Blind Deconvolution

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Outline

- Image Formation
- Deconvolution Schemes overview

  Linear and Non-Linear
- Other PSF calibration techniques
  - Blind deconvolution

Why Deconvolution?

- Better looking image
- Improved identification

  Reduces overlap of image structure to more easily identify features in the image
- PSF calibration

  Removes artifacts in the image due to the point spread function (PSF) of the system,.
- Higher resolution

Image Formation - Convolution

Shift invariant imaging equation (Image & Fourier Domains)

Image Domain - $g(\vec{r}) = f(\vec{r}) * h(\vec{r}) + n(\vec{r})$

Fourier Domain - $G(\vec{f}) = F(\vec{f})H(\vec{f}) + N(\vec{f})$

- $g(\vec{r})$ – measurement
- $f(\vec{r})$ – source (object)
- $h(\vec{r})$ – blur (Point Spread Function)
- $n(\vec{r})$ – noise contamination (photon noise & detector noise)
- Fourier transform: $\mathcal{F}\{g(\vec{r})\} = G(\vec{f})$ etc.
- $*$ denotes convolution

Deconvolution

The convolution equation is inverted.

Given the measurement *g(r)* and the PSF *h(r)* the object *f(r)* is computed.

e.g. $$F(\vec{f}) = |F(\vec{f})| \exp[i\varphi(\vec{f})] = \frac{G(\vec{f})}{H(\vec{f})}$$

and inverse Fourier transform to obtain *f(r).*

Problem:

The PSF and the measurement are both band-limited due to the finite size of the aperture.

The object/target is not.

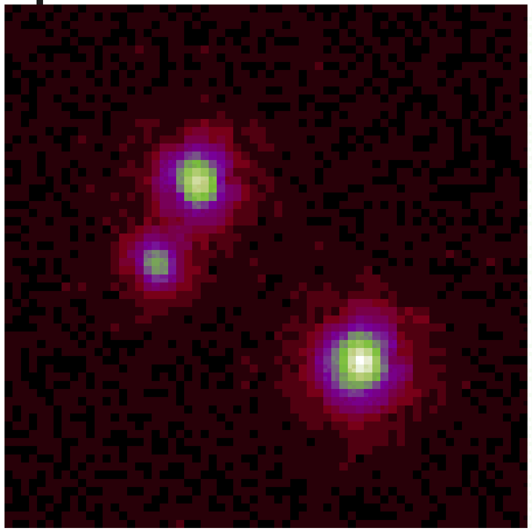
Deconvolution via Linear Inversion

$$\hat{f}(\vec{r}) = \mathcal{F}^{-1}\left\{\frac{G(\vec{f})}{H(\vec{f})}\Phi(\vec{f})\right\}$$

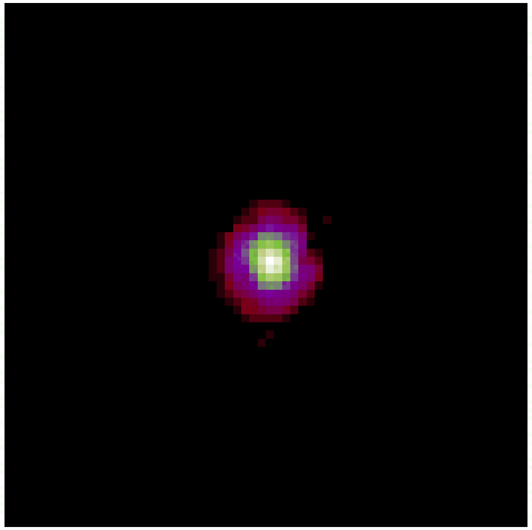
- Inverse Filtering: $\Phi(\vec{f})$ is a bandpass-limited attenuating filter, e.g. a *chat* function which is 0 for $H(\vec{f}) = 0$, i.e. for $f > f_c$.
- Wiener Filtering: Noise-dependent filter –

