Transfer Functions

Miloš Šrámek

Light Attenuation

- Interaction with matter results in absorption
  - Beer-Lambert law:

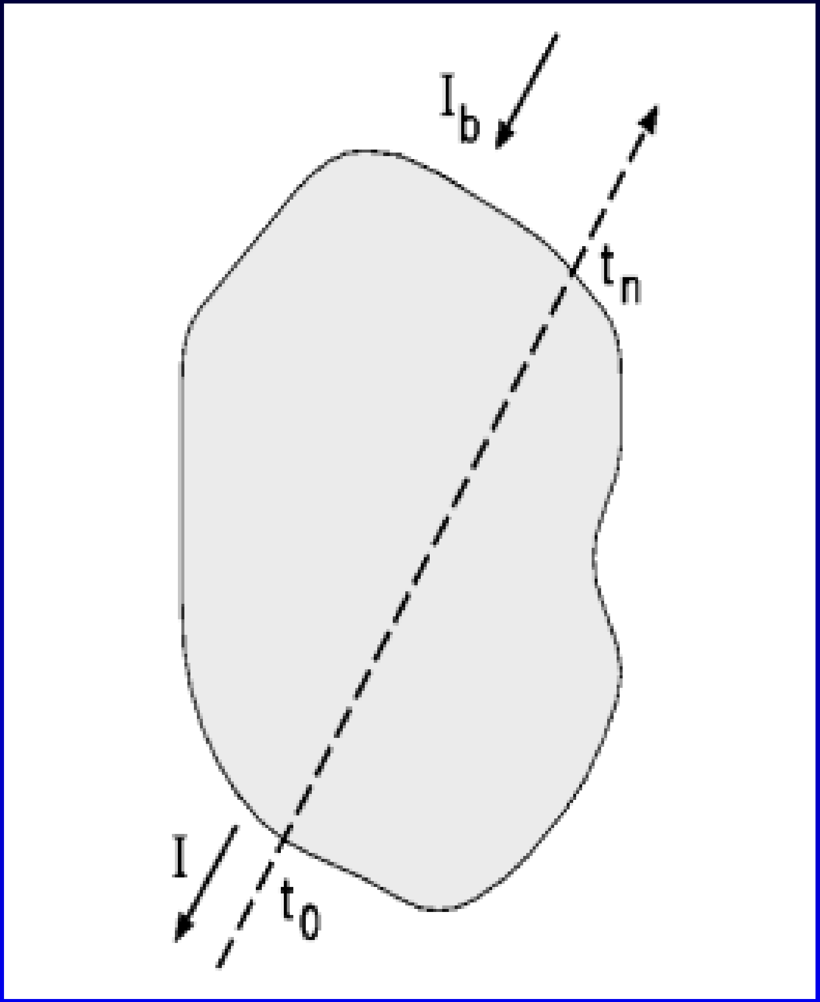
$$\frac{dI(t)}{dt} = \rho(t)I(t) - k(t)\rho(t),$$

  - $\rho(t)$: optical density, a measure of attenuation
  - $k(t)$: chromacity, color

Volume Rendering Integral

- Integral form of the Beer's law

$$I = \int_{t_0}^{t_n} k(t)\rho(t)e^{-\int_{t_0}^{t} \rho(u)\,du}\,dt + I_b e^{-\int_{t_0}^{t_n} \rho(t)\,dt}$$



Per Segment Evaluation of the VRI

- FTB
$$I_m = I_{m-1} + (1-\beta_{m-1})C_m$$
$$\beta_m = \beta_{m-1} + (1-\beta_{m-1})\alpha_m$$
under operator
- BTF
$$I_m = \alpha_m C_m + (1-\alpha_m) I_{m-1}$$
over operator

where:

- segment opacity $\alpha_i = 1 - e^{-\int_{t_i}^{t_{i+1}} \rho(u)\,du}$
- *Accumulated* opacity of m segments β_m
- *Color of i*-th segment
$$C_i = \int_{t_i}^{t_{i+1}} k(t)\rho(t)e^{-\int_{t_i}^{t} \rho(u)\,du}\,dt$$

Approximations

- **Aproximation of ρ and *k* by a constant:**

$$\alpha_i = 1 - e^{-\rho_i \Delta t_i}$$

$$C_i = k_i(1 - \alpha_i),$$

- **ρ is a li**

$$\alpha_i = 1 - e^{-\frac{\rho(t_i)+\rho(t_{i+1})}{2}\Delta t_i}.$$

Volume Rendering Equation

- We need opacities and chromacities.
- We have only densities.
- What shall we do?

