Rational Bezier solids

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Abstract

In this short paper we will focus on definition of rational Bezier solids and also we present algorithm for visualization of rational Bezier solids. These solids can be defined with recursive de Casteljau algorithm or by analytical expression. For visualization purposes we used net of points and described approximations of rational Bezier solids as these nets. We will also present practical output of our visualization algorithm.

Keywords: Bezier solids, rational, point net, visualization

1 Introduction

We know two basic types of Bezier solids, tetrahedral and tensor. In this paper we will focus on generalized set of Bezier solids called rational Bezier solids. These can be defined in two ways, using analytical expression or with recursive formula called de Casteljau algorithm. These solids are given by control net of vertexes and they can be manipulated using only these vertexes. Here we introduce these solids and describe one way how to visualize them.

2 Rational Bezier tetrahedras

Bezier tetrahedra is defined with degree, domain, control net of points and for each point real number (weight). Degree is an positive integer number, domain is non-degenerated tetrahedron $ABCD$ in E^3 and control net with weights is tetrahedral structure of points in E^3, that can be written following way:

$$V_{\mathbf{i}} \in E^3; w_{\mathbf{i}} \in R$$

$$\mathbf{i} = (i, j, k, l); |\, \mathbf{i} \,| = i + j + k + l = n;\ i, j, k, l \geq 0$$

where n is degree of rational Bezier tetrahedra. Now we can define point of this solid $RB^n(\mathbf{u})$with recursive de Casteljau algorithm:

$$V_{\mathbf{i}}^0(\mathbf{u}) = V_{\mathbf{i}}; w_{\mathbf{i}}^0(\mathbf{u}) = w_{\mathbf{i}}$$

$$w_{\mathbf{i}}^r(\mathbf{u}) = uw_{\mathbf{i}-\mathbf{e_1}}^{r-1}(\mathbf{u}) + vw_{\mathbf{i}-\mathbf{e_2}}^{r-1}(\mathbf{u}) + ww_{\mathbf{i}-\mathbf{e_3}}^{r-1}(\mathbf{u}) + tw_{\mathbf{i}-\mathbf{e_4}}^{r-1}(\mathbf{u})$$

$$w_{\mathbf{i}}^r(\mathbf{u})V_{\mathbf{i}}^r(\mathbf{u}) = uw_{\mathbf{i}-\mathbf{e_1}}^{r-1}(\mathbf{u})V_{\mathbf{i}-\mathbf{e_1}}^{r-1}(\mathbf{u}) + vw_{\mathbf{i}-\mathbf{e_2}}^{r-1}(\mathbf{u})V_{\mathbf{i}-\mathbf{e_2}}^{r-1}(\mathbf{u}) +$$

$$ww_{\mathbf{i}-\mathbf{e_3}}^{r-1}(\mathbf{u})V_{\mathbf{i}-\mathbf{e_3}}^{r-1}(\mathbf{u}) + tw_{\mathbf{i}-\mathbf{e_4}}^{r-1}(\mathbf{u})V_{\mathbf{i}-\mathbf{e_4}}^{r-1}(\mathbf{u})$$

$$B^n(\mathbf{u}) = V_{\mathbf{0}}^n(\mathbf{u})$$

where $r = 1, \ldots, n; |\, \mathbf{i} \,| = n - r$, $\mathbf{u} = (u, v, w, t); u + v + w + t = 1$ are barycentric coordinates of some point U from domain $(U = uA + vB + wC + tD)$ and $\mathbf{e_1} = (1,0,0,0)$,$\mathbf{e_2} = (0,1,0,0)$,$\mathbf{e_3} = (0,0,1,0)$,$\mathbf{e_4} = (0,0,0,1)$. From this definition the analytical expression can be evaluated. So for barycentric coordinates $\mathbf{u}$ of any point U from domain we have:

$$B^n(\mathbf{u}) = \frac{\sum_{|\mathbf{i}|=n} w_{\mathbf{i}} V_{\mathbf{i}} B_{\mathbf{i}}^n(\mathbf{u})}{\sum_{|\mathbf{i}|=n} w_{\mathbf{i}} B_{\mathbf{i}}^n(\mathbf{u})}$$

where $B_{\mathbf{i}}^n(\mathbf{u}) = \frac{n!}{i!j!k!l!} u^i v^j w^k t^l$ are generalized Bernstein polynomials.

