Fractals

Part 2 : More Classical fractals

Martin Samuelčík
Department of Applied Informatics

Self-similarity

- Similar transformations
- Decimal numbers
- Geometric series
- There is transformation from part of object to whole object
- Statistical self-similarity
- Extended to affine transformations

$$A = \bigcup_{i=1}^{n} \phi_i(A)$$

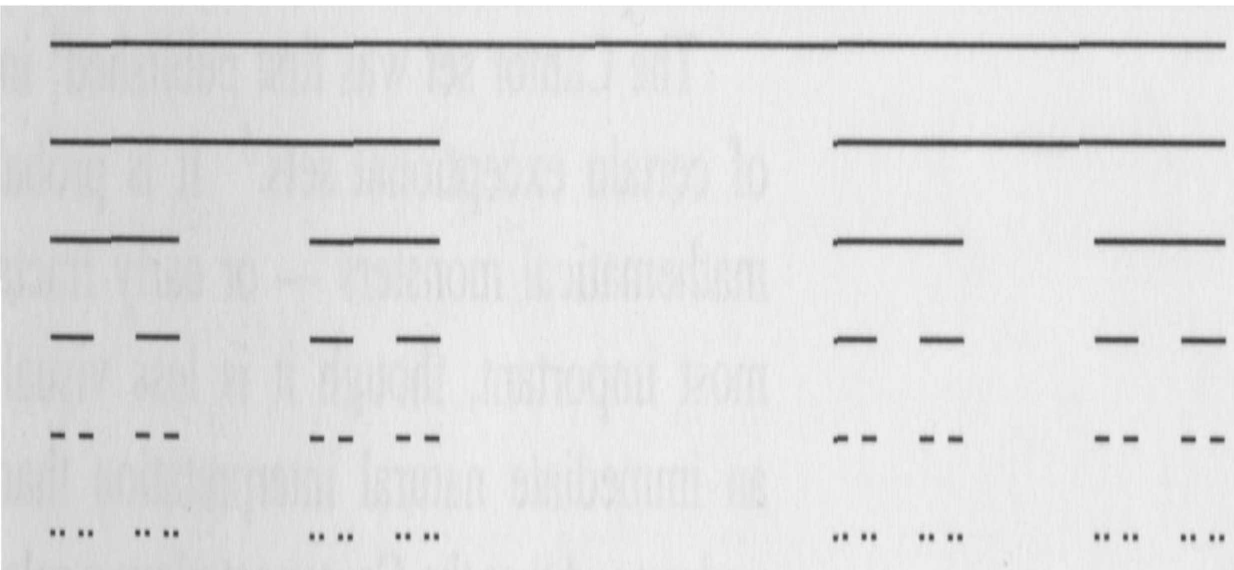
Classical fractals

- Deterministic
- Self-similarity
- Approx. 100 years old
- Good for modeling
- Worst approximation of natural objects
- Monsters

Cantor's set

- Georg Cantor, set theory
- Initiator
- Generator
- Base for model other fractals
- Chaotic dynamical systems

Cantor’s set 2



Triadic conversion of CS

- Triadic numbers
- X in [0,1]
- $X = a_1 * 3^{-1} + a_2 * 3^{-2} + a_3 * 3^{-3} + \ldots$
- $a_i = 0,1,2$
- Triadic expansion does not contain ‘1’
- Addressing system

Properties of CS

- End points of intervals are part of CS
- Cardinality is equal to [0,1]
- Every point is accumulation point of set – perfect set
- Uncountable
- Length of 0
- None point is interior point
- D=log(2)/log(3) =0,6309
- Complete metric space
- Compact

$$\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+\frac{8}{81}+\cdots=\frac{1}{3}\sum_{n=0}^{\infty}\frac{2^n}{3^n}$$

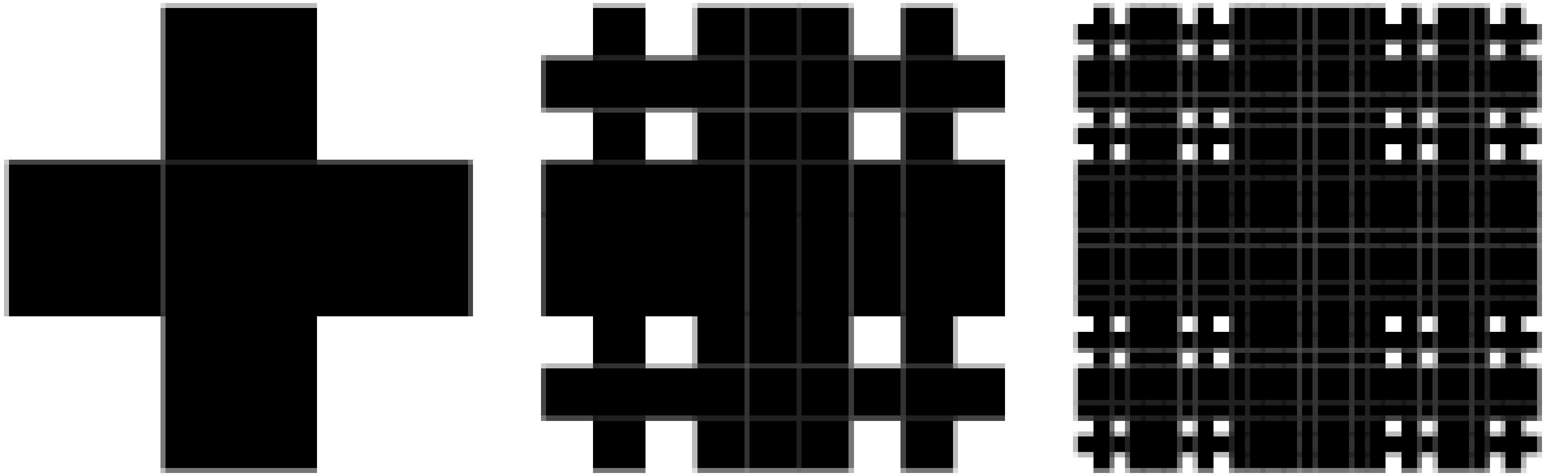
$$=\frac{1}{3}\left(\frac{1}{1-\frac{2}{3}}\right)$$

$$=\frac{1}{3}\times 3=1$$

Cantor Square Fractal

- Dimension 2
- Not a fractal

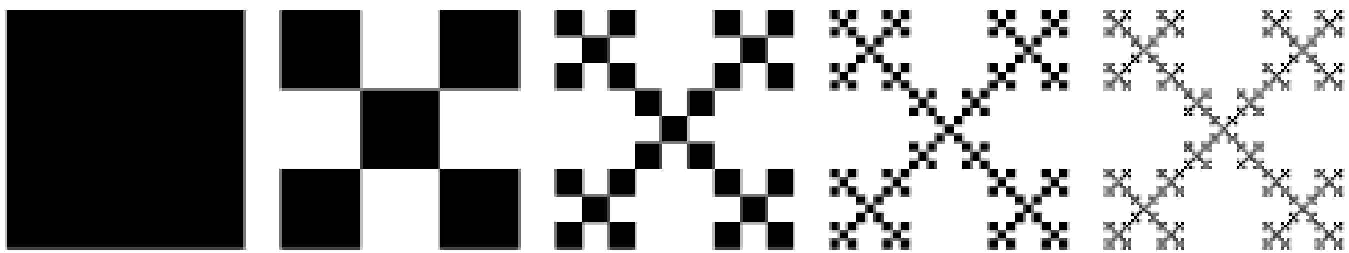
$$\left\{0 \to \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}, 1 \to \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\right\}.$$