How to Get Opacity/Chromacity from Density?

- Unshaded projection
- Density transfer functions
- Edge amplification
- Shaded projection
- Special transfer functions

Unshaded Projection, Reprojection

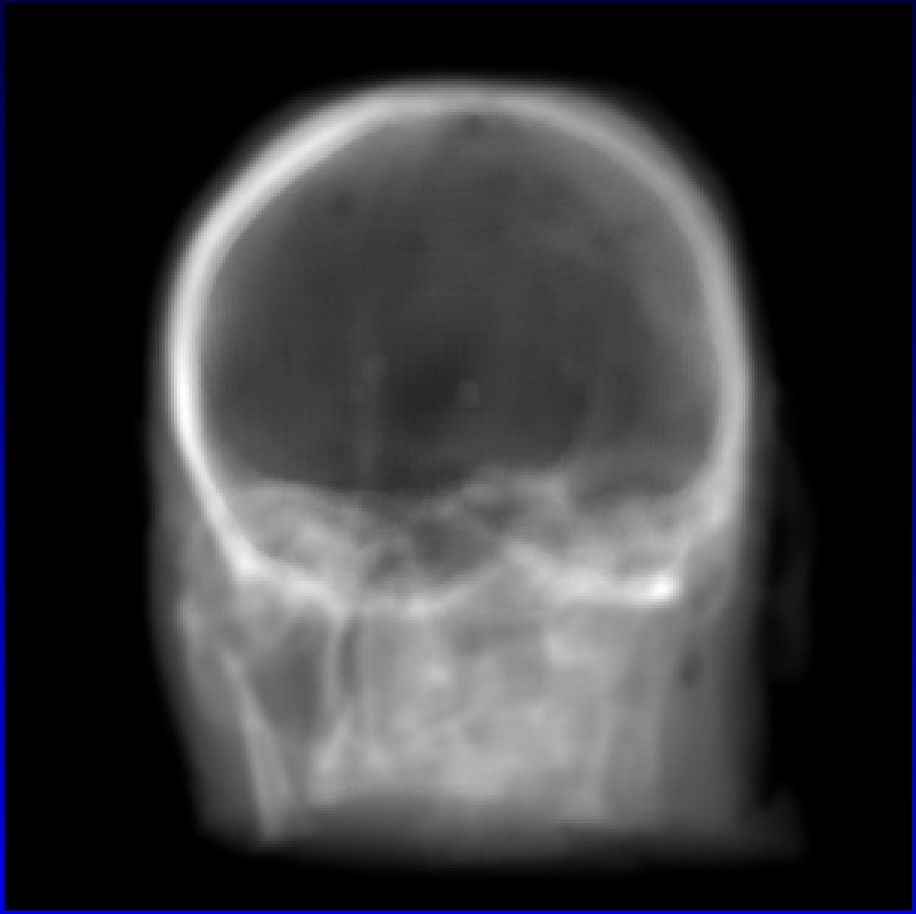
- Use densities directly, i.e.:

$$\rho(p) = d(p)$$
$$k(p) = 1$$

p **is a position in the volume**

d(p) is density at p

Unshaded Projection



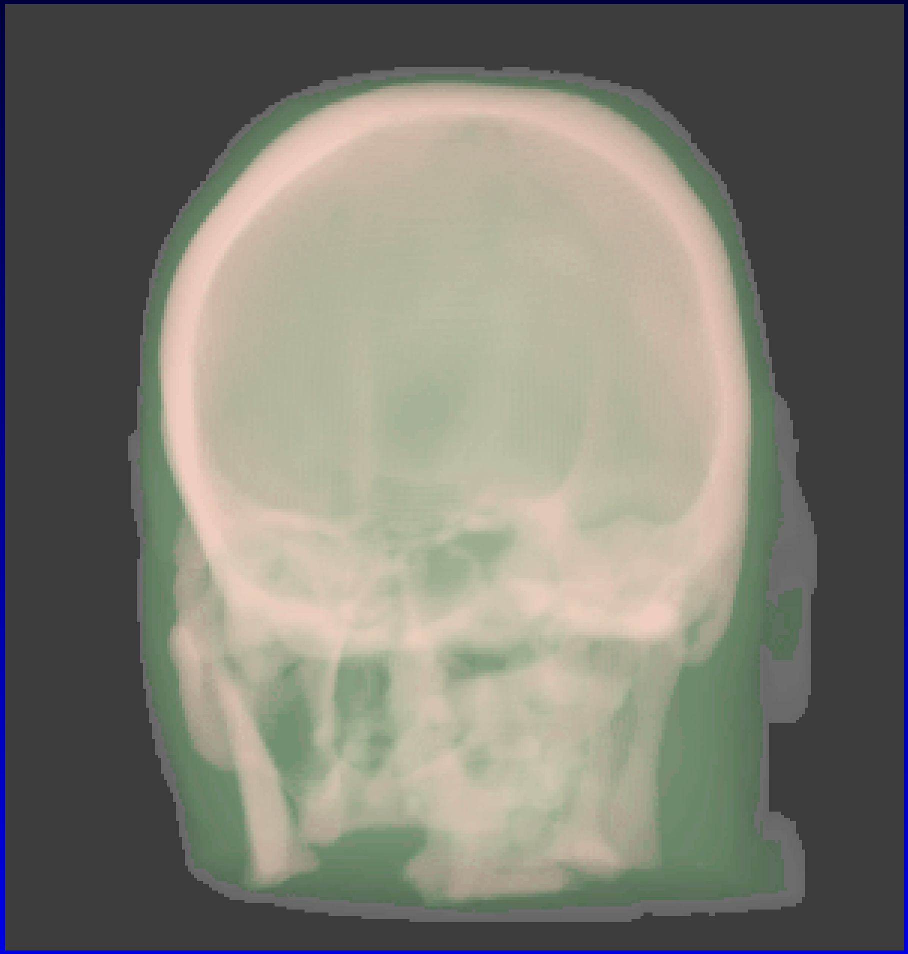
Density Transfer Functions

- **Use functions to assign opacities and chromacities to each density, i.e.:**

$$\rho\left(p\right)=f\left(d\left(p\right)\right)$$
$$k\left(p\right)=g\left(d\left(p\right)\right)$$

- **Where *p* is a position in the volume**
- **d(p) is density at p**
- ***f* and *g* are transfer functions**

Density Transfer Functions



Density Transfer Functions

- Transfer functions can be of any shape and complexity.

- But they assign the same value to a particular density regardless of its position and environment.

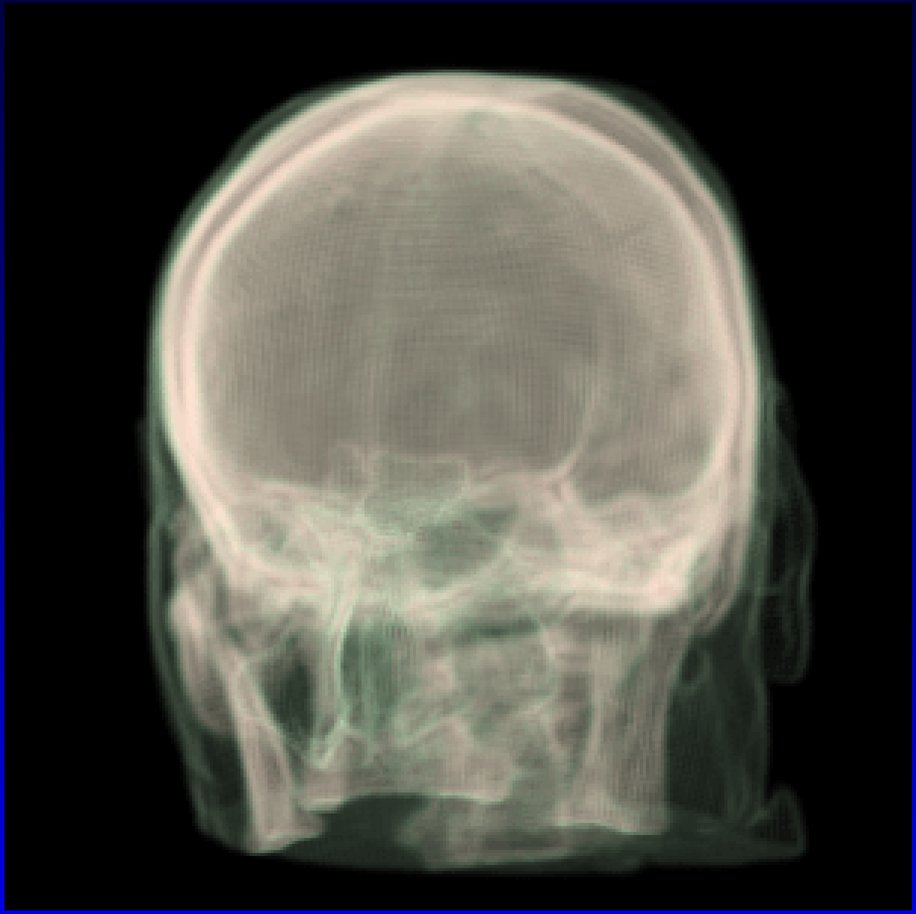
Edge Enhancement

- Transfer functions assign opacities and chromacities to gradient magnitudes:

$$\rho(p) = f\left(\left|\vec{g}(p)\right|\right)$$
$$k(p) = g\left(\left|\vec{g}(p)\right|\right)$$

- This results in edge enhancement, but only as good as the edge detector/gradient operator used.

Edge Enhancement



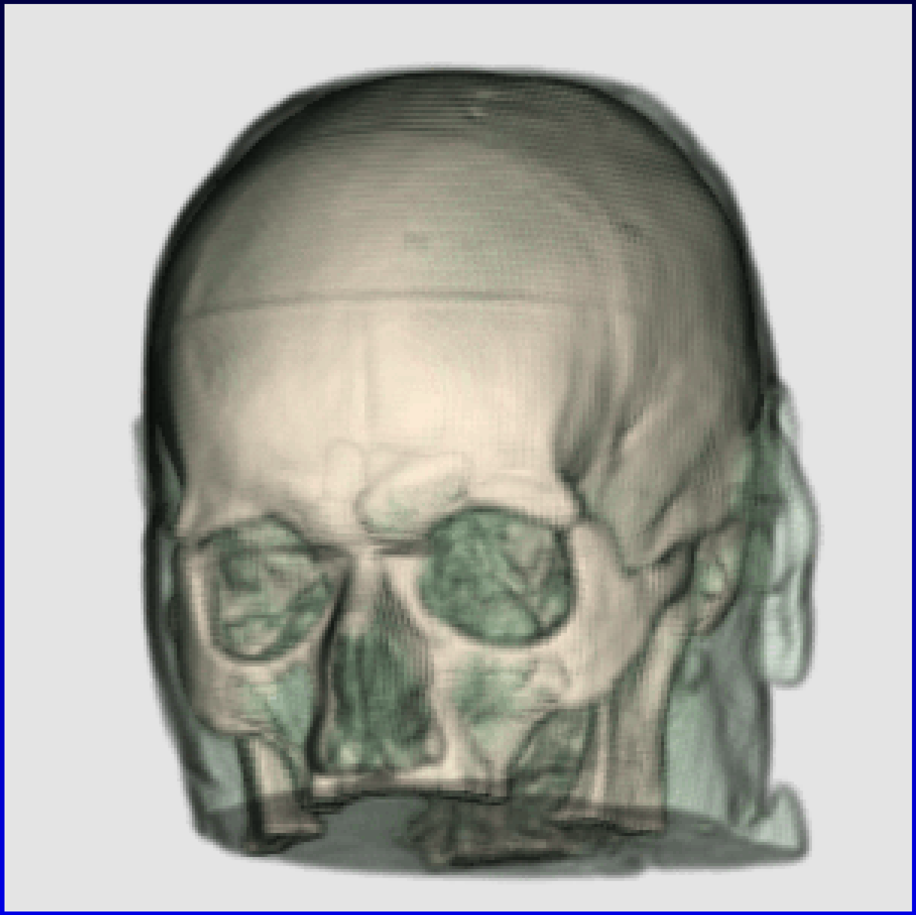
Shaded Projection

- Use gradients to compute colors, basically evaluating the shading equation at each volume sample point using the gradient there as surface normal.

$$\rho(p) = f\left(\frac{\vec{g}(p)}{|\vec{g}(p)|}\right)$$

- This results in view-dependent surface enhancement.

Shaded Projection



Special Transfer Functions

- These are functions that assign opacities/chromacities to some property derived at the sample points, e.g. curvature.

- The problem is how to choose the optimal one among the many possible shapes.

How to approximate the VRI?

