3D deconvolution

Marek Zimányi
Faculty of Mathematics, Physics and Informatics
Comenius University, Bratislava

Intro

- Inverse problems and data reconstruction
- Using of parameters of confocal microscope in reconstruction process
- Application of this method in
  - Medical image reconstruction and enhancement
  - Confocal imaging

Contents

- Intro
- Confocal microscopy
- Image Restoration
- Deconvolution in Confocal Microscopy

Schematic 3-D Analysis

3D image acquisition

System Identification

↓

Image Restoration

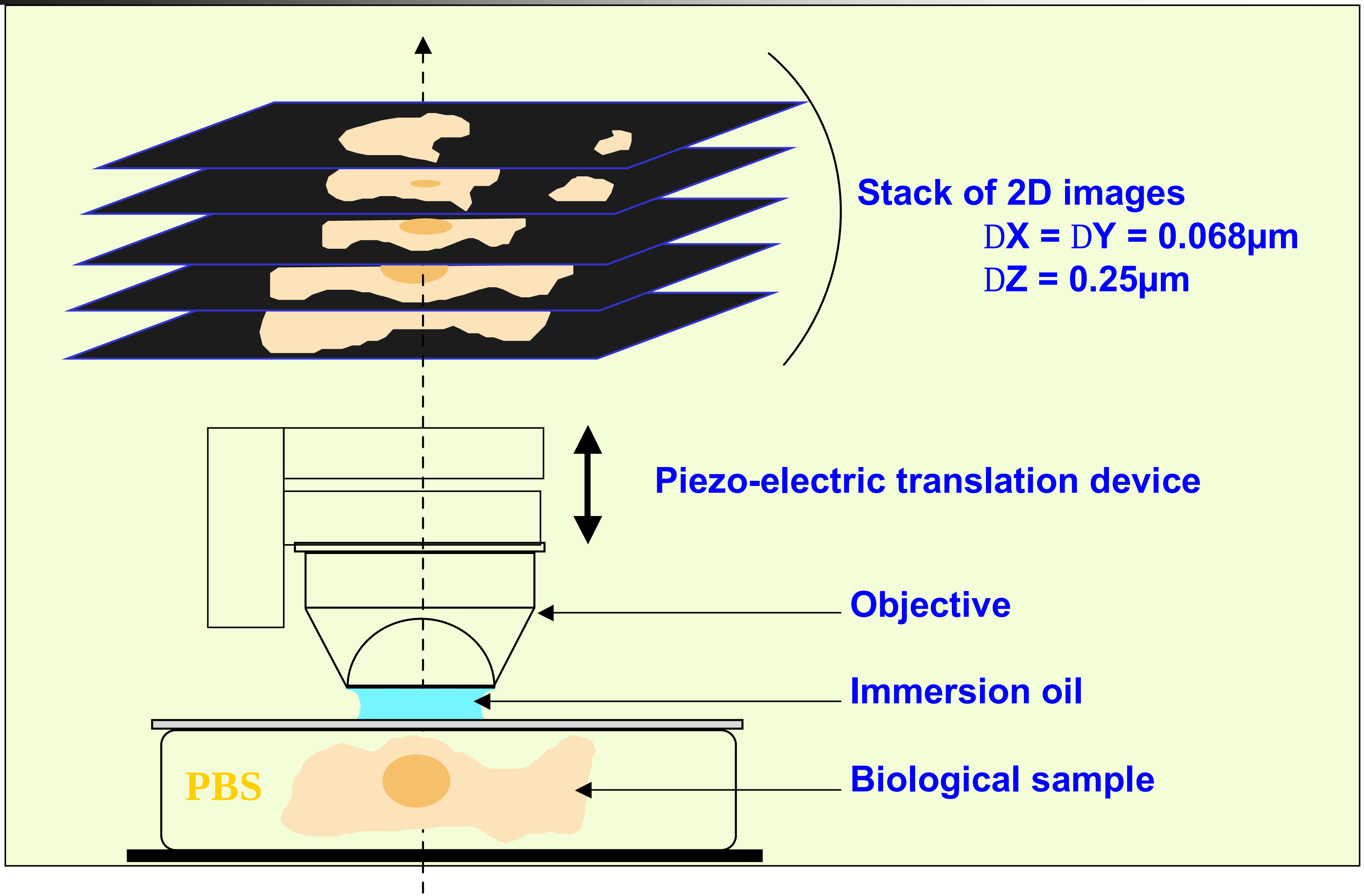
↓

Segmentation and Measurements

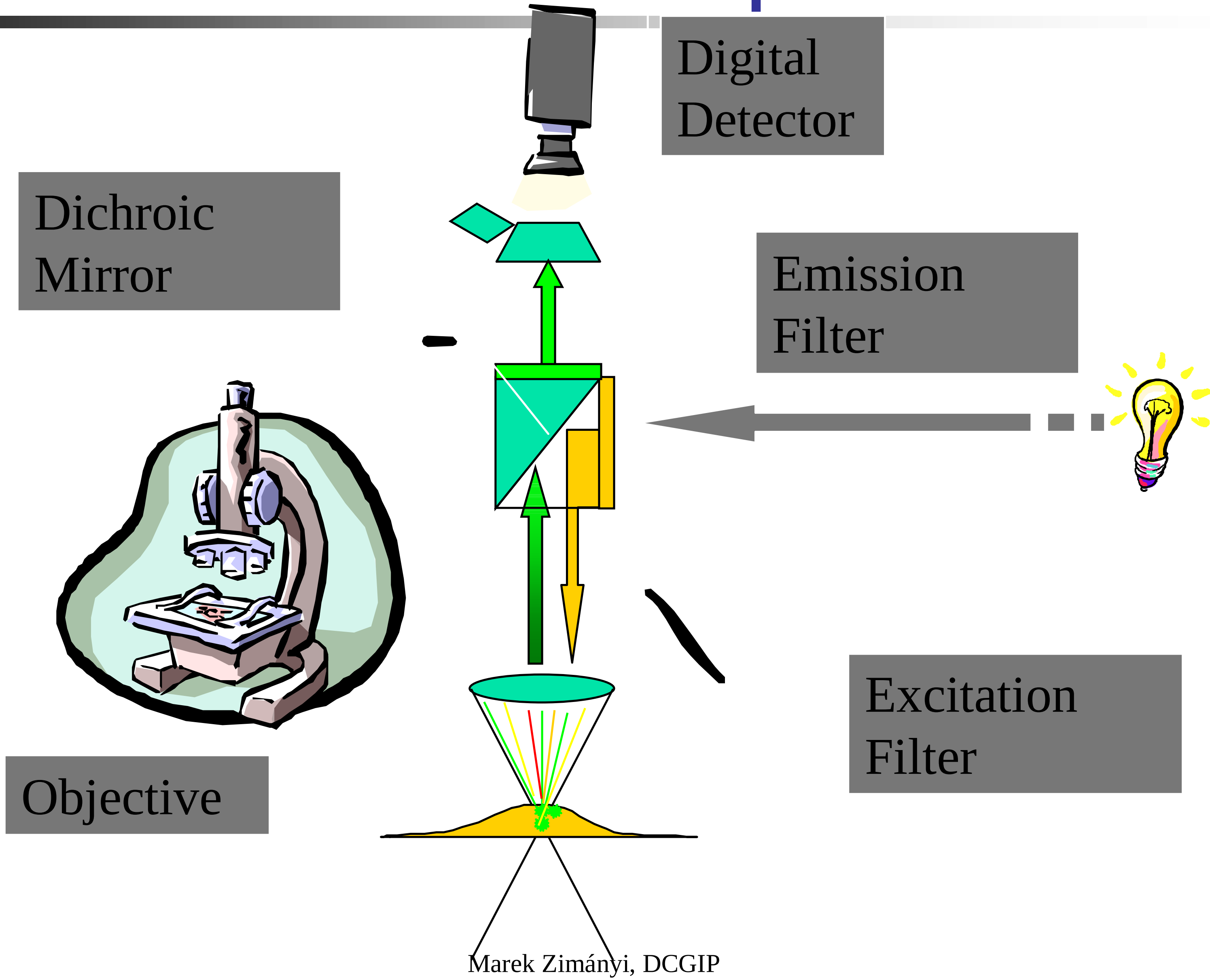
↓

Biologist -> Interpretation

Specific equipment - Optical

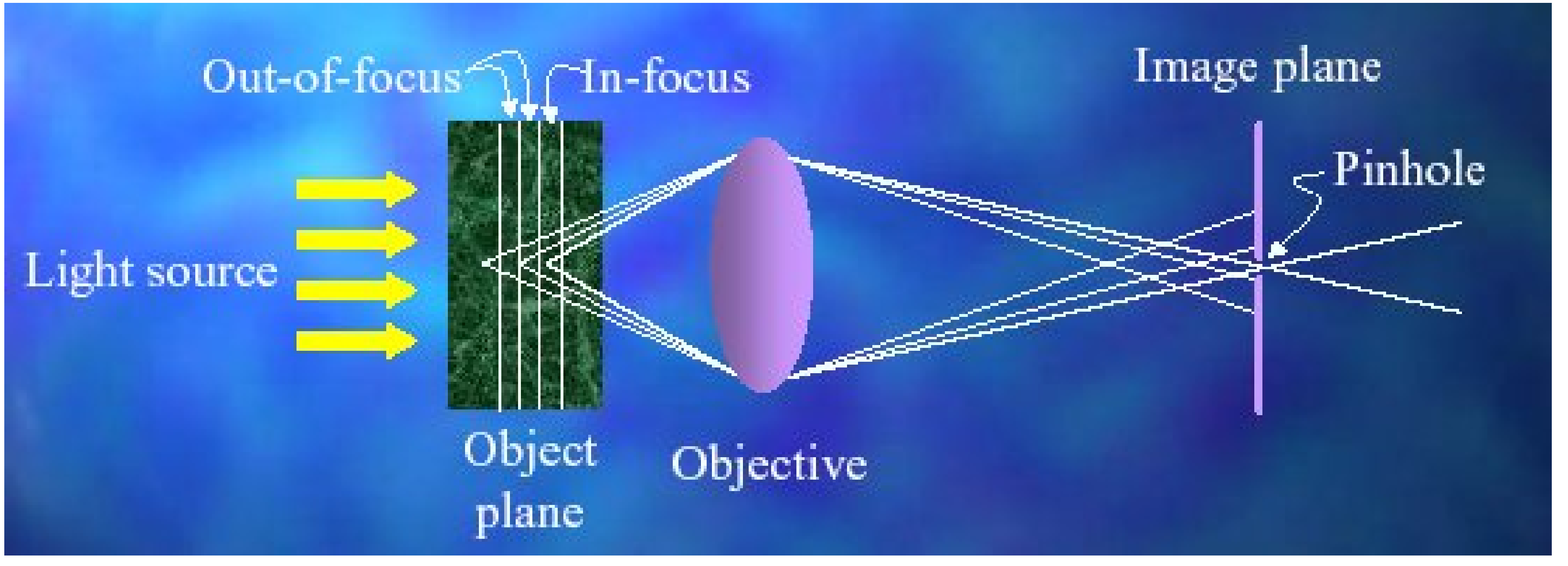


