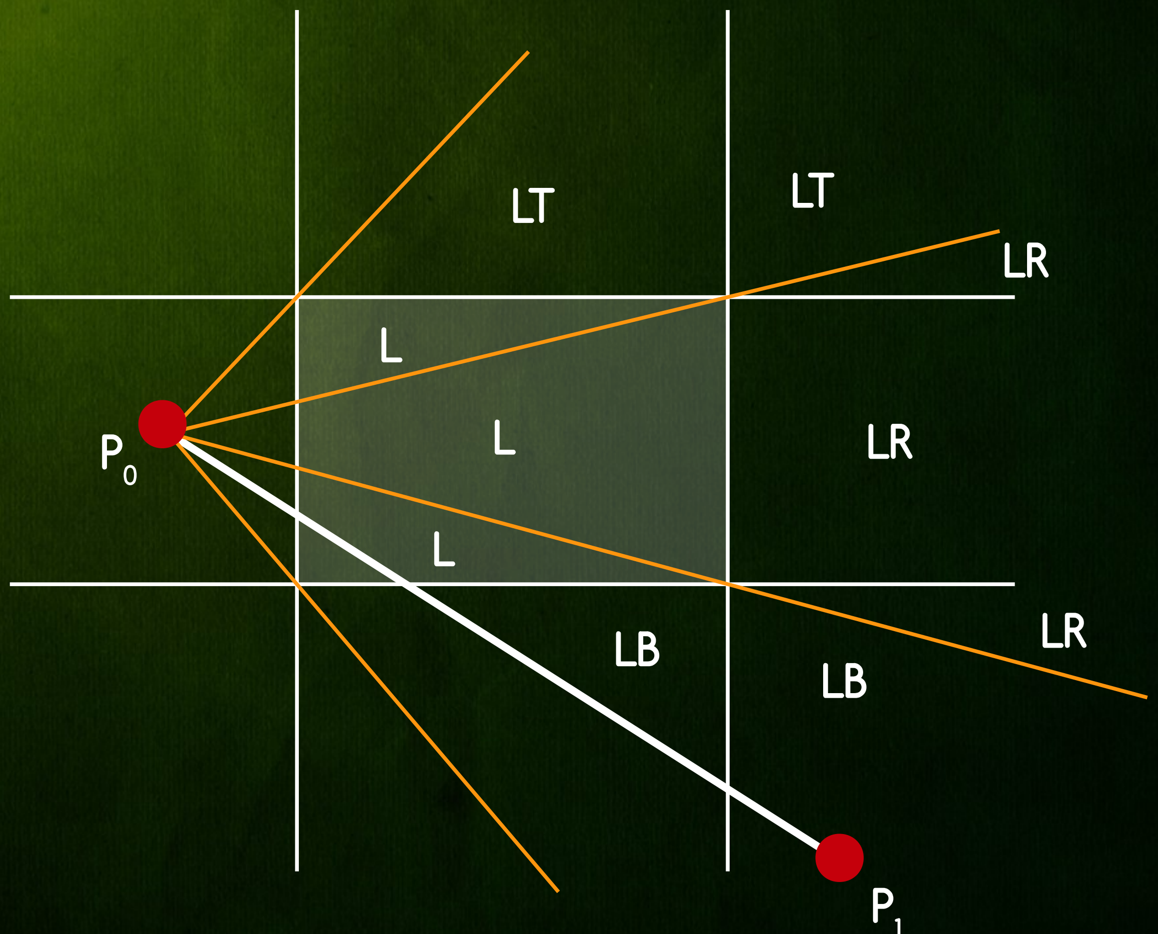
Nicholl-Lee-Nicholl

- Edge region



Nicholl-Lee-Nicholl

- Edge region Example



Nicholl-Lee-Nicholl

```
procedure LeftEdgeRegionCase (ref real x1, y1, x2, y2; ref boolean visible)
begin
    real dx, dy;

    if  x2 < xmin
        then  visible := false
        else  if  y2 < ymin
            then  LeftBottom (xmin,ymin,xmax,ymax,x1,y1,x2,y2,visible)
            else  if  y2 > ymax
                then
                    begin
                        { Use symmetry to reduce to LeftBottom case }
                        y1 := –y1;   y2 := –y2;   { reflect about x-axis }
                        LeftBottom (xmin,–ymax,xmax, –ymin,x1,y1,x2,y2,visible);
                        y1 := –y1;   y2 := –y2;   { reflect back }
                    end
                else
                    begin
                        dx := x2 – x1;   dy := y2 – y1;
                        if  x2 > xmax  then
                            begin
                                y2 := y1 + dy*(xmax – x1)/dx;    x2 := xmax;
                            end;
                        y1 := y1 + dy*(xmin – x1)/dx;    x1 := xmin;
                        visible := true;
                    end
end;
```

Nicholl-Lee-Nicholl

```
procedure LeftBottom (      real  xmin, ymin, xmax, ymax;
                        ref real  x1, y1, x2, y2; ref boolean  visible)
begin
    real dx, dy, a, b, c;

    dx := x2 – x1;              dy := y2 – y1;
    a   := (xmin – x1)*dy;   b   := (ymin – y1)*dx;
    if  b > a
        then  visible := false  { (x2,y2) is below ray from (x1,y1) to bottom left corner }
        else
            begin
                visible := true;
                if  x2 < xmax
                    then
                        begin   x2 := x1 + b/dy;    y2 := ymin;    end
                    else
                        begin
                            c := (xmax – x1)*dy;
                            if  b > c
                                then   { (x2,y2) is between rays from (x1,y1) to
                                         bottom left and right corner }
                                    begin   x2 := x1 + b/dy;    y2 := ymin;    end
                                else
                                    begin   y2 := y1 + c/dx;    x2 := xmax;   end
                        end;
            end;
    y1 := y1 + a/dx;   x1 := xmin;
end;
```

Clipping Algorithms Summary

- Cohen-Sutherland
  - Repeated clipping is expensive
  - Best when trivial accepts/rejects occur often
- Cyrus-Beck
  - Cheap intersection parameter calculation
  - Points are clipped only once at the and
  - Best when most lines have to be clipped
- Liang-Barsky - optimized Cyrus-Beck for window
- Nicholl et. al. - Fastest, not applicable in 3D


The End