- Ray-casting algorithms
  - Image order traversal
- Projection algorithms
  - Object order traversal

Brute-force Ray-casting

- For each pixel:
  - For each sample along a ray:
    - Compute color
    - Weight color by opacity
    - Accumulate color and opacity
    - Determine next sample
  - Pixel gets accumulated color

Ray-casting Fundamentals

- Front-to-back traversal:

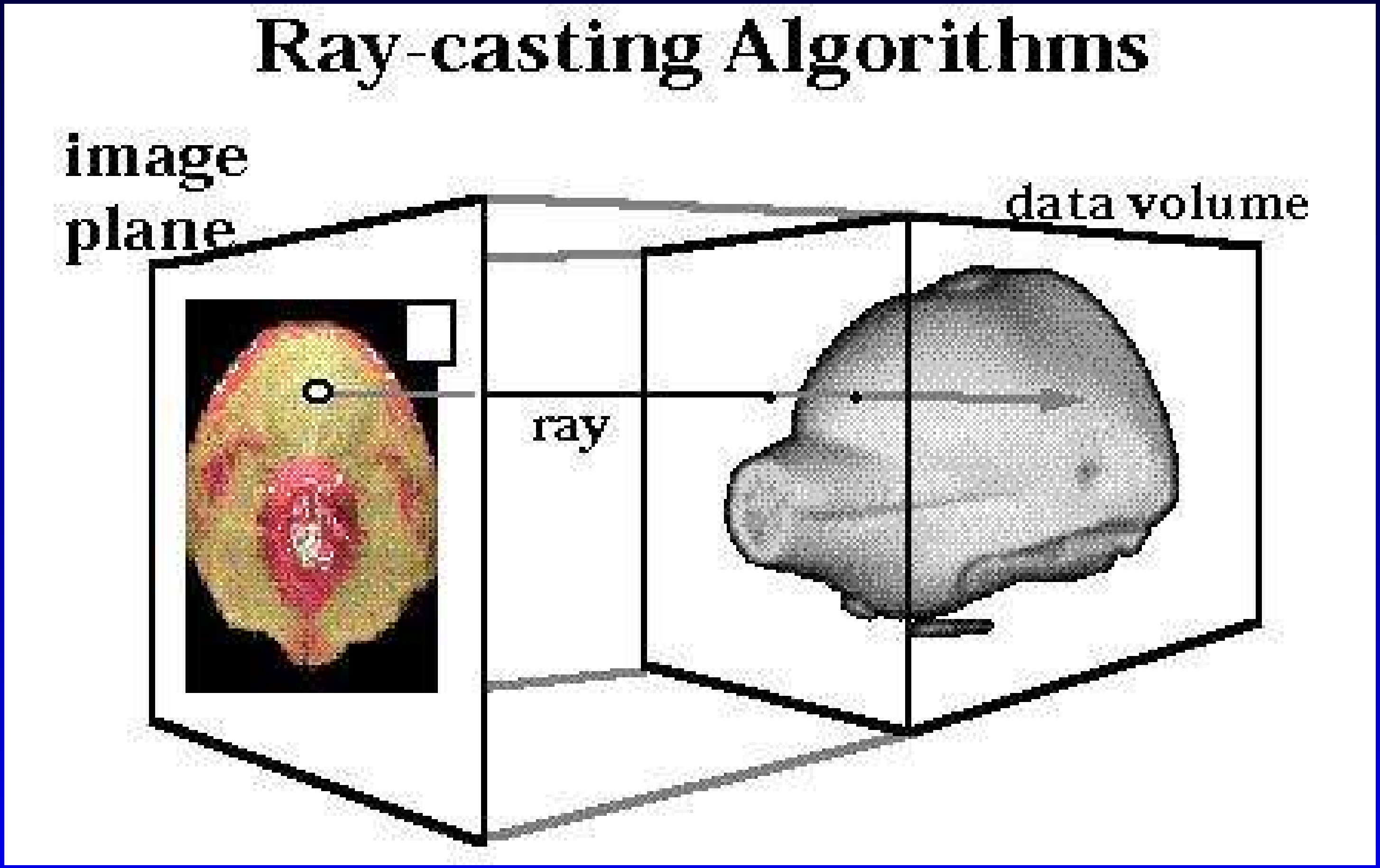
$$I_m = I_{m-1} + (1 - \beta_{m-1}) C_m$$
$$\beta_m = \beta_{m-1} + (1 - \beta_{m-1}) \alpha_m$$

under operator

- Early termination if $\alpha_i \geq 1$
- Final step:

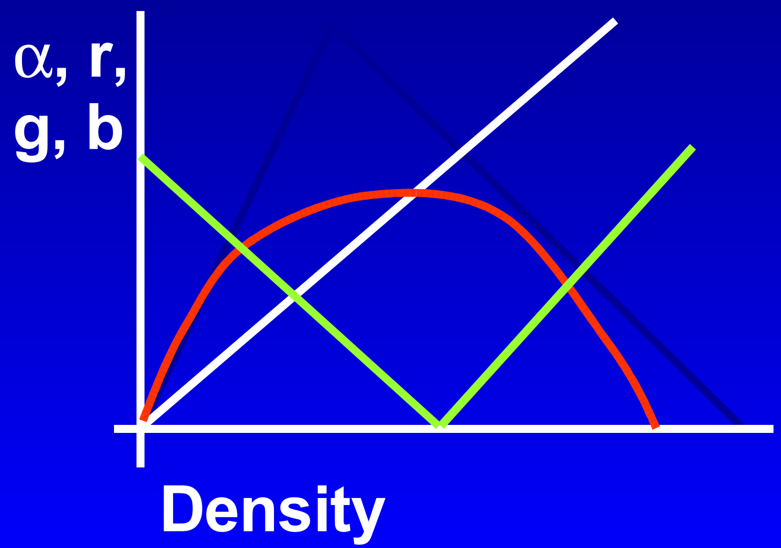
$$c_f = c_i \alpha_i$$

Ray-casting Fundamentals



Definition of Transfer Functions

- **Purpose: enhancement / suppression of desired / unwanted features of data**
- **Problem: too many degrees of freedom:**