Confocal Microscope



Confocal microscope

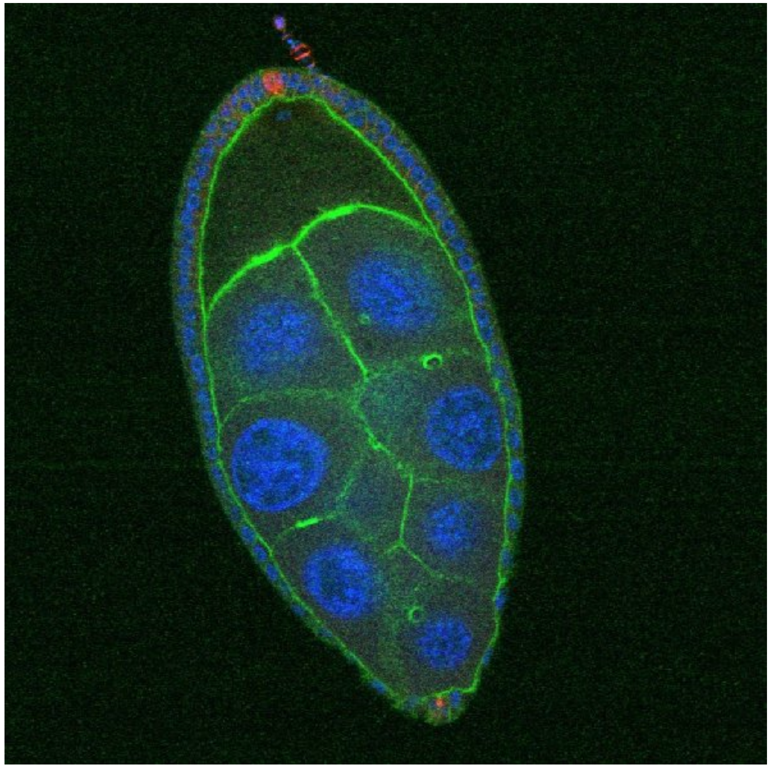
- Confocal Imaging

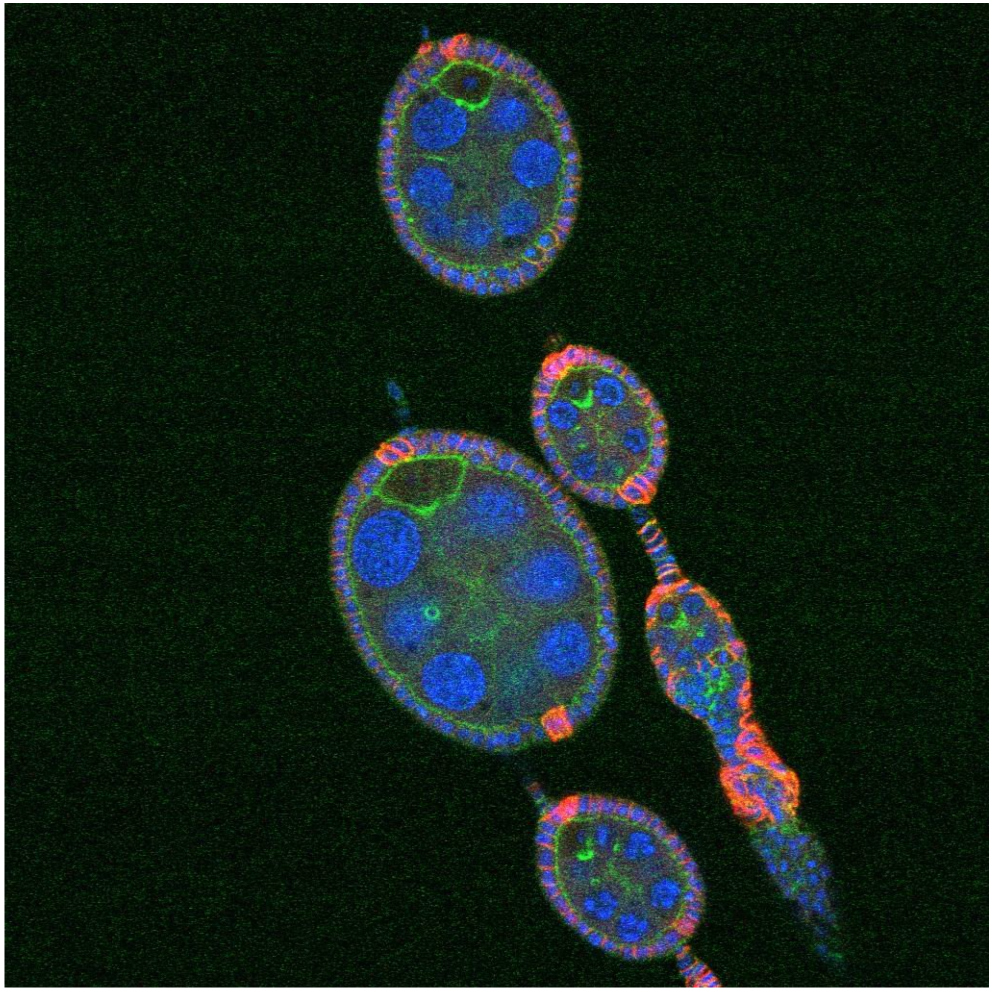


From Bowe Ellis presentation "A Review of Confocal Microscopy"

Confocal microscope

- Confocal Imaging





Sources of microscopic image degradation

Microscopic images can be degraded by:

1. Instrumental imaging properties:
   - Shading,
   - Finite resolution (diffraction),
   - Glare,
   - Geometrical distortion,
   - (Projection of 3D object to 2D image).
   - (Which is the prime reason for using a confocal microscope)
2. Object induced:
   - Object influences shape of the PSF
   - Variable absorption or scattering
3. Noise:
   - Additive (Gaussian noise),
   - Multiplicative (Poisson noise)