$$\Phi(\vec{f}) \sim \frac{|G(\vec{f})|^2 - |N(\vec{f})|^2}{|G(\vec{f})|^2}$$

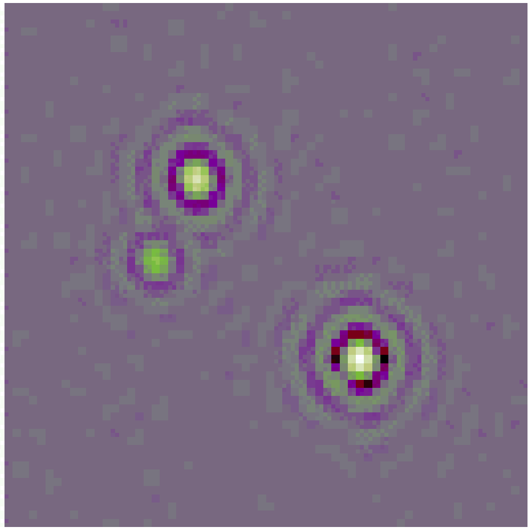
Deconvolution via Linear Inversion with a Wiener Filter - Example



measurement



PSF



reconstruction

Note the negativity in the reconstruction – not physical

Bayes Theorem on Conditional Probability

$$P(A|B)\,P(B) = P(B|A)\,P(A)$$

P – Probabilities
A & *B* – Outcomes of random experiments
P(A|B) - Probability of *A* given that *B* has occurred

For Imaging:

P(B|A) - Probability of measuring image *B* given that the object is *A*

Fitting a probability model to a set of data and summarizing the result by a probability distribution on the model parameters and observed quantities.

Bayes Theorem on Conditional Probability

- Setting up a *full probability model* – a joint probability distribution for all observable and unobservable quantities in a problem,
- Conditioning on observed data: calculating and interpreting the appropriate *posterior distribution* – the conditional probability distribution of the unobserved quantities.
- Evaluating the fit of the model. How good is the model?

Maximum *a posteriori* (MAP)

Regularized Maximum-likelihood

The *posterior* probability comes from Bayesian approaches, i.e. the probability of f being the object given the measurement g is:

$$P(f \mid g) = \frac{P(g \mid f)\, P(f)}{P(g)}$$

where $P(g|f) = \prod \frac{\exp\{-\sum_{j} h_{kj} f_j\}(\sum_{j} h_{kj} f_j)^{g_k}}{g_k!}$

and *P(f)* is now the *prior probability distribution* (prior)

Richardson-Lucy Algorithm

Discrete Convolution

$$g_i = \sum_j h_{ij} f_j \quad \text{where} \quad \sum_j h_{ij} = 1 \quad \text{for all } j$$