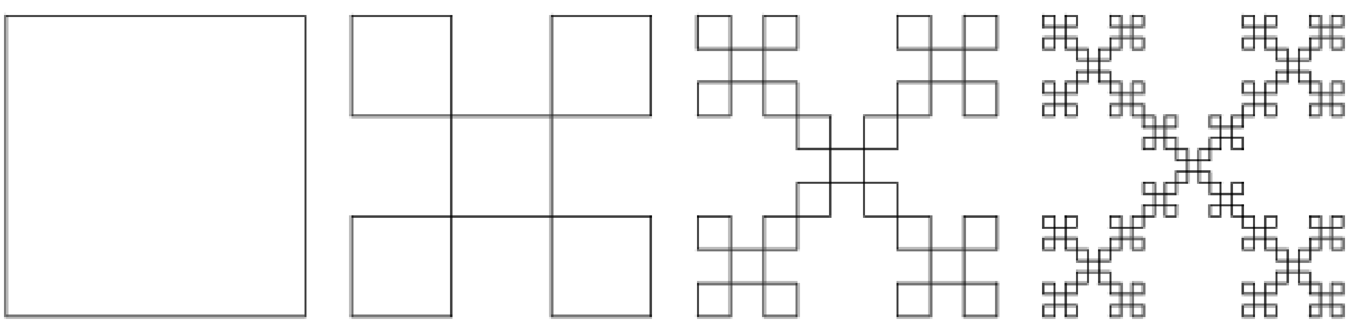


Box Fractal

- D = log(5)/log(3)=1.464973521…

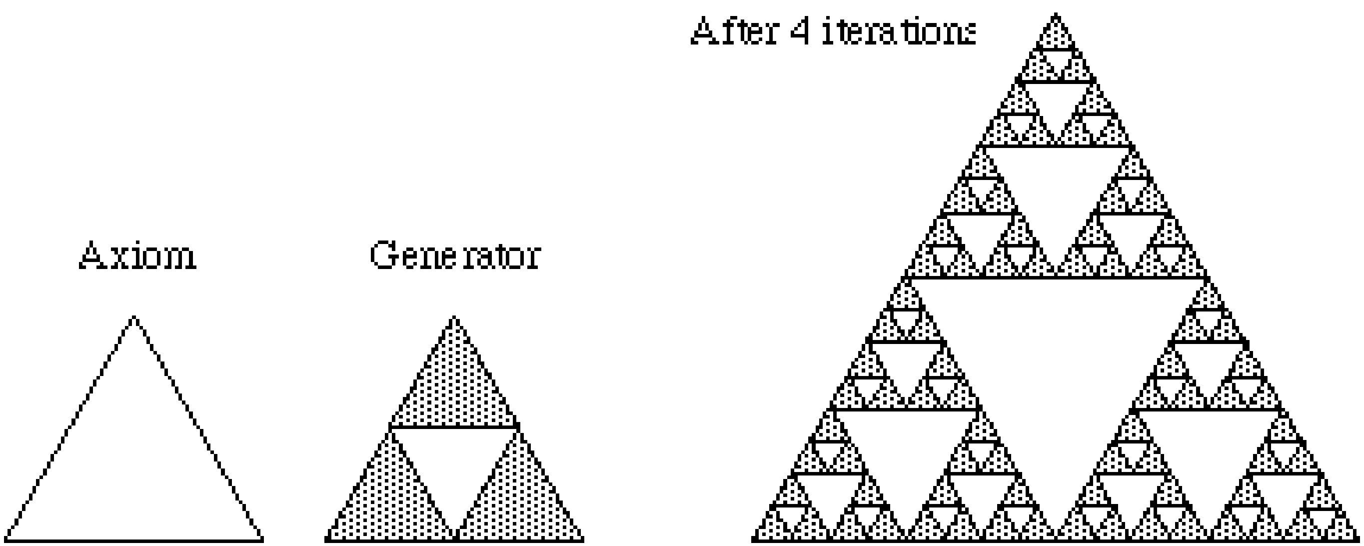
$$\left\{0 \to \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, 1 \to \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}\right\}.$$





Sierpinski gasket

- Waclaw Sierpinski
- Moon crater
- Definition with points, curves, areas



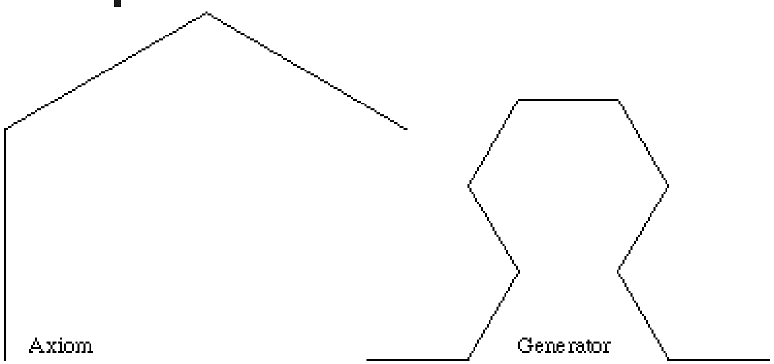
Sierpinski gasket 2

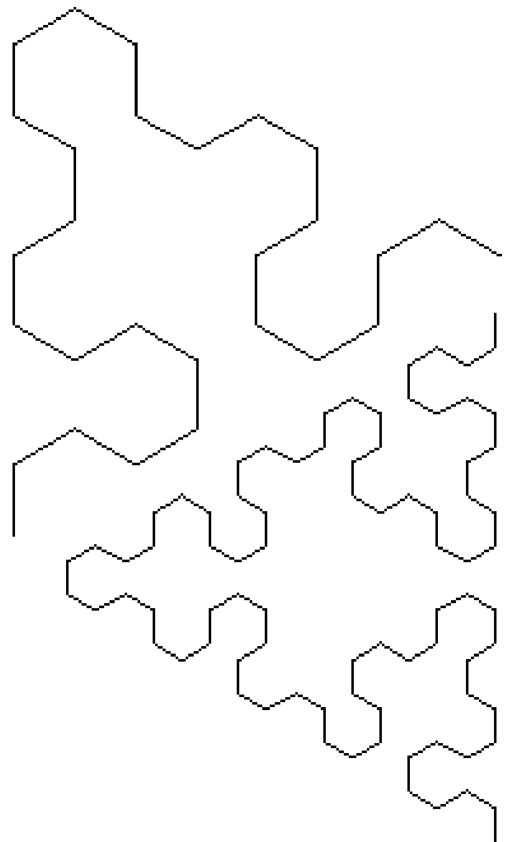
- D=log(3)/log(2)
- Length of border = infinite
- Area (total is 0)