3D Image Formation

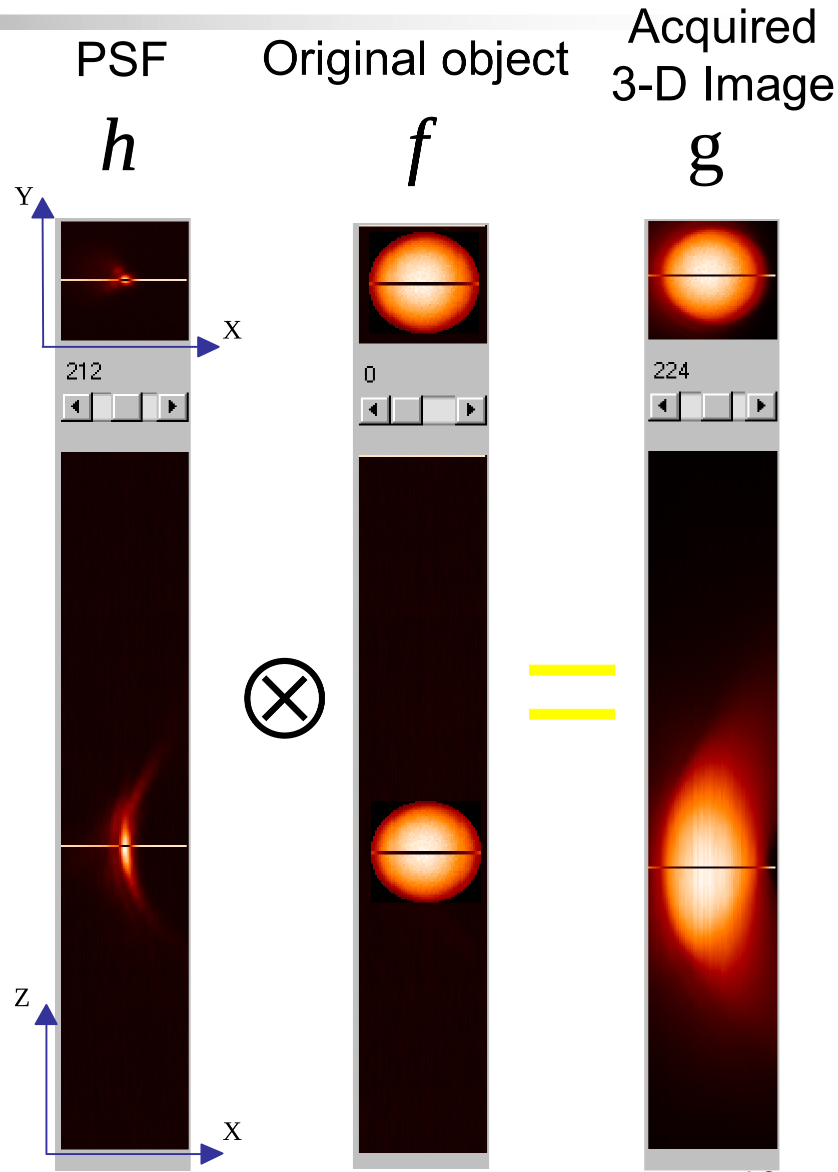
Modelization

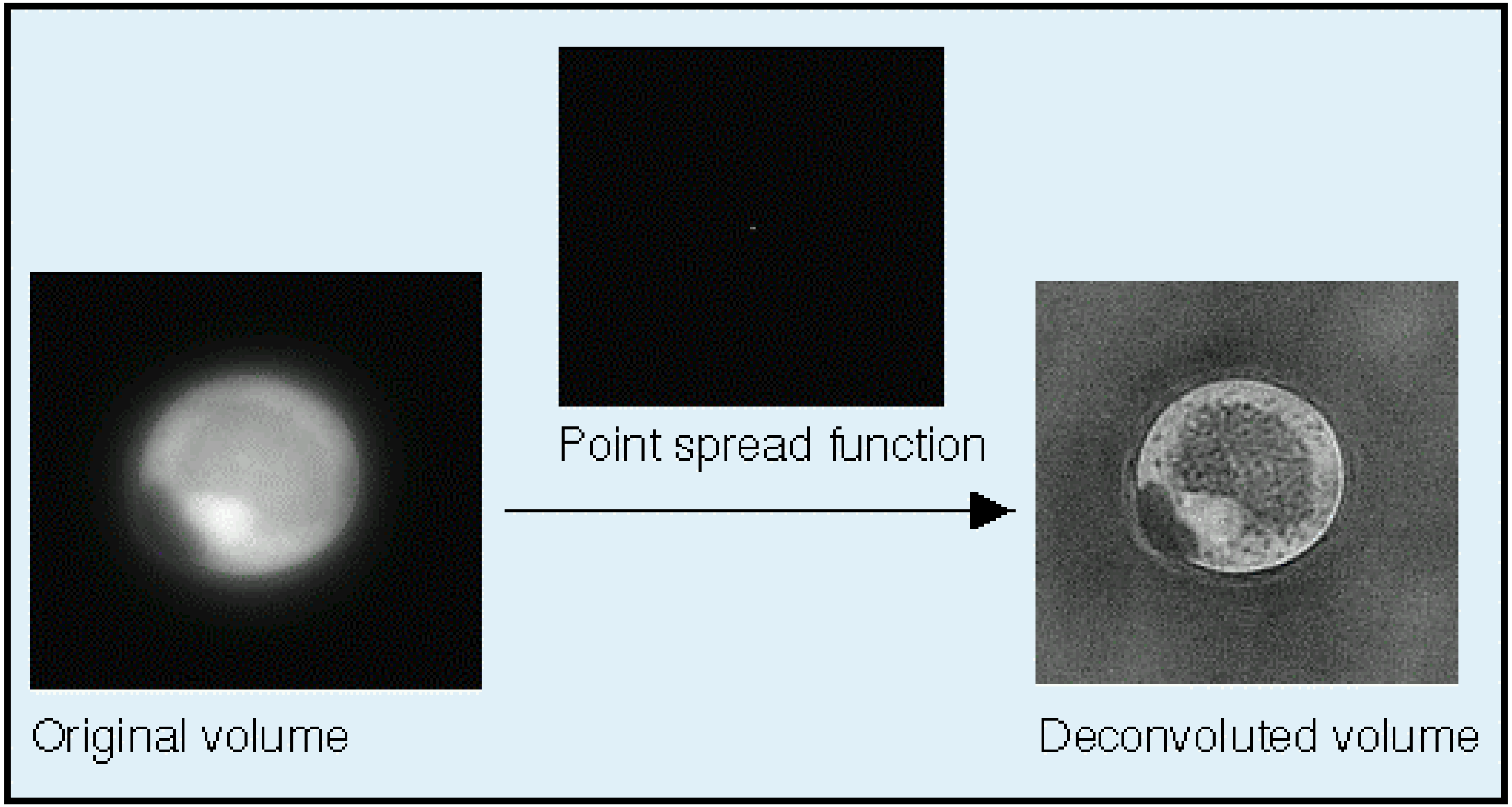
$$g = h \otimes f$$

Convolution   Additive noise

Point Spread Function (PSF)

- Determines image quality
- Acquisition system dependent:
  - confocal
  - conventional




Point spread function
Original volume
Deconvoluted volume

Traditional reconstructions and their artifacts

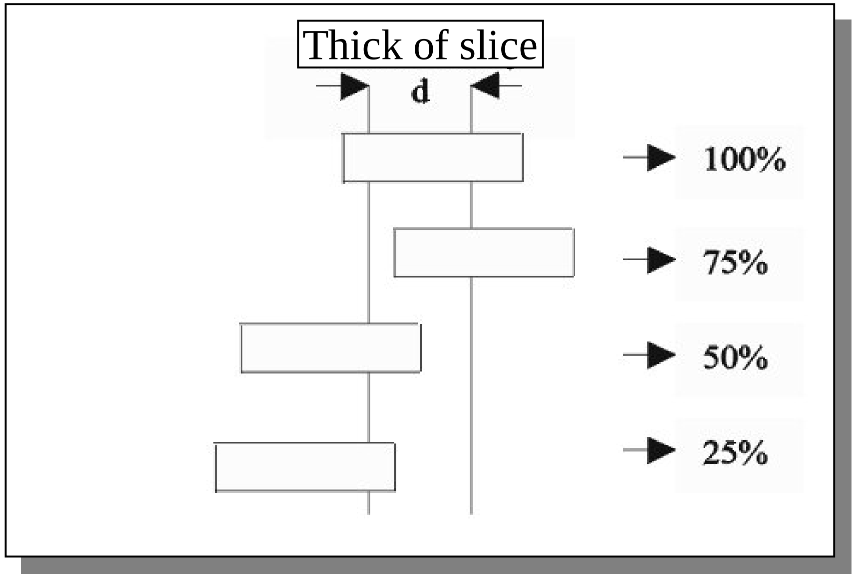
Reconstructions are based on the convolution scanned data with a filter.