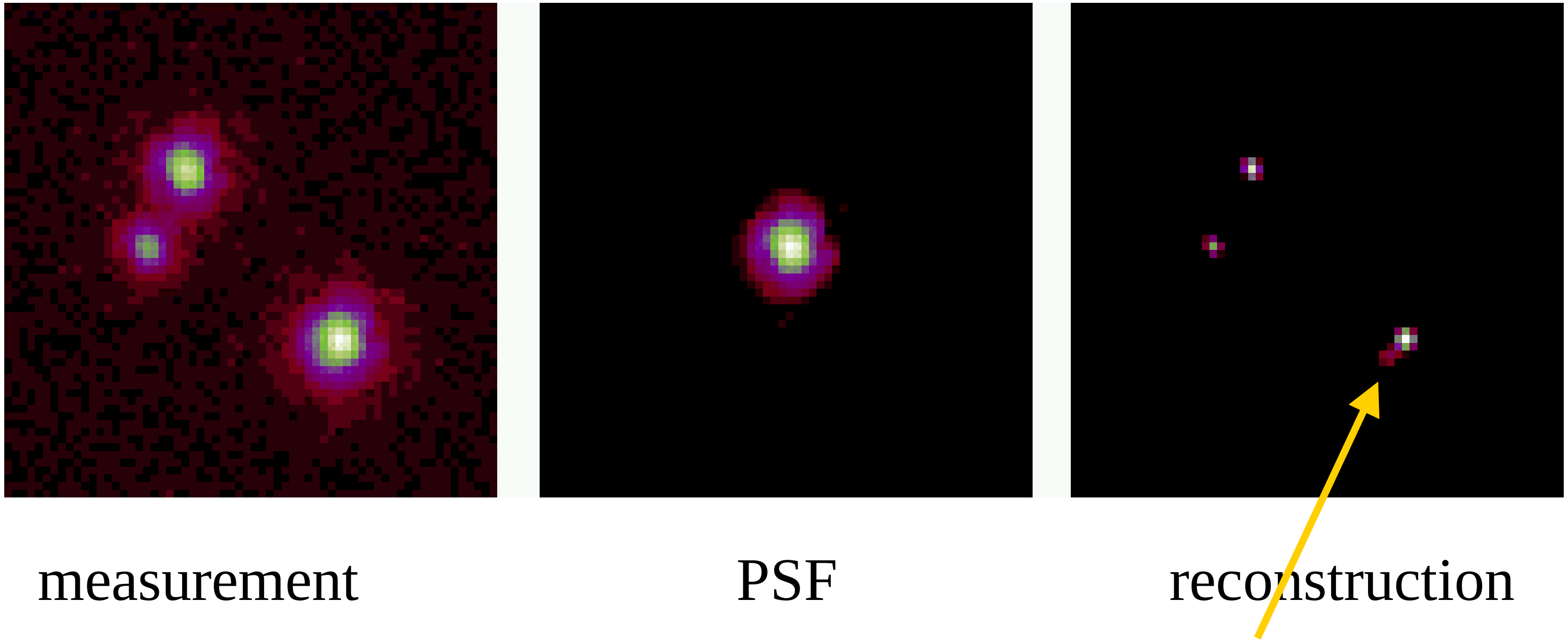
From Bayes theorem $P(g_i|f_j) = h_{ij}$ and the object distribution can be expressed iteratively as

$$f_j = f_j \sum_i \left(\frac{h_{ij} g_i}{\sum_k h_{jk} f_k} \right)$$

so that the RL kernel approaches unity as the iterations progress

Richardson, W.H., “Bayesian-Based Iterative Method of Image Restoration”, *J. Opt. Soc. Am.*, **62**, 55, (1972).
Lucy, L.B., “An iterative technique for the rectification of observed distributions”, *Astron. J.*, **79**, 745, (1974).

Richardson-Lucy Application



Super-resolution means recovery of spatial frequency information beyond the cut-off frequency of the measurement system.

Blind Deconvolution

- Poor or no PSF estimate – Blind Deconvolution

$$g(\vec{r}) = f(\vec{r}) * h(\vec{r}) + n(\vec{r})$$

Measurement

unknown object

unknown or poorly known PSF

contamination

Need to solve for both object & PSF

"It's not only impossible, it's hopelessly impossible"

PSF

- **Exact PSF** – known exactly
- **In situ PSF** – depends on properties of sample

Multiple Frame Blind Deconvolution

m independent observations of the same object.

$$g_1(\mathbf{r}) = f(\mathbf{r}) * h_1(\mathbf{r}) + n_1(\mathbf{r})$$
$$g_2(\mathbf{r}) = f(\mathbf{r}) * h_2(\mathbf{r}) + n_2(\mathbf{r})$$
$$\vdots$$
$$g_m(\mathbf{r}) = f(\mathbf{r}) * h_m(\mathbf{r}) + n_m(\mathbf{r})$$