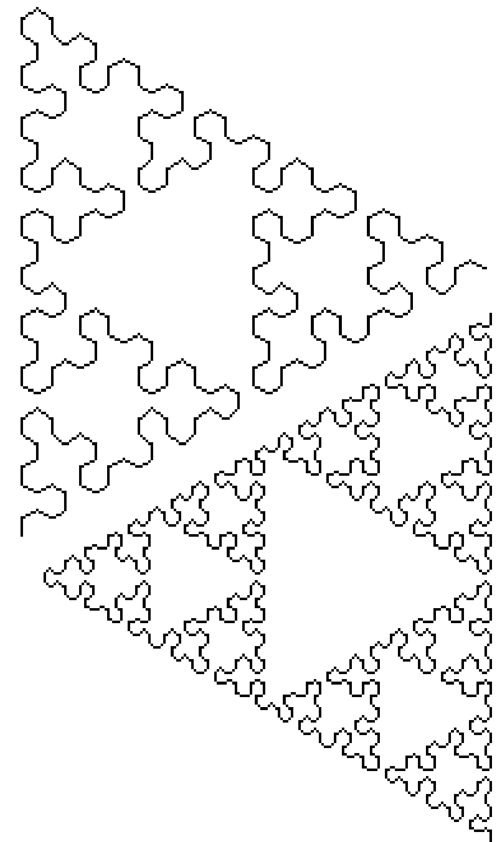
$$A_N = \frac{1}{3}\sum_{i=1}^{N}\left(\frac{3}{4}\right)^i \qquad A_\infty = 1$$

- Addressing system (L,R,T)

Sierpinski gasket 3

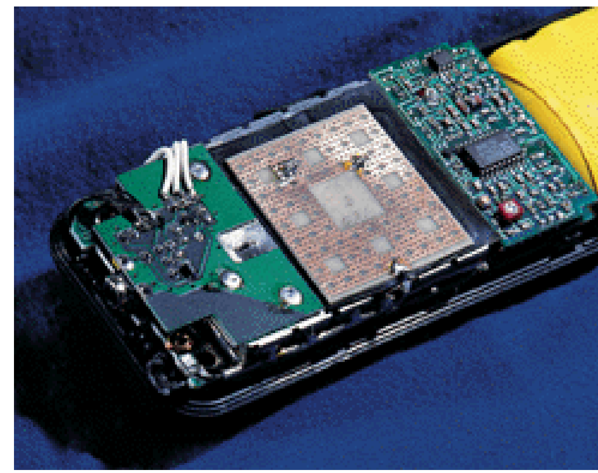


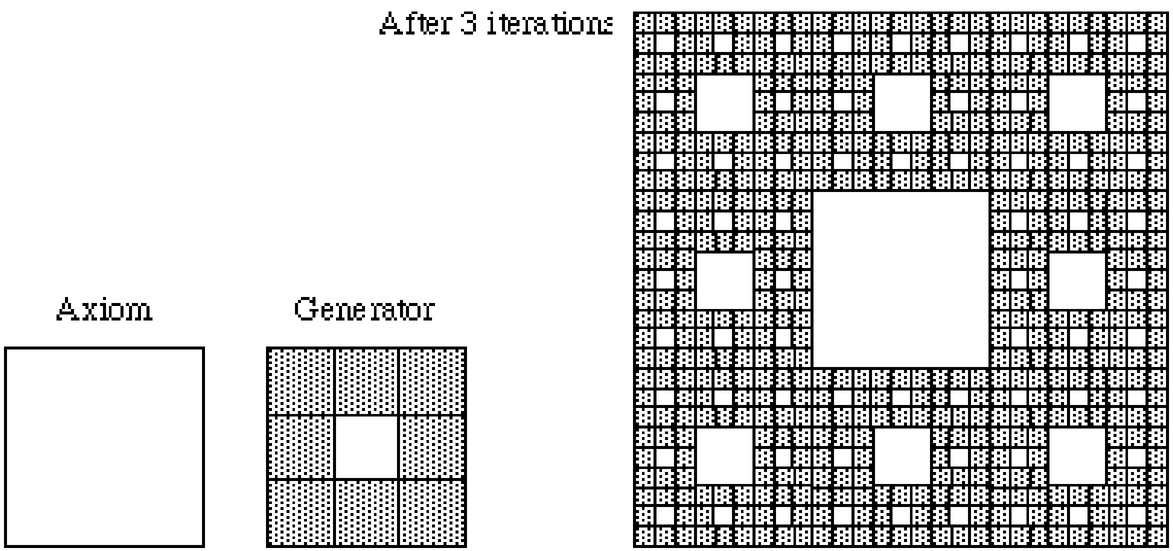




Sierpinski carpet

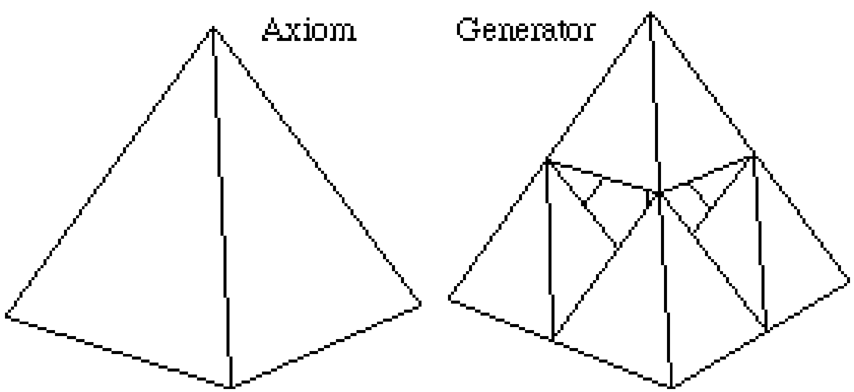
- Subdividing square
- D=log(8)/log(3)
- Area=0