Fact

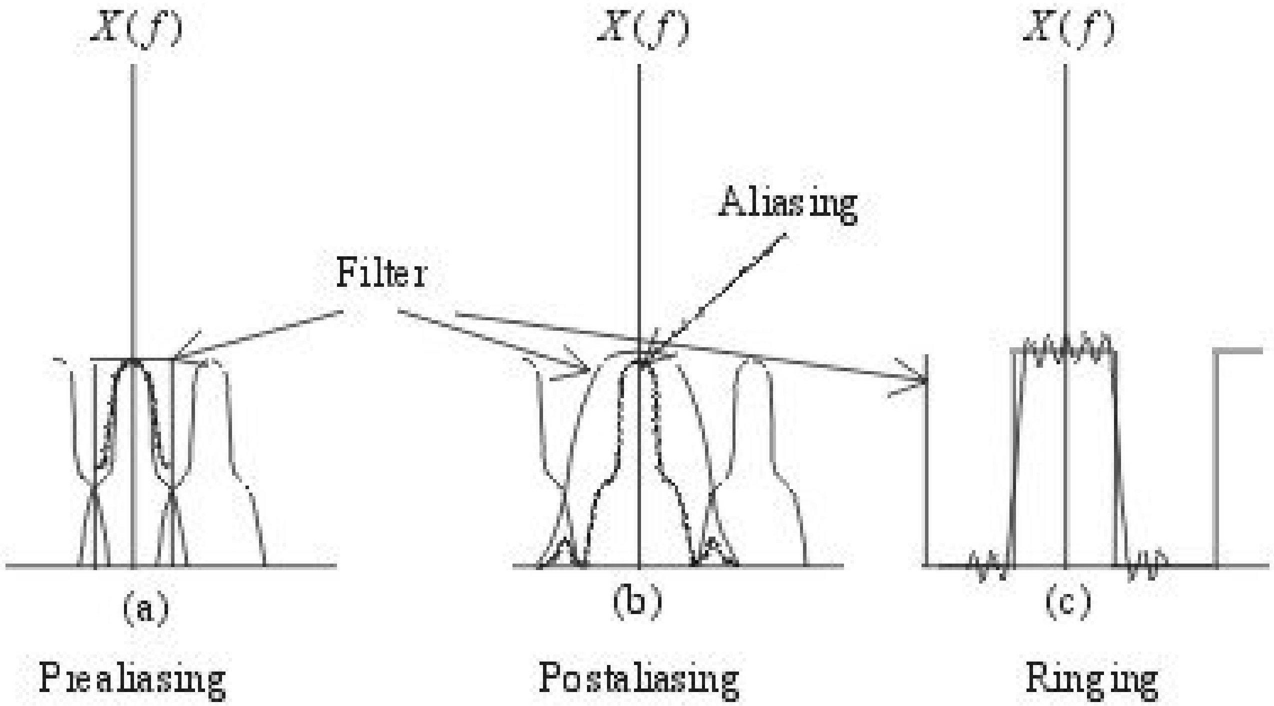
- *Disadvantage sampling of* the object (under Nyquist frequency)

Fact

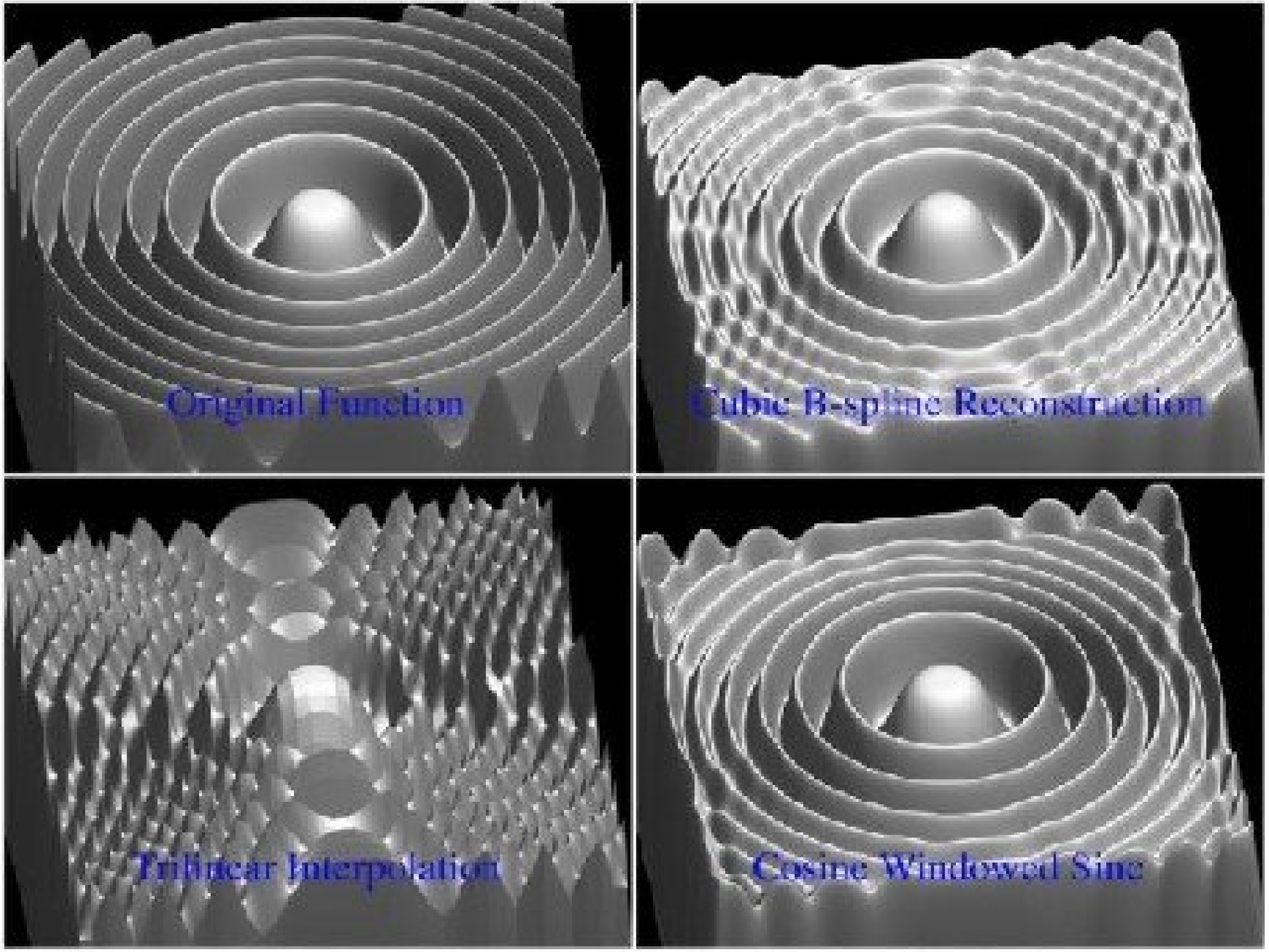
- *Thickness of* the slices
- Overlapping of two slices



Traditional reconstructions and their artifacts 2



Traditional reconstructions and their artifacts 3



Marschner and Lobb data set [4]

Inverse problem

- Model space **(reconstructed space … e.g. cube)**
- Data space **(defined by slices)**
  - **Result of the forward modeling**
- Forward problem
  - Mapping of the model space to data space **(generation of images)**

$$d = F(m) + n$$

- Inverse problem
  - Reconstruction of object from slices and scanner parameters

Solutions of Inverse problem

Non iterative methods (inverse filtering):

- Tikhonov-Miller (TM) regularized inversion (Wiener filter)

Iterative forms of inverse filtering:

- van Cittert: iterative form of direct inversion, with or without positivity constraint.
- Landweber method: steepest descent optimization form of TM with(out) positivity
- Conjugate gradients form of TM, with(out) positivity (ICTM).

(MLE)

- Expectation-maximization algorithm (EM).
- Maximum Entropy.
- Blind deconvolution (...including PSF estimation)

Restoration criteria

Minimize a functional (such as the Tikhonov functional)

$$\underbrace{\left| g - \overbrace{\left(\hat{f} \otimes h + b \right)}^{\text{forward modeling}} \right|^2}_{\text{error}} + \lambda \underbrace{\left| \hat{f} \right|^2}_{\text{regularization}}$$

The functional consists of two conflicting terms balanced by the regularization parameter λ.