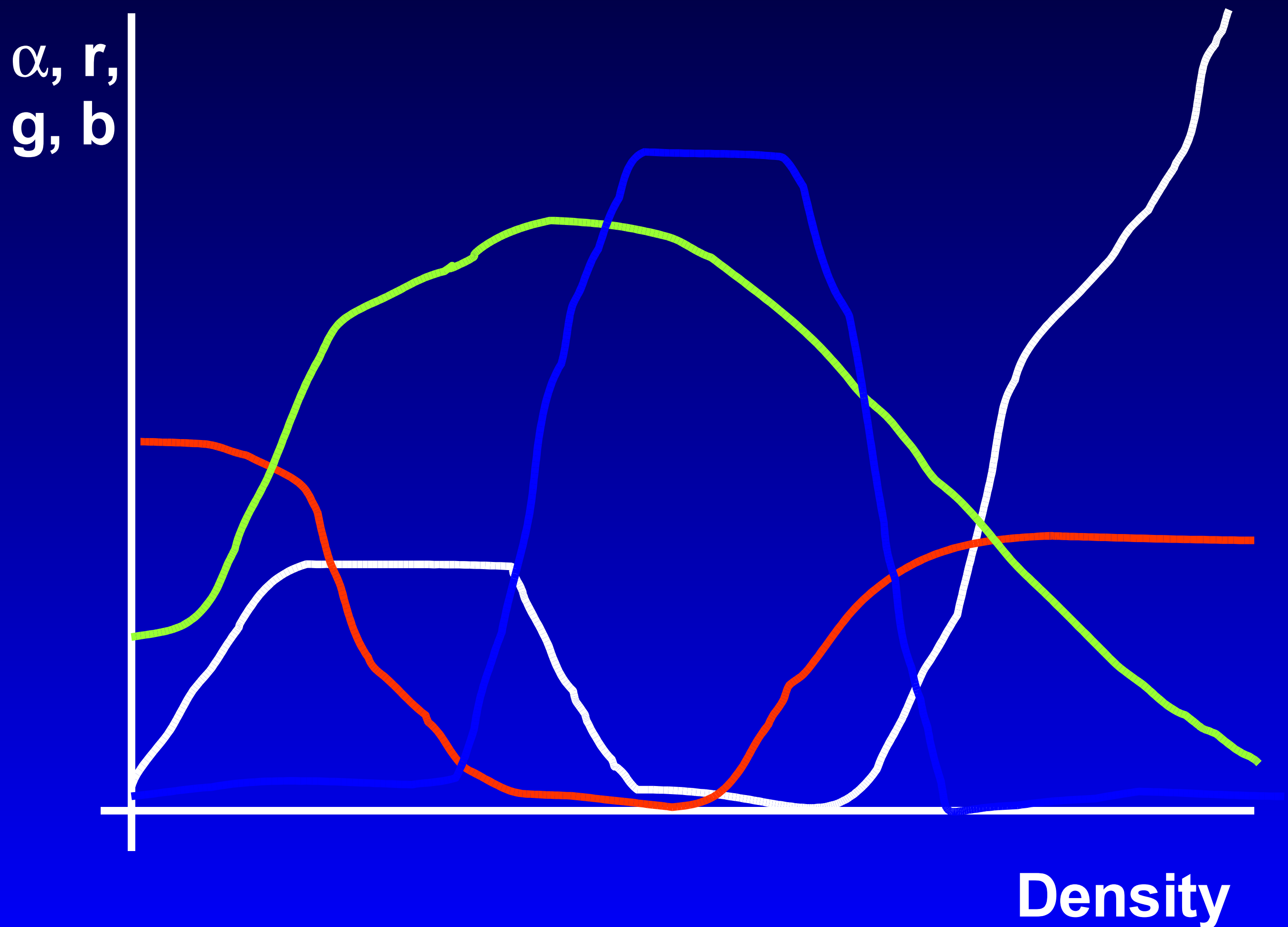


Definition of Transfer Functions (TF)

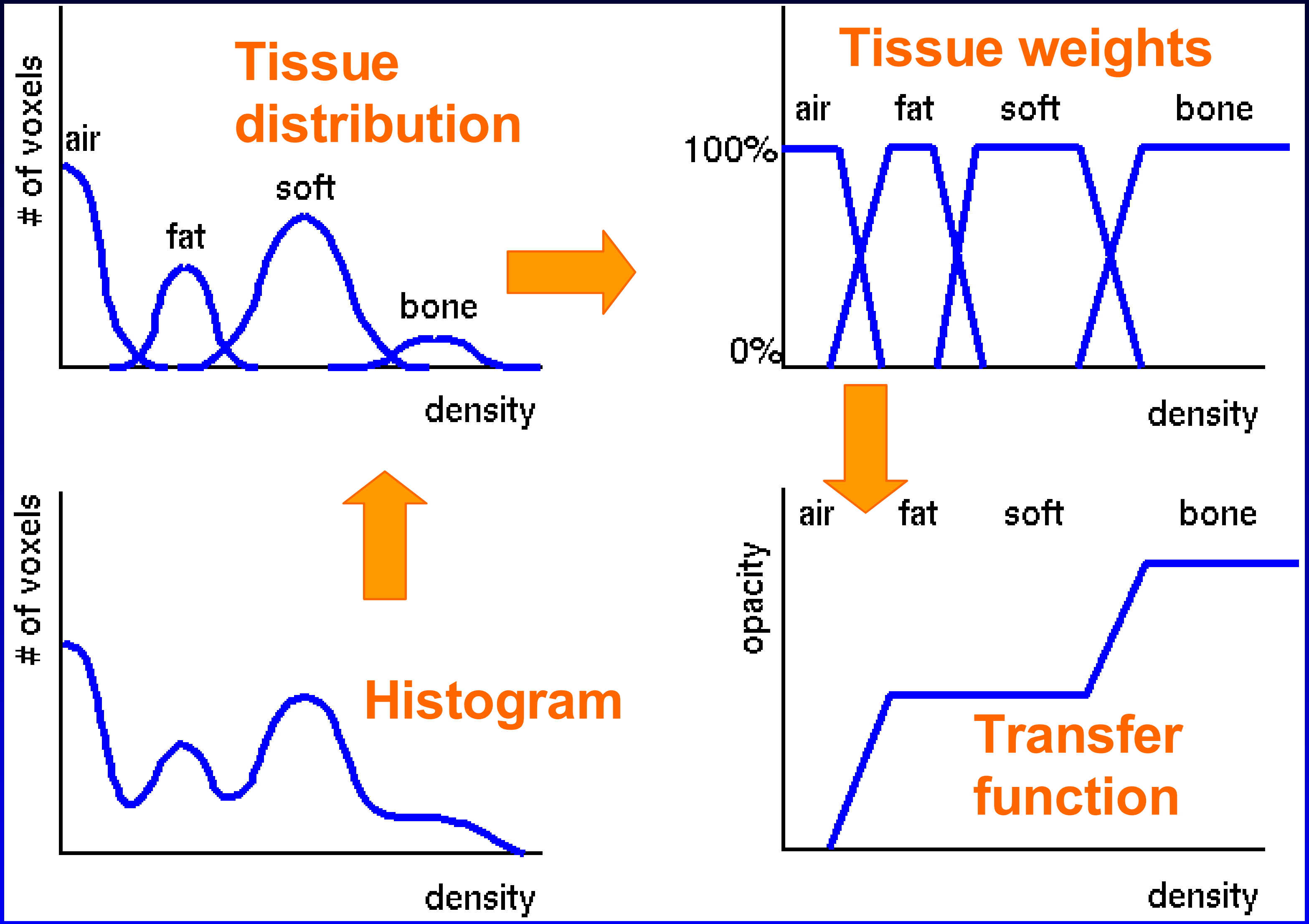
- Manual (trial & error) setting:
  - Freely (hand) drawn curves: Hard to achieve meaningful result
  - Piecewise linear: Based on tissue classification
- Computer assisted setting:
  - Interactive evolution
  - Inverse design
  - Design galleries