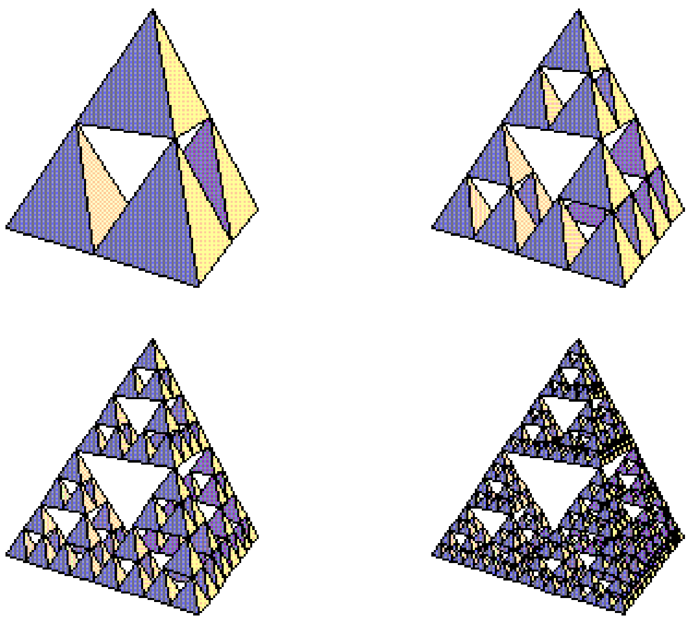




Sierpinski in 3D

- Based on tetrahedron,pyramid
- D=2, D=log(5)/log(2)
- Volume=0

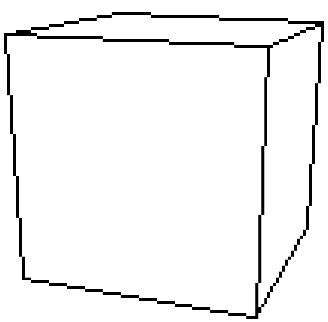




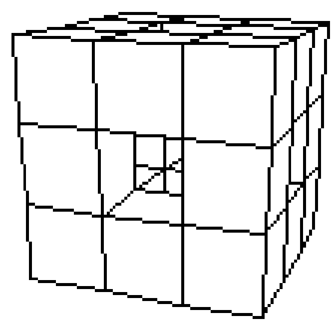
Sierpinski in 3D

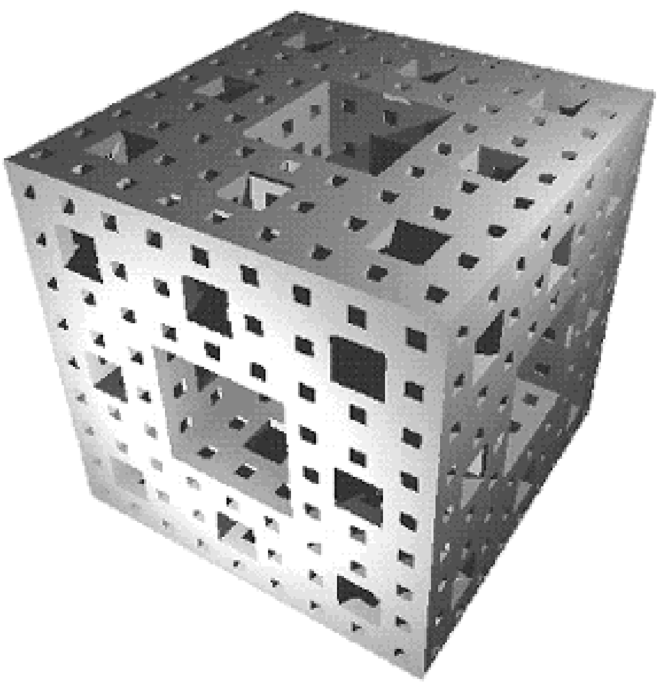
- Menger sponge
- D=log(20)/log(3) = 2.7268