- Error criterion between result and image

Gaussian noise:	mean-square-error
Poisson noise:	I-divergence

- Regularization term

  models a priori knowledge

  penalizes undesired (noise dominated) results

- These two terms are balanced by the regularization parameter λ

Wiener filter

- The Wiener restoration filter W minimizes the MSE between the restoration result and the ground truth.

$$\min_{W}\left\{\left|\hat{F}-F\right|^{2}\right\}$$

- Wiener requires an estimate of:
  - Transfer function H
  - Spectral density function of noise:
  - Spectral density function of the image: S_f

$$W(u,v)=\frac{H^{*}(u,v)}{\left|H(u,v)\right|^{2}+\frac{\sigma_n^2}{S_f(u,v)}}$$

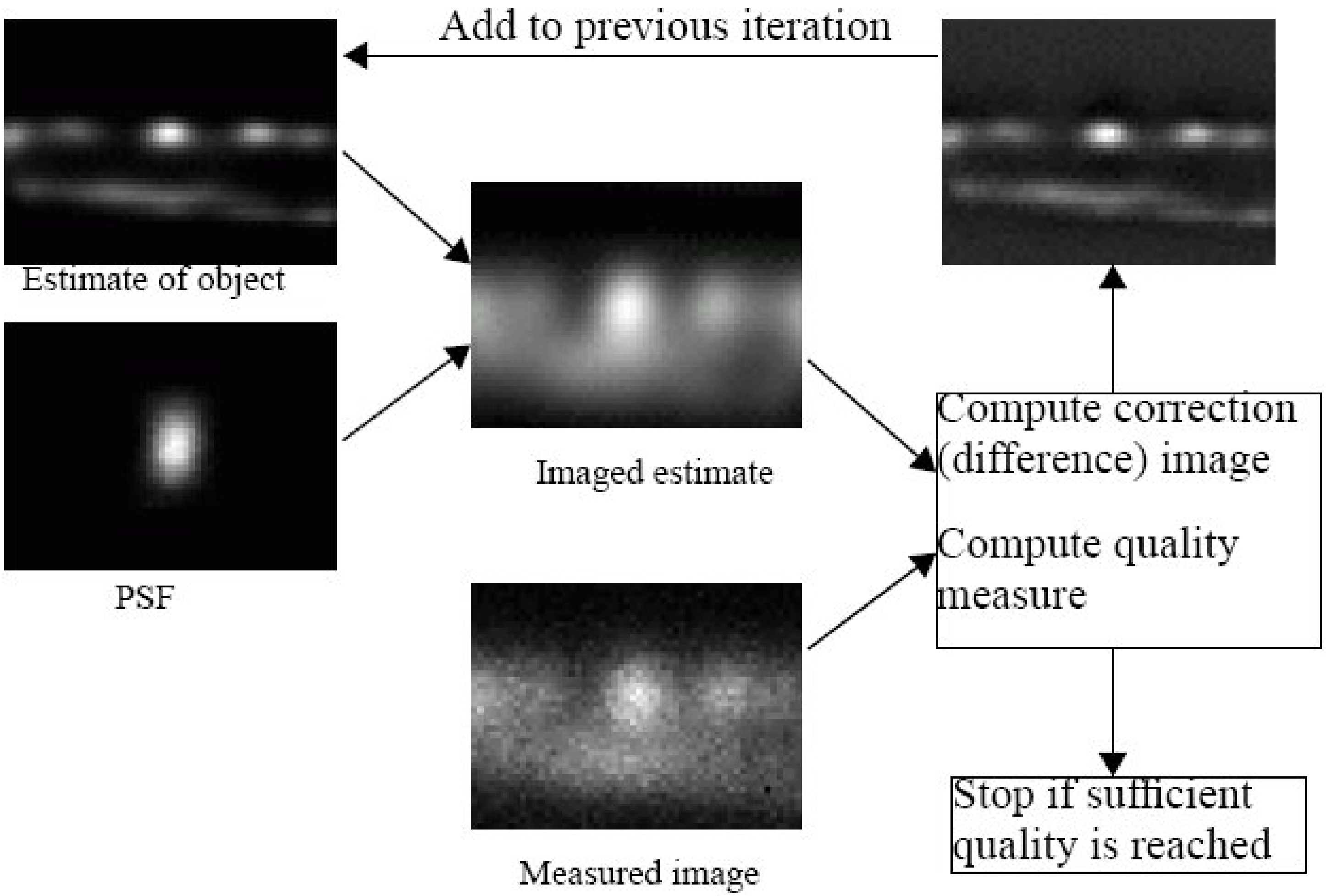
- If the spectral density functions are unknown, then

  Pseudo inverse
  - Question: How to choose K?

$$P(u,v)=\frac{H^{*}(u,v)}{\left|H(u,v)\right|^{2}+K}$$

The ‘van Cittert’ method I

- Optimize some quality measure of the estimate



The ‘van Cittert’ method I

The iterations are started for instance by setting the first estimate to the measured image.

It can be shown that the iterations converge to direct inversion. Still, with this technique we can now:

- Force a positivity constraint by clipping each new estimate. Importantly, this allows **recovery of lost spatial frequency components**.
- Stop if noise amplification becomes too severe

Iterative Constrained Tikhonov-Miller (ICTM) restoration

- The quality measure in the 'van Cittert' method is based on the difference between the measured image and the imaged estimate:

$$D_k = g - (h \otimes \hat{e}_k)$$

Quality measures for image restoration

The direct T-M and the ICTM method both optimize a (weighted) *Mean Square Error* criterion:

$$\mathrm{MSE}\ (g, e) = \|e - g\|^2$$

However, for solutions which are required to be non-negative (fluorescent objects!), Csiszár showed the *I-Divergence* distance measure (also known as cross-entropy) to be an optimal criterion:

$$I(e, g) = \sum e_i \ln(e_i / g_i) - \sum (e_i - g_i)$$

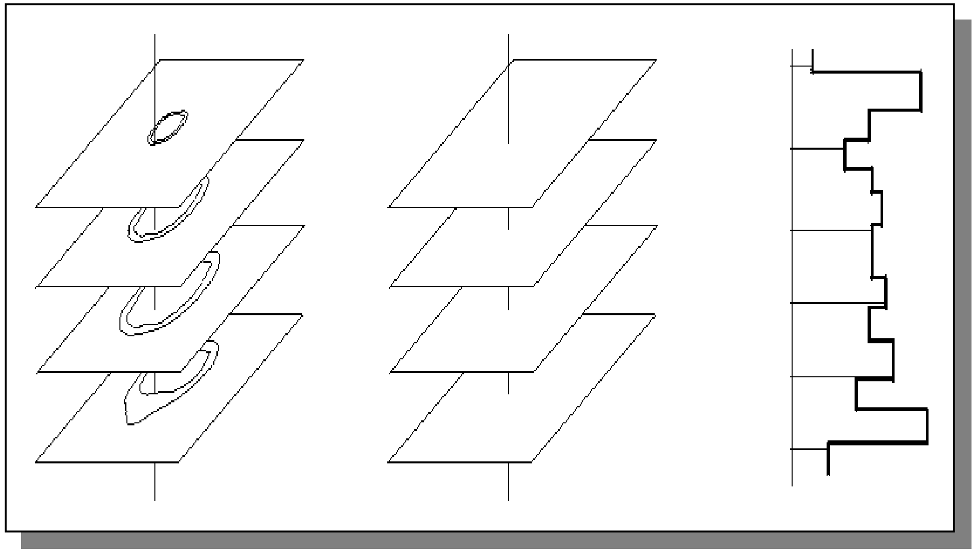
with the sum over all measured voxels.

Maximum likelihood estimation

- Forward problem - Information about scanner.

  - **Simplification:** *Transformation*

    to the 1D problem



- **Inverse problem** - Data object (1D function) modeling by MRF and simulated annealing

Interpolation and the probability

Solution of inverse problem: Bayesian relation form

d - scanned data

f - data model

$$p(f/d) = \frac{p(d/f) \times p(f)}{p(d)}$$

$p(f/d)$ … its maximization gives solution:

a posteriori information $f^* = \arg\max p(f/d)$

$p(d/f)$ … obtained from simulation of CT.

by forward modeling

$p(f)$ ……. main information about the object.