3 Rational Bezier tensor solids

Bezier tensor solid is defined with three degrees and control net of points and for each point real number (weight). Degrees are an positive integer numbers, domain is non-degenerated box $ABCDEFGH$ in E^3 and control net with weights is box structure of points in E^3, that can be written following way:

$$V_{(i,j,k)} \in E^3; w_{(i,j,k)} \in R$$

$$0 \leq i \leq n, 0 \leq j \leq m, 0 \leq k \leq o$$

where n, m, o are degrees of rational Bezier tensor solid. Now we can define point of this solid $RB^{n,m,o}(u,v,w)$ with analytical expression:

$$B^{n,m,o}(u,v,w) =$$

$$\frac{\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{o} w_{(i,j,k)} V_{(i,j,k)} B_i^n(u) B_i^m(v) B_i^o(w)}{\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{o} w_{(i,j,k)} B_i^n(u) B_i^m(v) B_i^o(w)}$$

where $0 \leq u,v,w \leq 1$ and $B_i^n(u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-i}$ are Bernstein polynomials.
This type of solid also posses properties like Bezier tetrahedra, also there exists Casteljau algorithm for it, but it is less generalized.

4 Point nets

For purpose of visualization of Bezier solids we use data structure of points in E^3, with included connection information between points and weights for points. We used two basic types of solid nets:

- tetrahedral point net, it can be written in the form $V_{\mathbf{i}} \in E^3; w_{\mathbf{i}} \in R \mathbf{i} = (i,j,k,l); |\ \mathbf{i}\ | = i+j+k+l = n;\ i,j,k,l \geq 0$.
- box point net, it can be written in the form $V_{i,j,k} \in E^3; w_{i,j,k} \in R 0 \leq i \leq n, 0 \leq j \leq m, 0 \leq k \leq o$.

In each type connection between points is established only if these two points are neighbors in data structure. We can see that with these structures we can describe control nets of rational Bezier solids. For visualization we use three modes: visualization of vertexes, visualization of edges as connection between points and visualization of faces as triangles with as three connected points.

5 Visualization

There are several ways to approximate Bezier solids using point nets. In our approach we sampled domain of solids and for each sampled value we computed point on Bezier solid and inserted it into solid net based on topological information from structure of sampled points in domain. We can then visualize resulting solid net and get visualization of rational Bezier solid approximation.
Mentioned algorithm was implemented using Visual Studio .Net environment and OpenGL library. Below are examples of solids rendered this way.

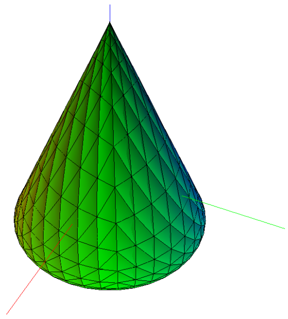

Figure 1: Cone as rational Bezier tetrahedron.

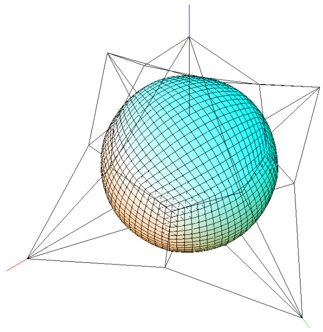

Figure 2: Sphere as Rational Bezier tensor solid with control net.

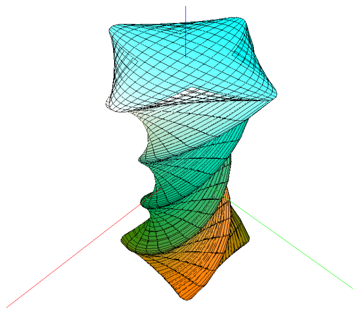

Figure 3: Twisted rational Bezier tensor solid.

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