Axiom

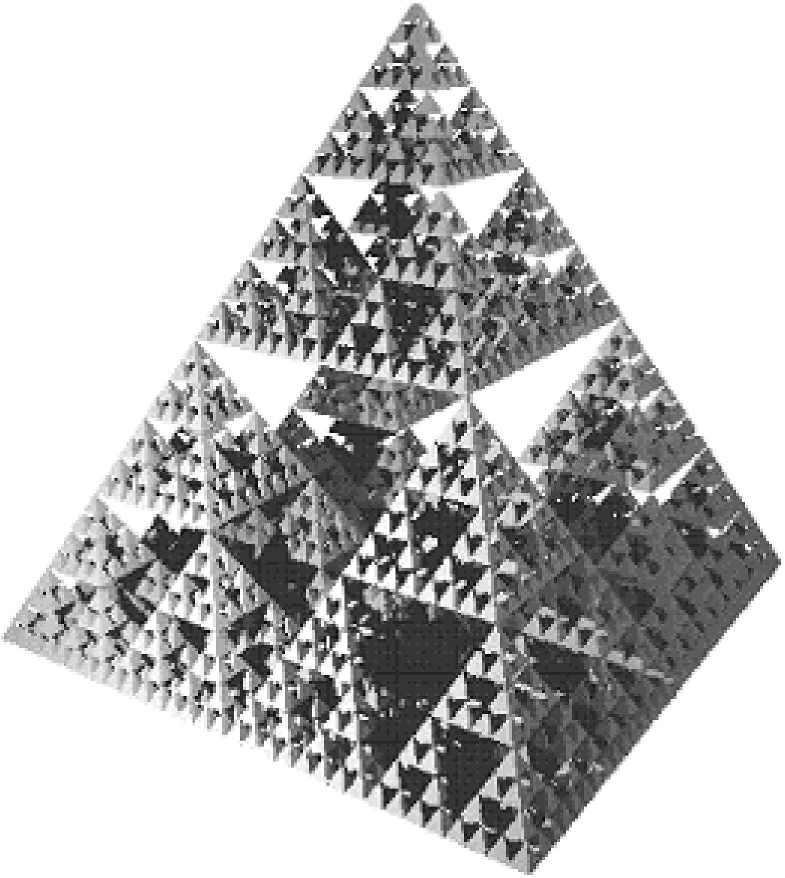


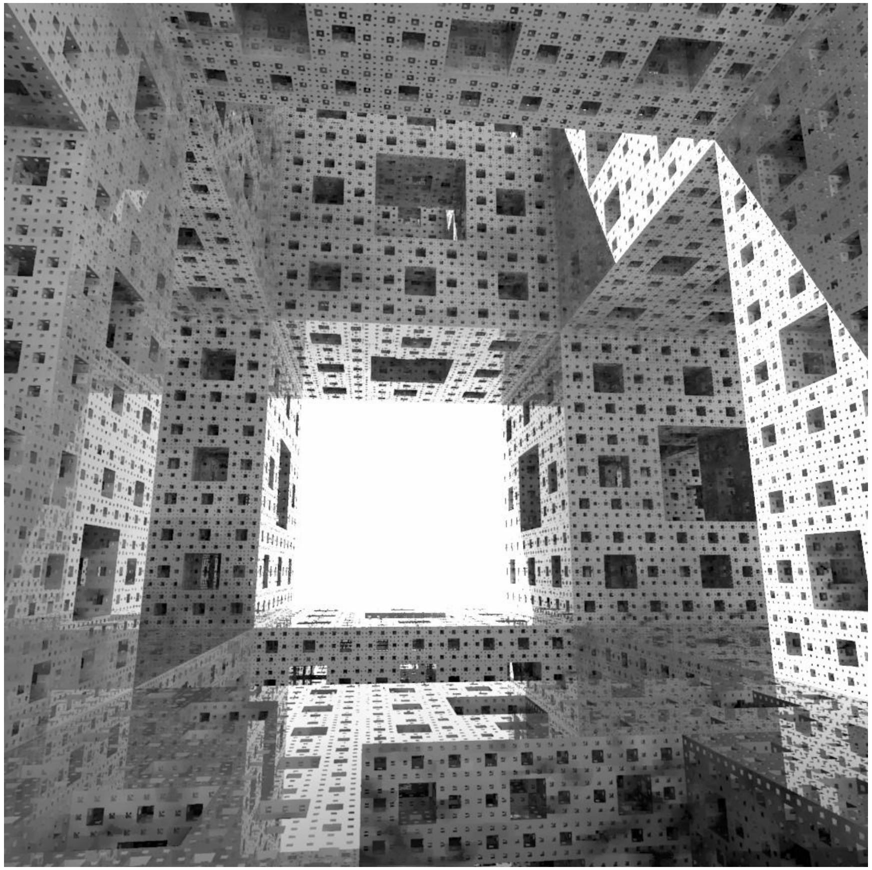
Generator





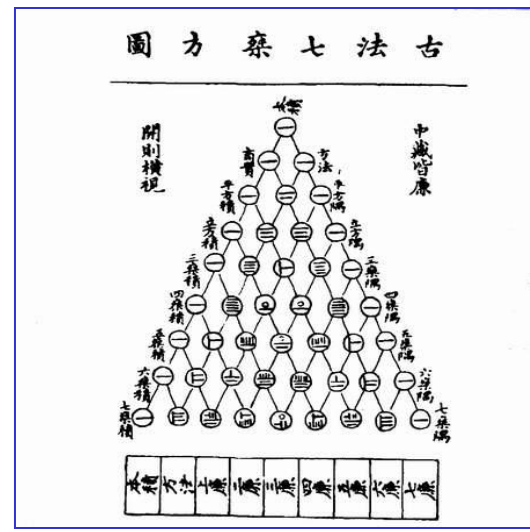
Sierpinski in 3D

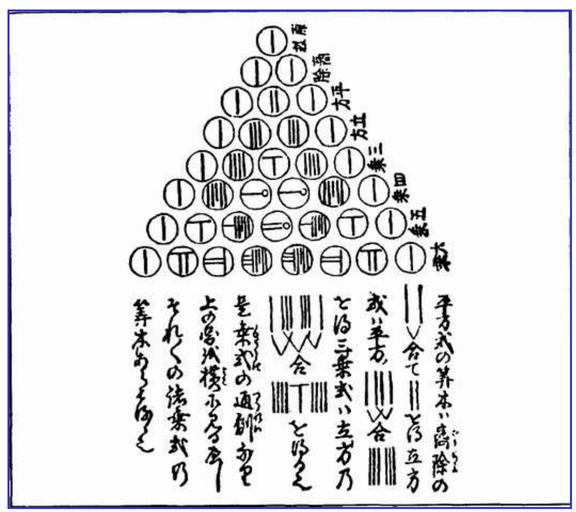


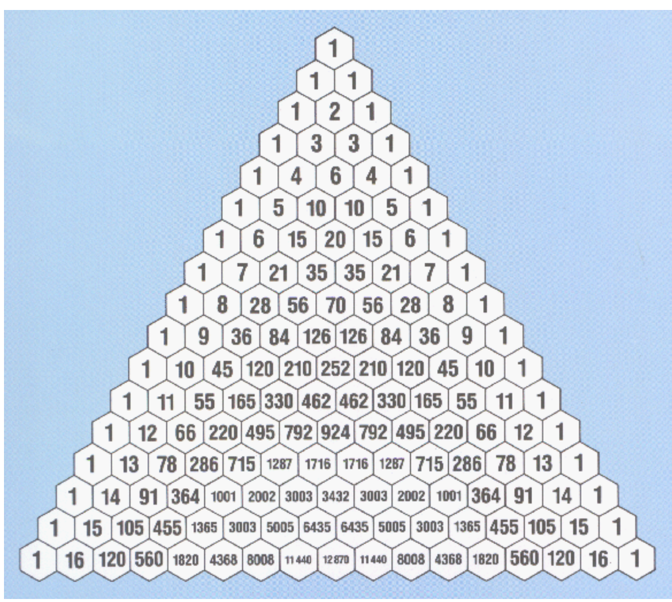


Pascal's triangle

- 1303 China, 1527 Europe
- Blaise Pascal (17th century)
- Binomial coefficients

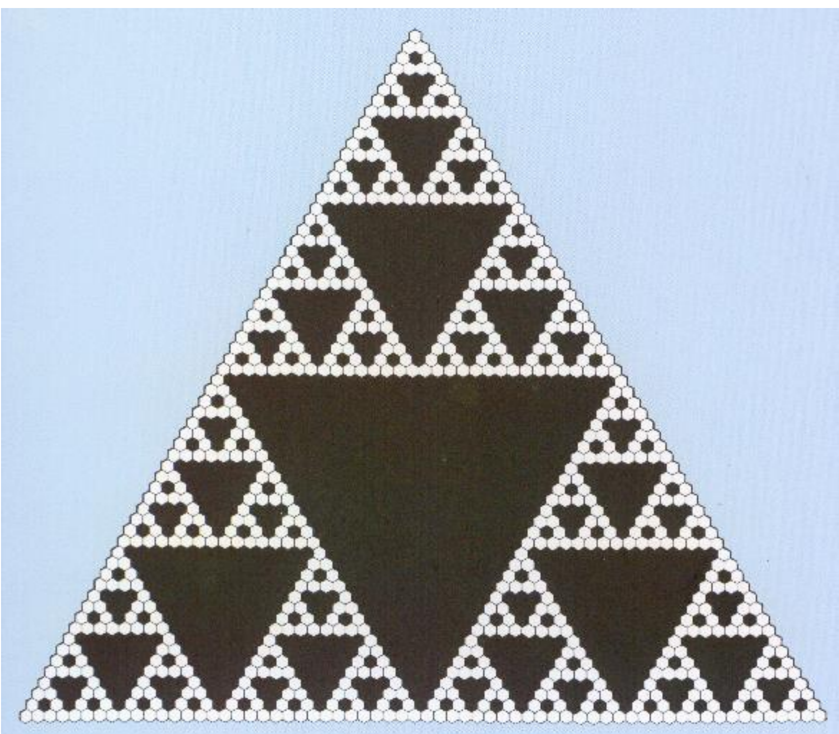


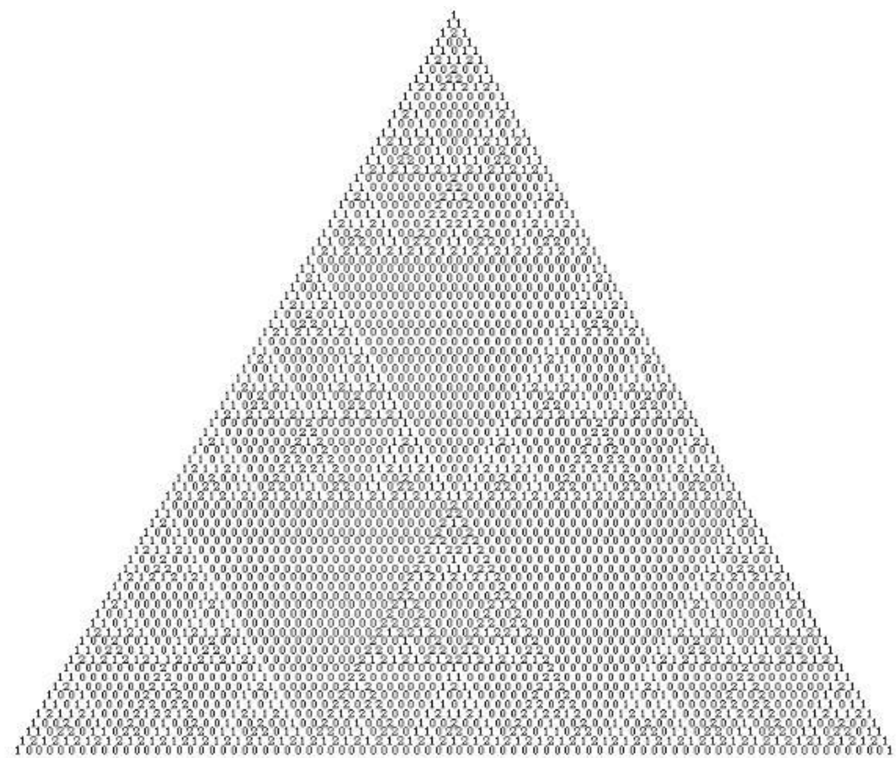




Pascal’s triangle as fractal

- Coloring with mod 2
- Other modules





Space filling curves

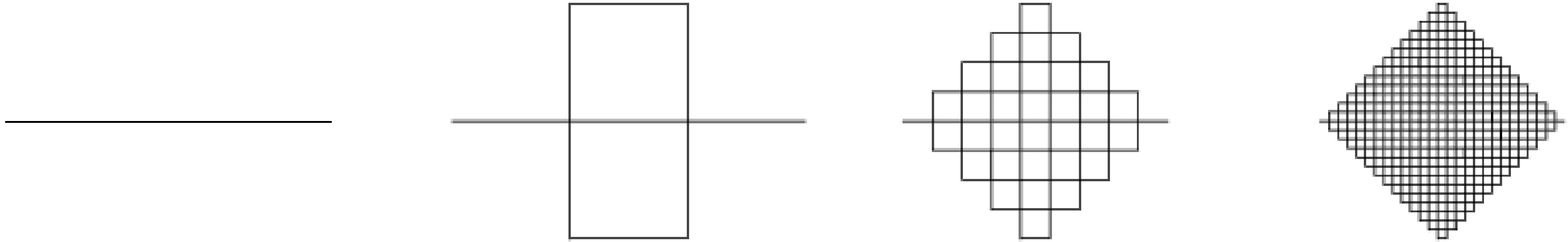
- 1878, George Cantor: there exists a bijective function between any two finite-dimensional smooth manifolds.