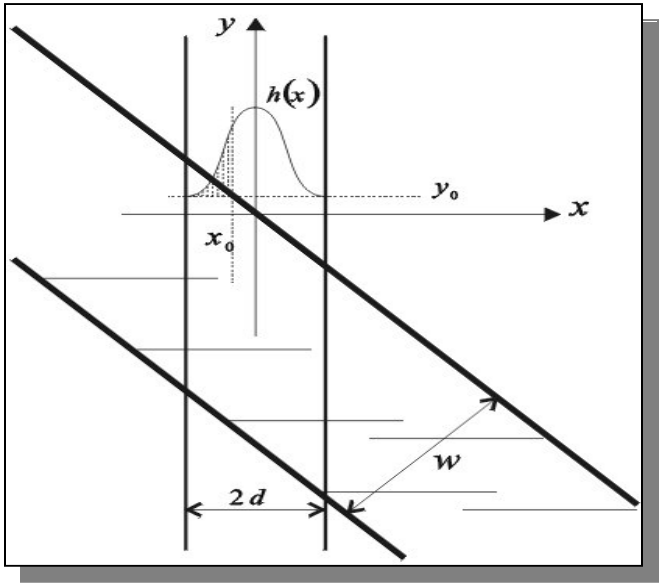
prior probability

$p(d)$ …… constant (*null information*)

Parameters of CT scanner

- Estimation of the Point Spread Function $h(x)$ of tomograph

$$f(y_0) = \int_{-\infty}^{x_0} h(x)dx$$



- We replace function $h(x)$ by *gaussian function* or *set of scanned data*.

- Expression of *p(d/f)* (it is Gaussian noise):

$$p(d/f) = \frac{e^{(-U(d/f)/T)}}{s\sqrt{2p}} \quad \text{where} \quad U(d/f) = \sum \frac{(fct_i - d_i)}{2s^2}$$

Point spread function

- Restoration algorithms require the PSF as input.
  The PSF can be obtained in the following ways:

- Theoretical PSF
  PSF calculated using diffraction theory
  PSF is noise-free, but does not account for optical aberrations

- Measured PSF
  PSF obtained from restoring images of beads of known size
  Measured PSF is not noise-free and may include restoration artifacts

- Blind Image Restoration does not require a PSF since it is estimated by the algorithm

Reconstruction by MRF

- *Markov random field* theory:

$$p(f) = \frac{e^{(-U(f)/T)}}{Z}$$ where $U(f) = \sum V_c(f);$

$V_c(f)$ is a potencial

- We assume Gaussian noise:

$$p(d/f) = \frac{e^{(-U(d/f)/T)}}{s\sqrt{2p}}$$ where $U(d/f) = \sum \frac{(fct_i - d_i)}{2s^2}$

$$\max[p(f/d)] \Leftrightarrow \max[p(f).p(d/f)]$$

$$\Downarrow$$

$$\min[U(f) + U(d/f)]$$

Energy function

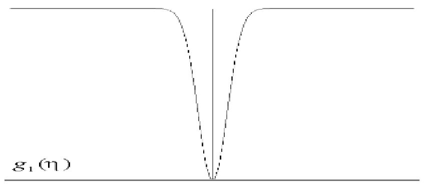
Smoothness constraint expressed by derivations. $V_c(f) = g(f')$

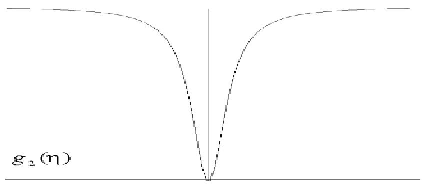
$$\min[\sum g(f') + \sum (fct - d)/2s^{2}]$$

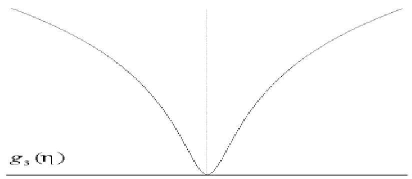
$$g_1(h) = -ge^{-(h^2/g)}$$

$$g_2(h) = -\frac{g}{1+\left(h^2/g\right)}$$

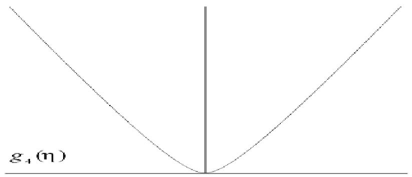
$$g_3(h) = g\ \ln(1+\frac{h^2}{g})$$



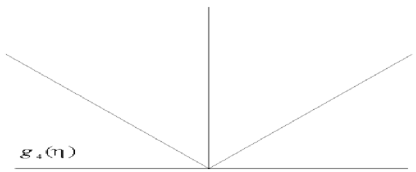




$$g_4(h) = g\ |h| - g^2 \ln(1+\frac{|h|}{g})$$



$$g_5(h) = |h|$$



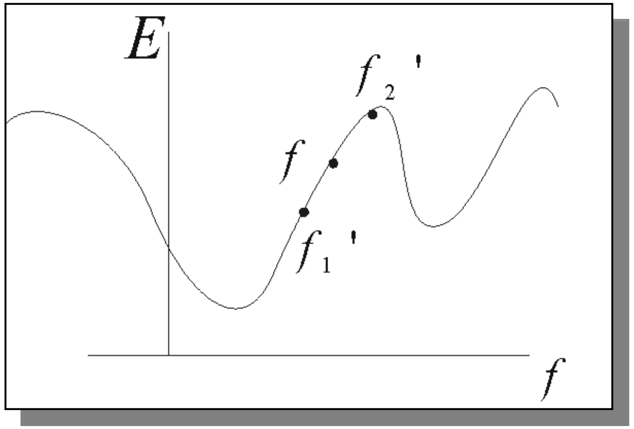
Minimization

- Simulated annealing
- Genetic algorithm
- Memetic Algorithm
- Mean-Field Annealing