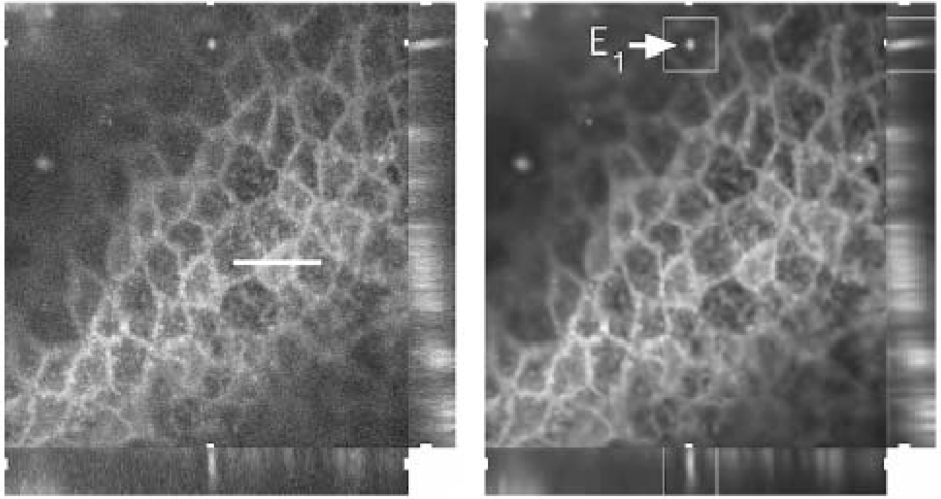
The problem reduces from

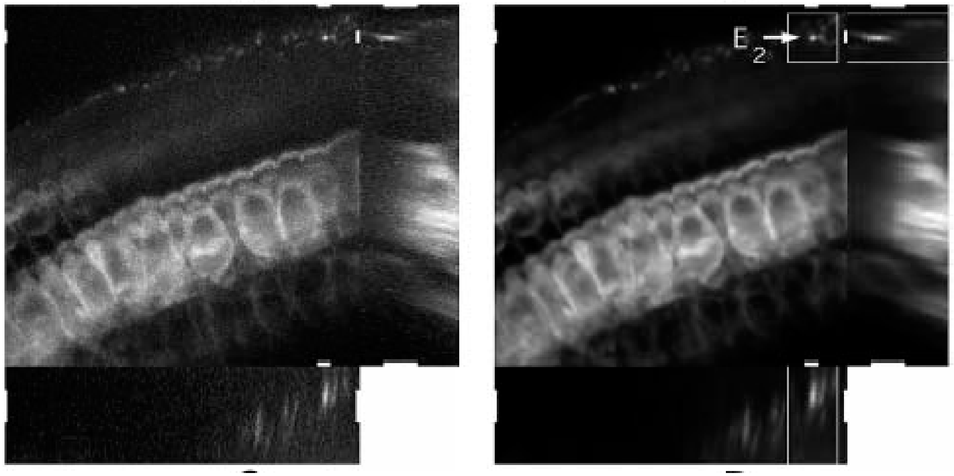
1 measurement & 2 unknowns

to

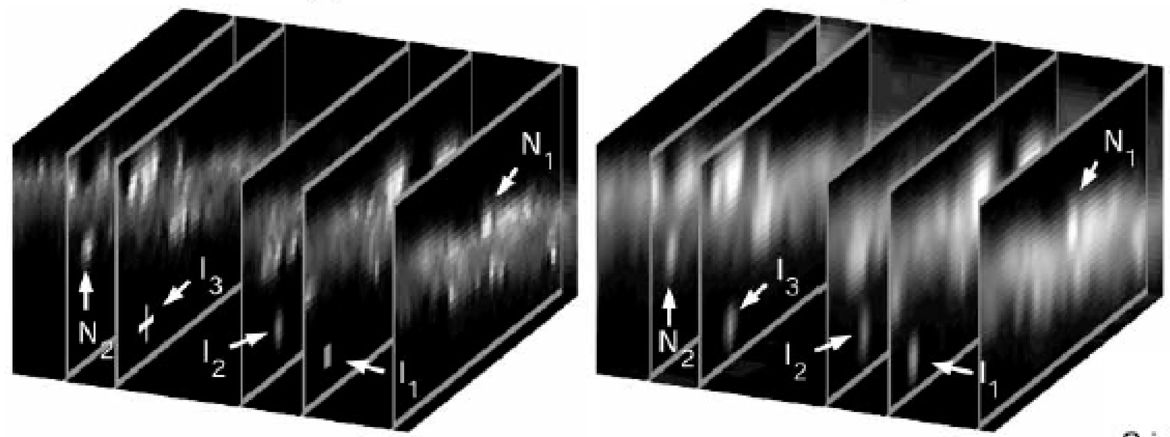
m measurements & m+1 unknowns

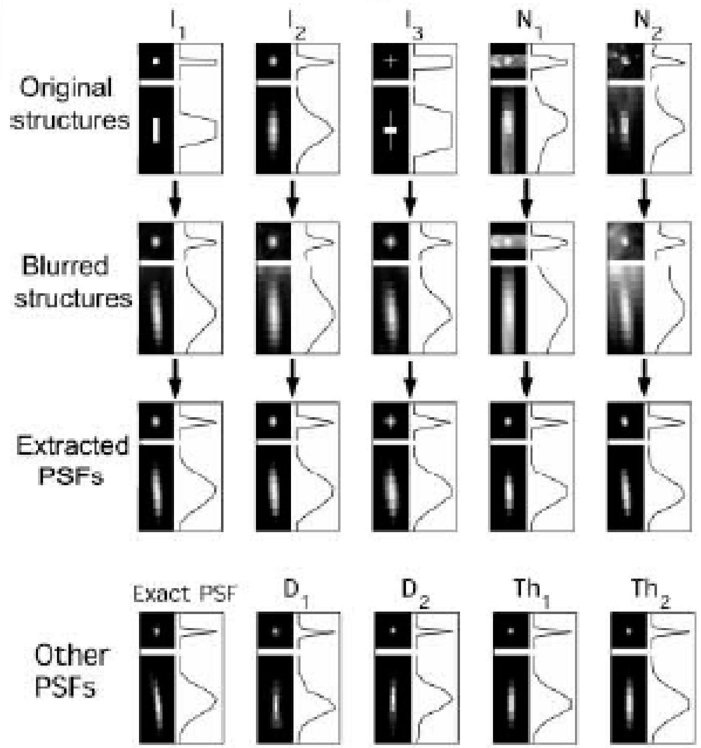
Get PSF from directly from object





Get PSF from directly from object





Physically Constrained Iterative Deconvolution

- "Blind" deconvolution solves for both **object *f(r)*** and **PSF *h(r)*** simultaneously.
  - Ill-posed inverse problem.
  - Under – determined: 1 measurement, 2 unknowns

- Uses Physical Constraints.
  - ***f(r)*** & ***h(r)*** are positive, real & have finite support.
    - Finite support reduces # of variables (symmetry breaking)
  - ***h(r)*** is band-limited – symmetry breaking

- *a priori* information - further symmetry breaking.
  - Noise statistics
  - PSF knowledge
  - Object & PSF parameterization
  - Multiple Frames:
    - Same object, different PSFs.
    - ***N*** measurements, ***N+1*** unknowns.

Summary

- Deconvolution is necessary for many applications to remove the effects of PSF
  - – PSF calibration
  - identification of sources in a crowded field
  - removal of asymmetric PSF artifacts etc.