- "I see it, but I don't believe it."
- mapping between a coordinate pair (x,y) and a real number z: Represent x and y in their decimal forms: x = 0.abcd… and y = 0.ABCD… The mapping is z = 0.aAbBcCdD…

Space filling curves

- Finding continuous mapping of [0,1] into R^2
- Peano
- Hilbert
- Lebesgue
- Moore
- Sierpinski
- Dragon
- Topological: 1, Fractal: 2

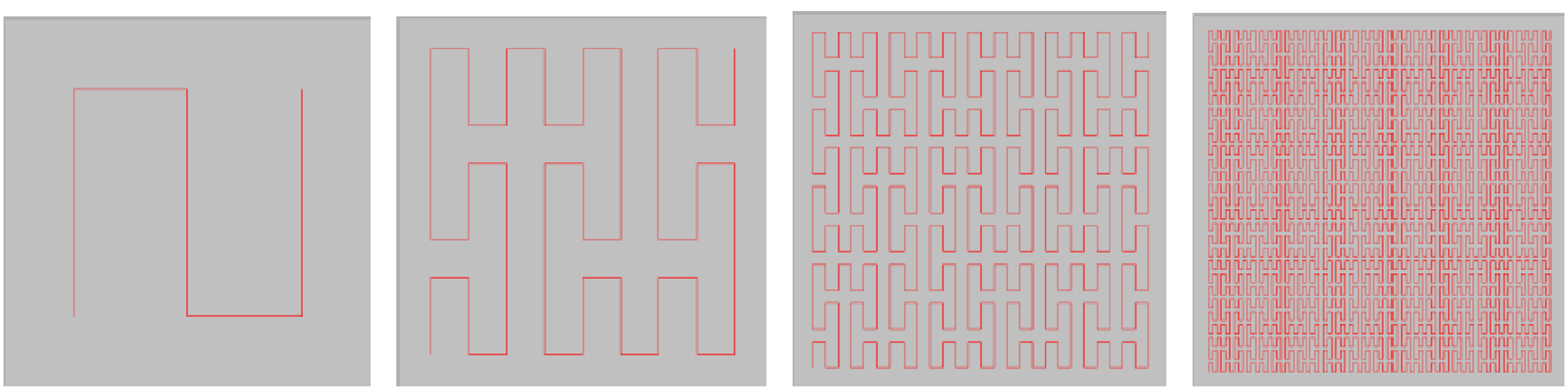
Peano curve

- Giuseppe Peano
- Similar to Koch
- D=log(9)/log(3)=2
- Length = 3^k



Peano curve

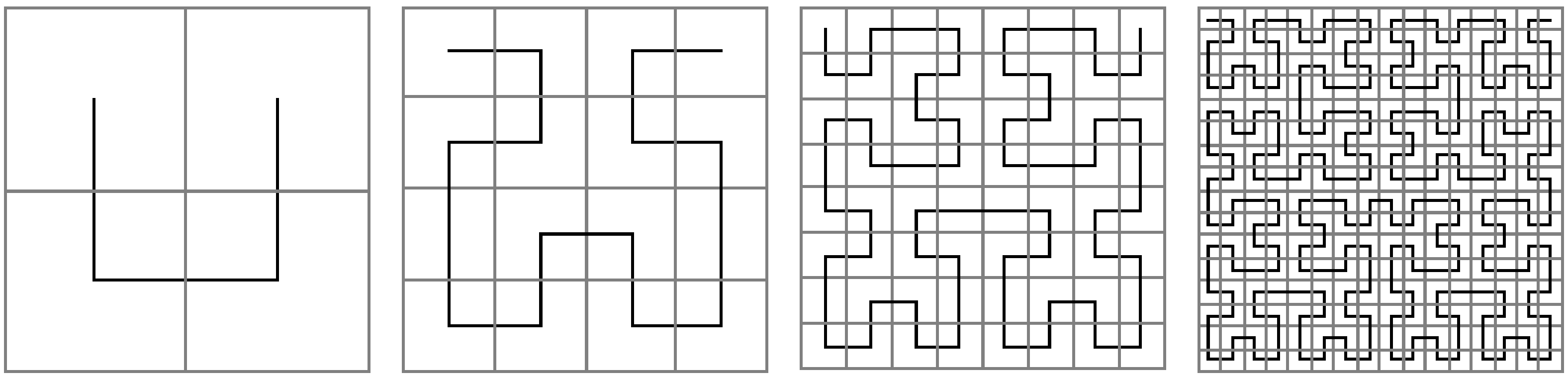
- While finding continuous mappings
- Jordan: curve (with endpoints) is a continuous function whose domain is the unit interval [0, 1]
- Range – multidimensional set