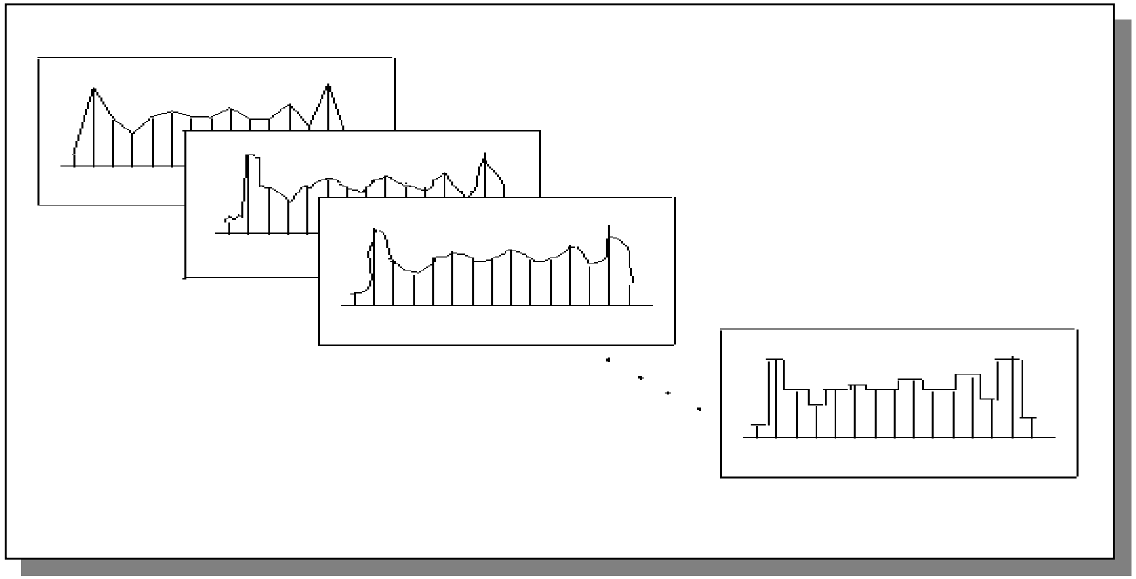
Minimization

Simulated annealing

$$\text{Prob}(E) = e^{-E/T}$$

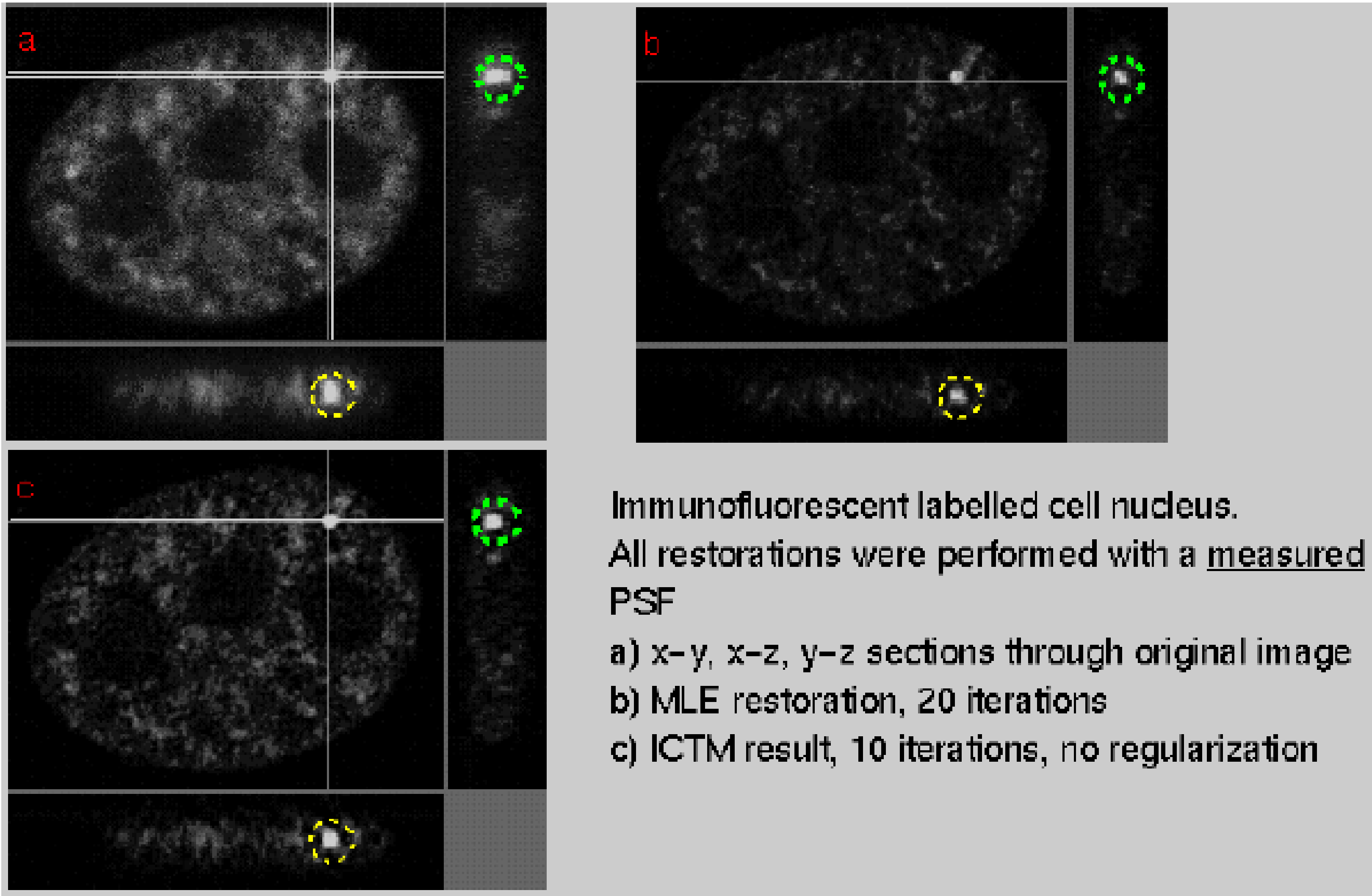


_SA_ algorithm:
initialize T and f
repeat
  Metropolis(f, T)
  decrease T
until ($T > T_{min}$)
return f



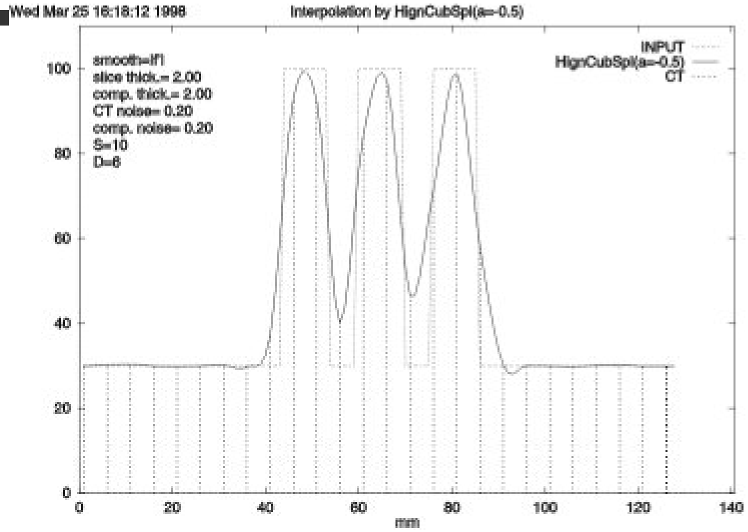
Metropolis sampler:
insert T, f
repeat
  $f \in$ rand(vinicity f)
  $\Delta E \leftarrow E(f') - E(f)$
  x $\in$ random (0,1)
  $p = e^{-\Delta E / T}$
  if ($x < p$) then f´ $\rightarrow$ f
until (equilibrium is reached)

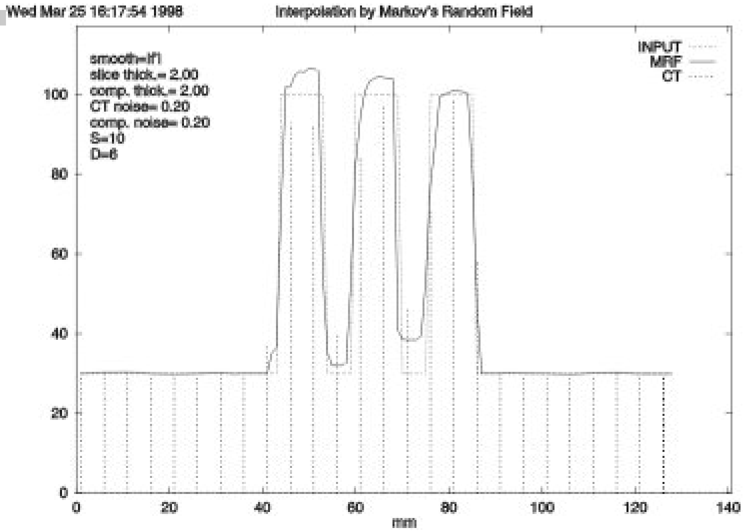
A comparison of ICTM and MLE restored images I