- A choice of algorithms available
  - Is any one algorithm the best?
    - different algorithms for different applications
    -
- What happens when the PSF is poorly determined?
  - This is a problem for many AO cases.

What happens when the PSF is spatially variable?

Blind Deconvolution – Key Papers

Ayers & Dainty, “Iterative blind deconvolution and its applications” , *Opt. Lett.* **13** , 547-549, 1988.

Holmes , “Blind deconvolution of speckle images quantum-limited incoherent imagery: maximum-likelihood approach” , *J. Opt. Soc. Am. A,* **9** , 1052-106, 1992.

Lane , “Blind deconvolution of speckle images” , *J. Opt. Soc. Am. A*, **9** , 1508-1514, 1992 .

Jefferies & Christou, “Restoration of astronomical images by iterative blind deconvolution” , *Astrophys. J.* **415**, 862-874, 1993.

Schultz , “Multiframe blind deconvolution of astronomical images” , *J. Opt. Soc. Am. A*, **10** , 1064-1073, 1993.

Thiebaut & Conan, “Strict *a priori* constraints for maximum-likelihood blind deconvolution” , *J. Opt. Soc. Am. A,* **12** , 485-492, 1995.

Conan et al., “Myopic deconvolution of adaptive optics images by use of object and point-spread function power spectra”, *Appl. Opt.*, **37**, 4614-4622, 1998 .