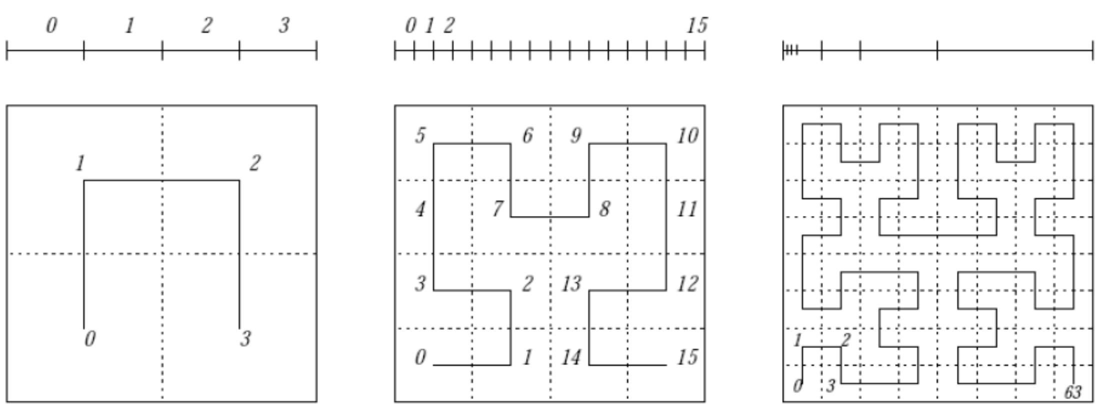


Level 1 Level 2 Level 3 Level 4

Hilbert curve

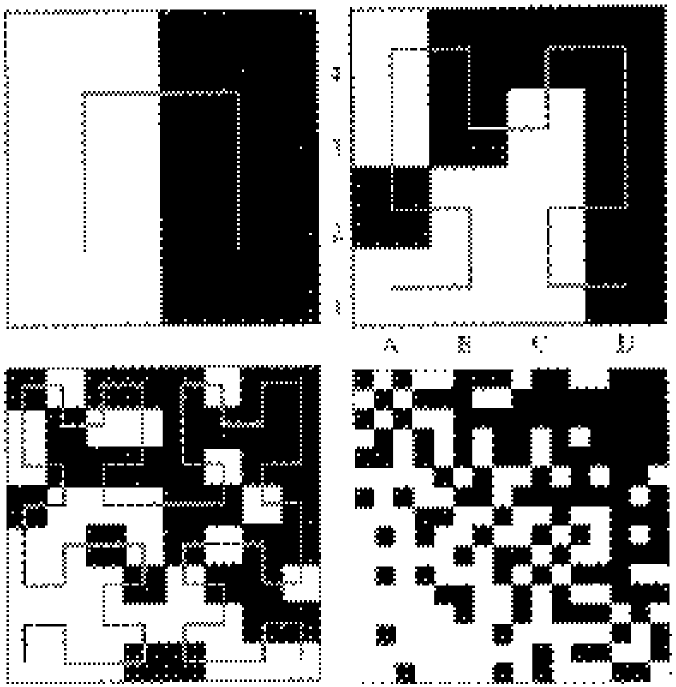
- Not-simple generator





Using Hilbert curve

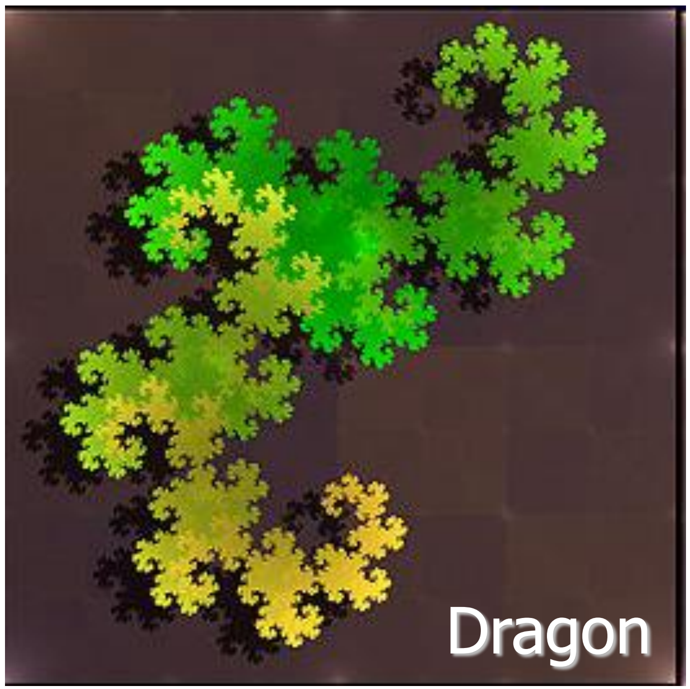
- Multidimensional DBMS
- Geographics research
- Image dithering