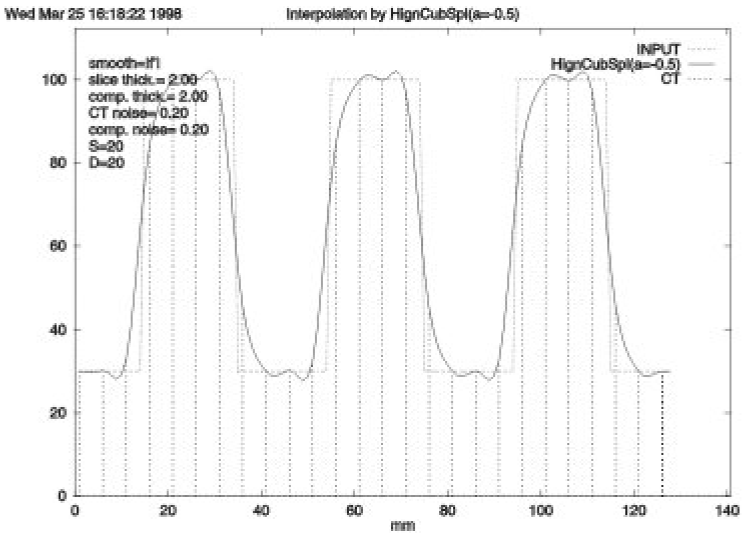


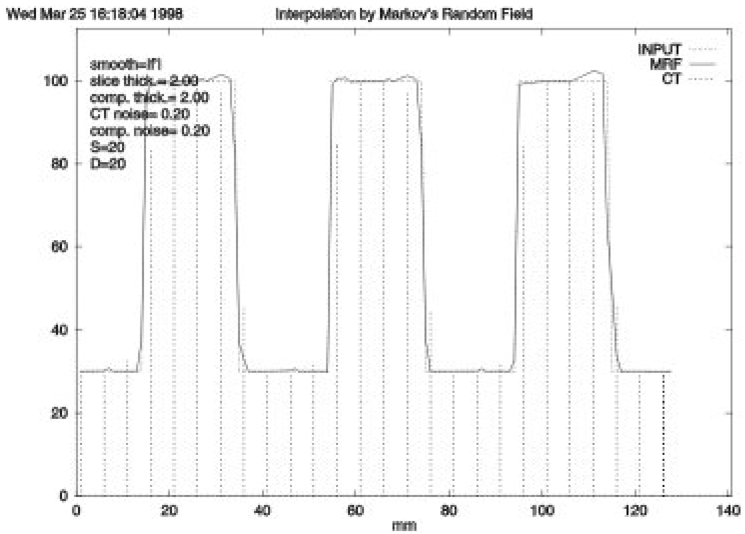
Results

Cubic interpolation | *Statistic interpolation*







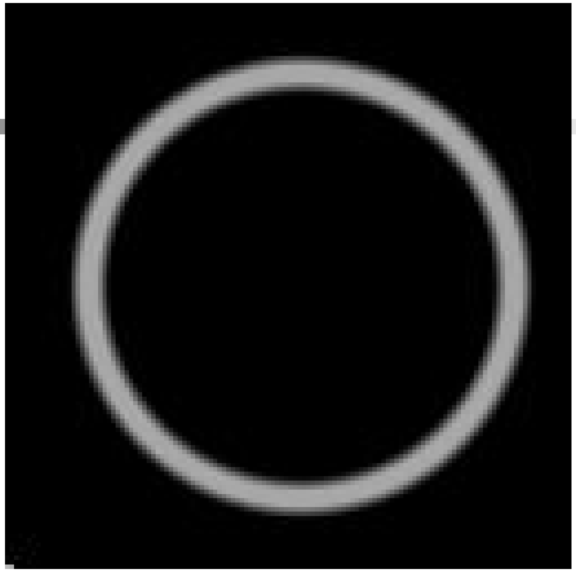


Overview

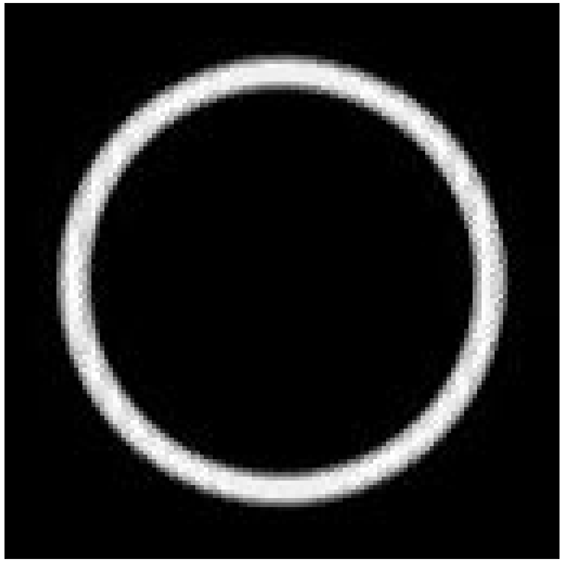
	Richardson-Lucy (+ many others)	ITCM, Carrington (+ many others)
Noise	Poisson	Additive Gaussian
Error criterion	I-divergence	mean-square-error
Functional	log-likelihood	Tikhonov
Regularization	—	Tikhonov (+ others)
Constraints	non-negativity	non-negativity
Algorithm	EM	Conjugate gradients

Results

Original object



Cubic interpolation

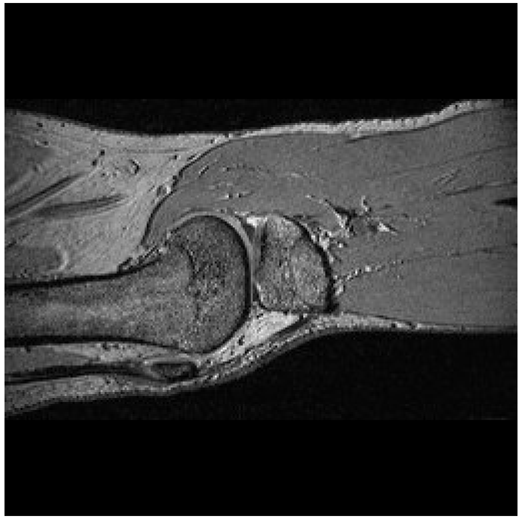


Statistic interpolation

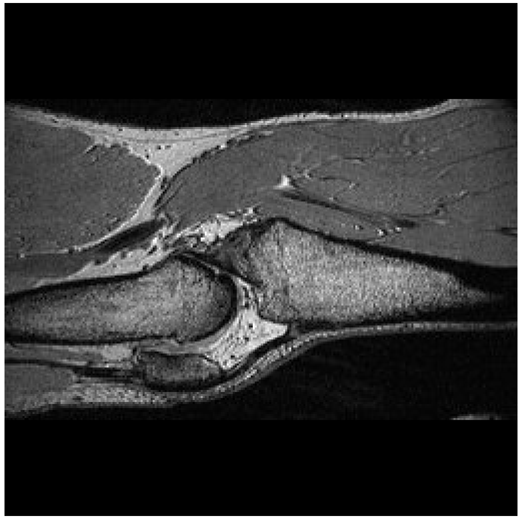
Results

1.

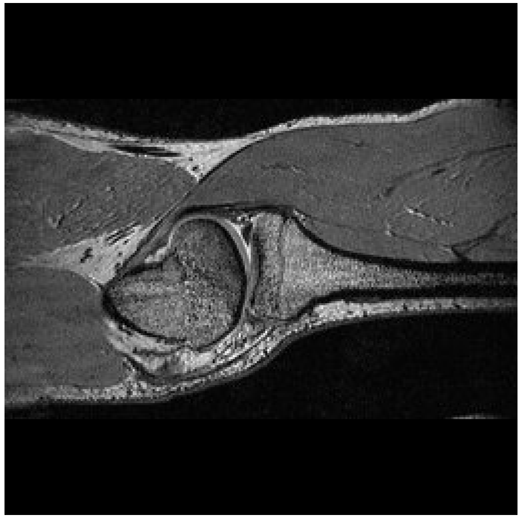
Original data:



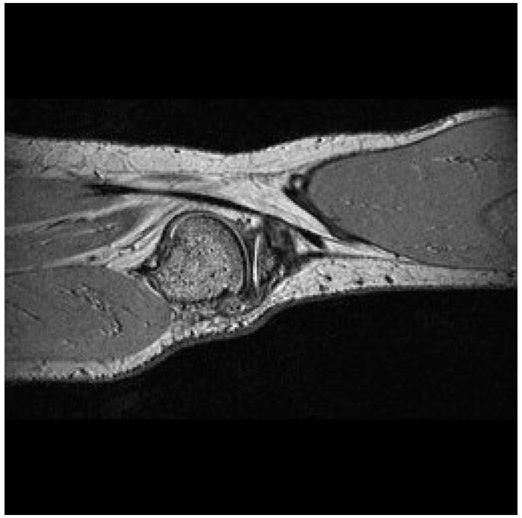
2.



3.



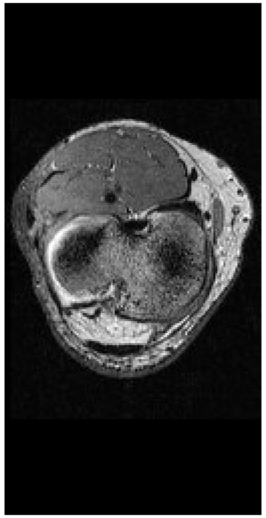
4.



Results

Original

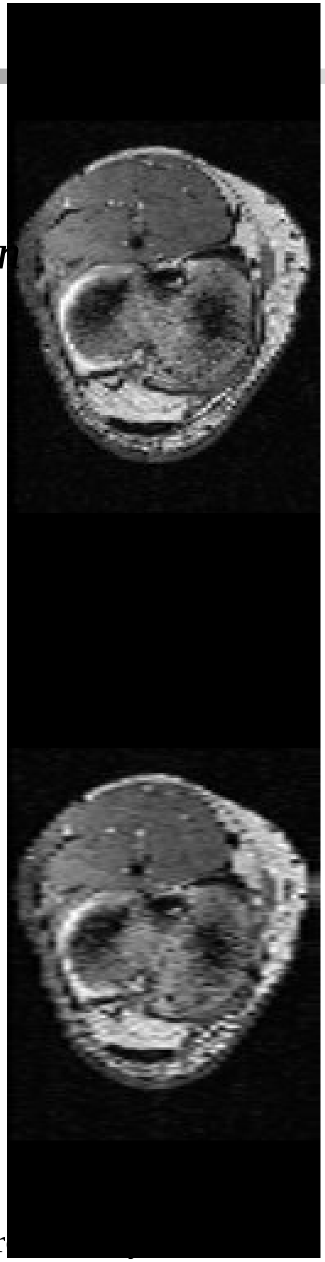
Statistic interpolation



Sampling = 1

Cubic interpolation

Vzorkovanie = 3



Vzorkovanie = 4