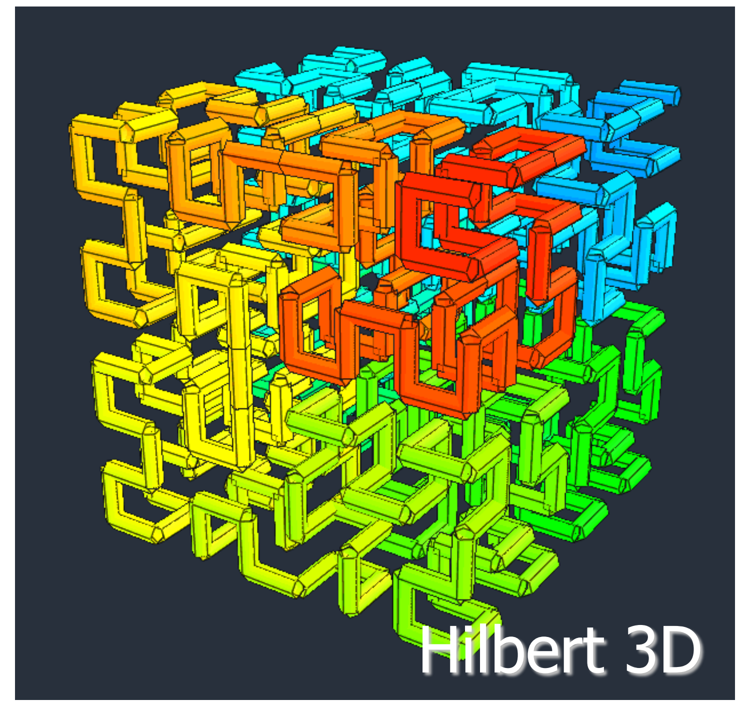


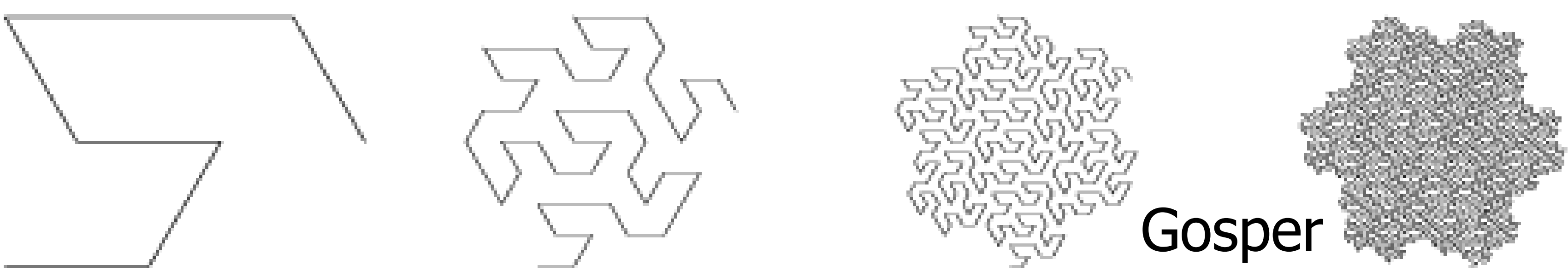
HC Art

http://www.donrelyea.com/hilbert_algorithmic_art_menu.htm

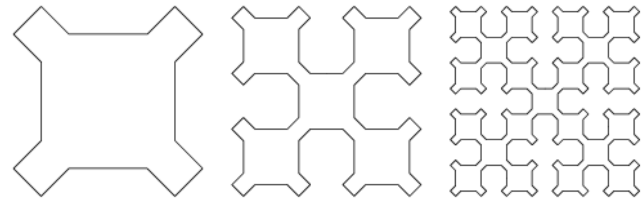
Other SFC



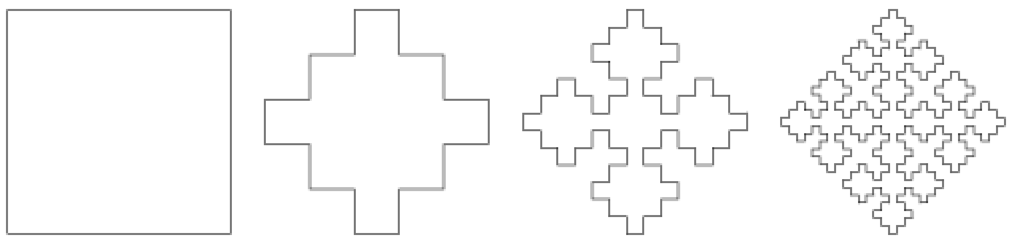


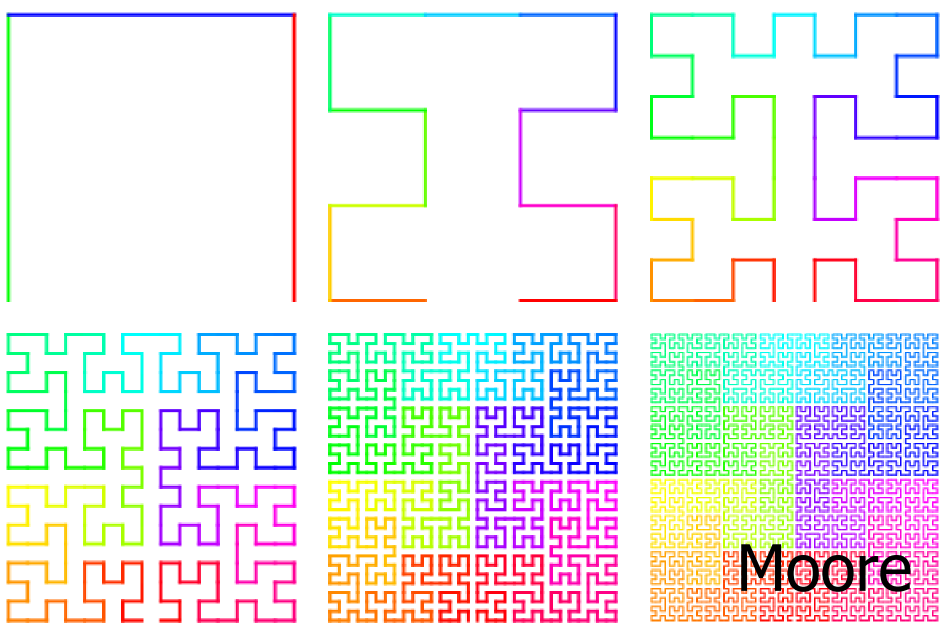


Other SFC



Sierpinski



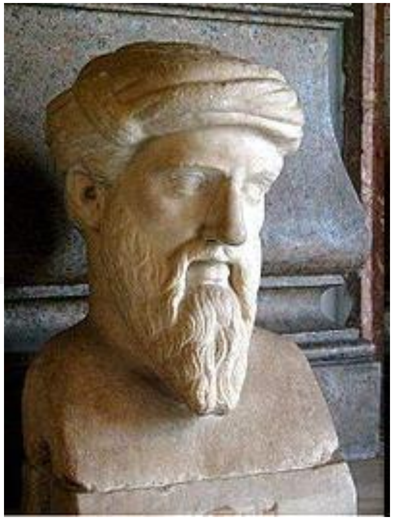


Generalized Koch curve

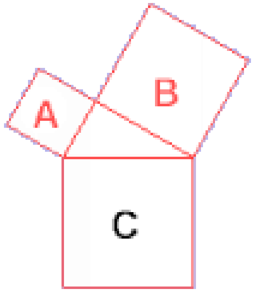
- For $a = 90$ º
- Self intersecting
- D = log(4)/log(2)=2
- Filling triangle

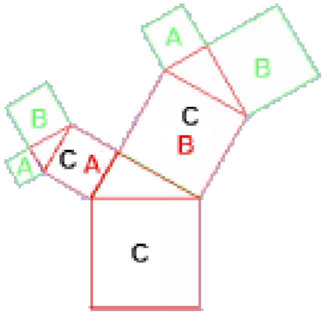
$$D = \frac{\log 4}{\log(2 + 2 * \cos\alpha)}$$

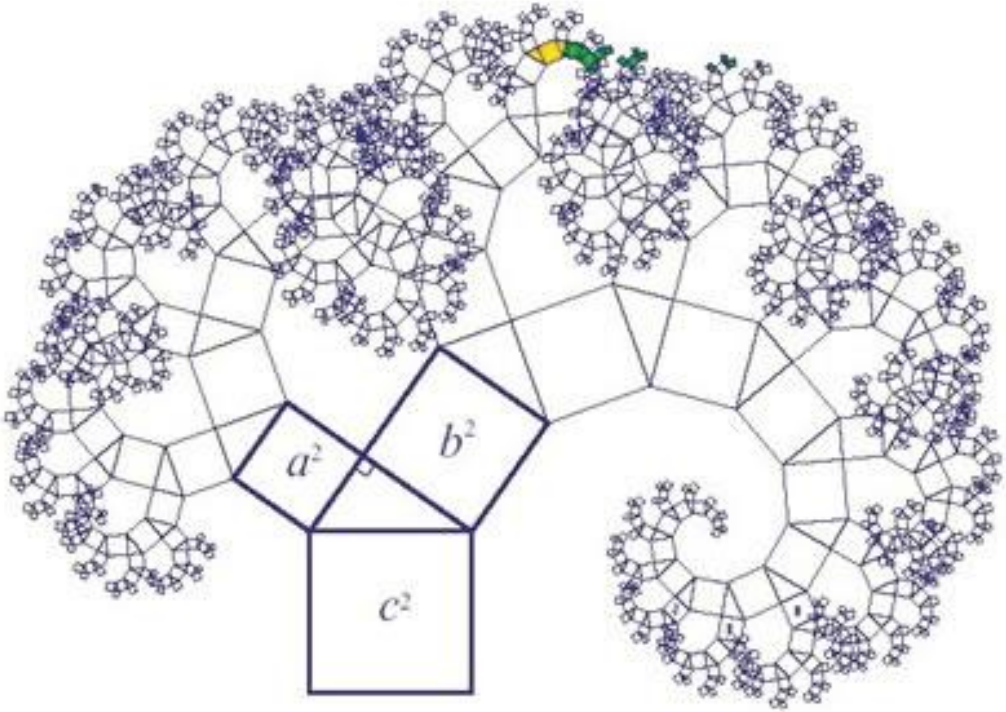
Pythagorean trees



- Based on Pythagorean theorem
- Can be generalized







Links

- http://www.root.cz/serialy/fraktaly-v-pocitacove-grafice/
- http://local.wasp.uwa.edu.au/~pbourke/fractals/
- http://www.sccg.sk/~samuelcik
- http://home.att.net/~Paul.N.Lee/Fractal_Software.html
- http://www.google.com
- http://en.wikipedia.org/wiki/List_of_fractals_by_Hausdorff_dimension

End

End of Part 2