Hand-drawn TF

- A typical result:

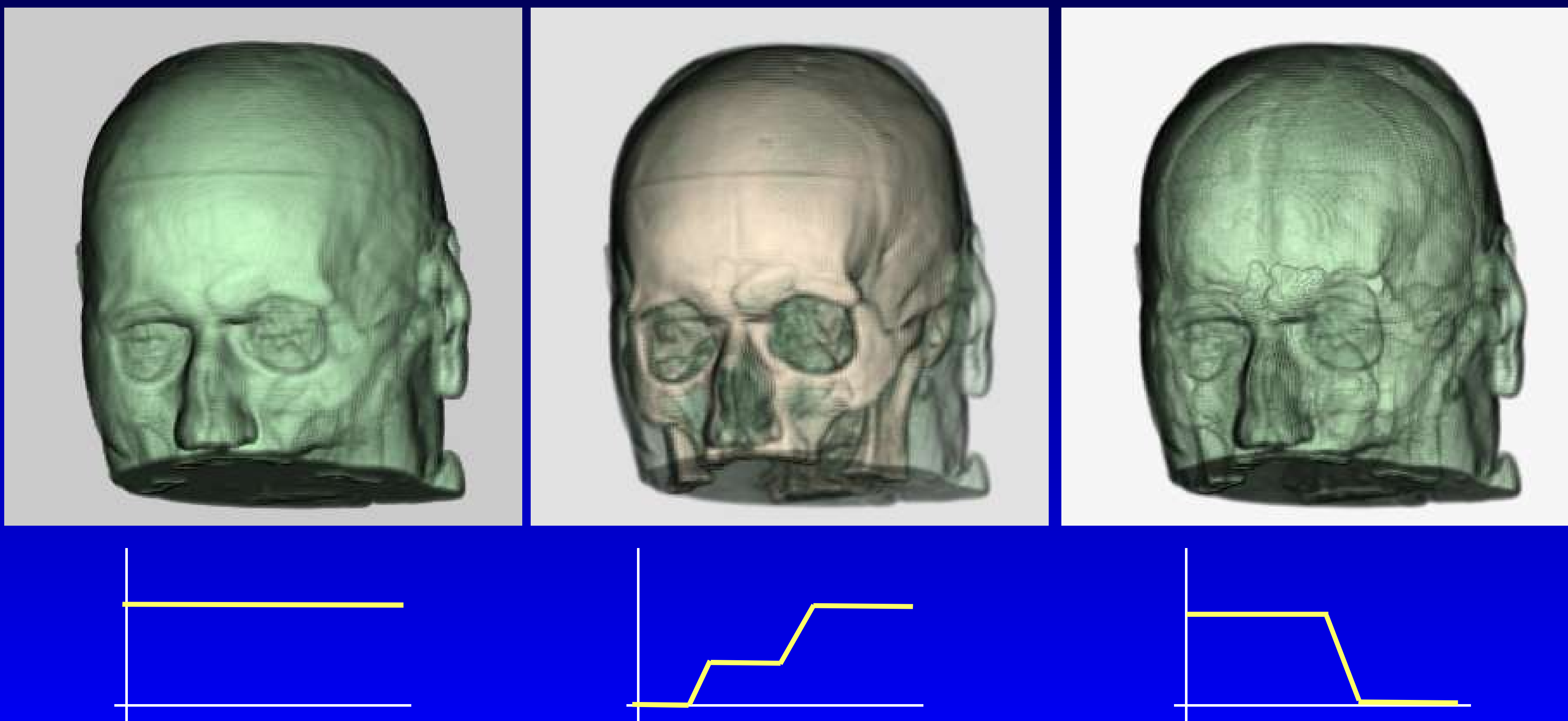


Piecewise Linear TF (with Tissue Classification)



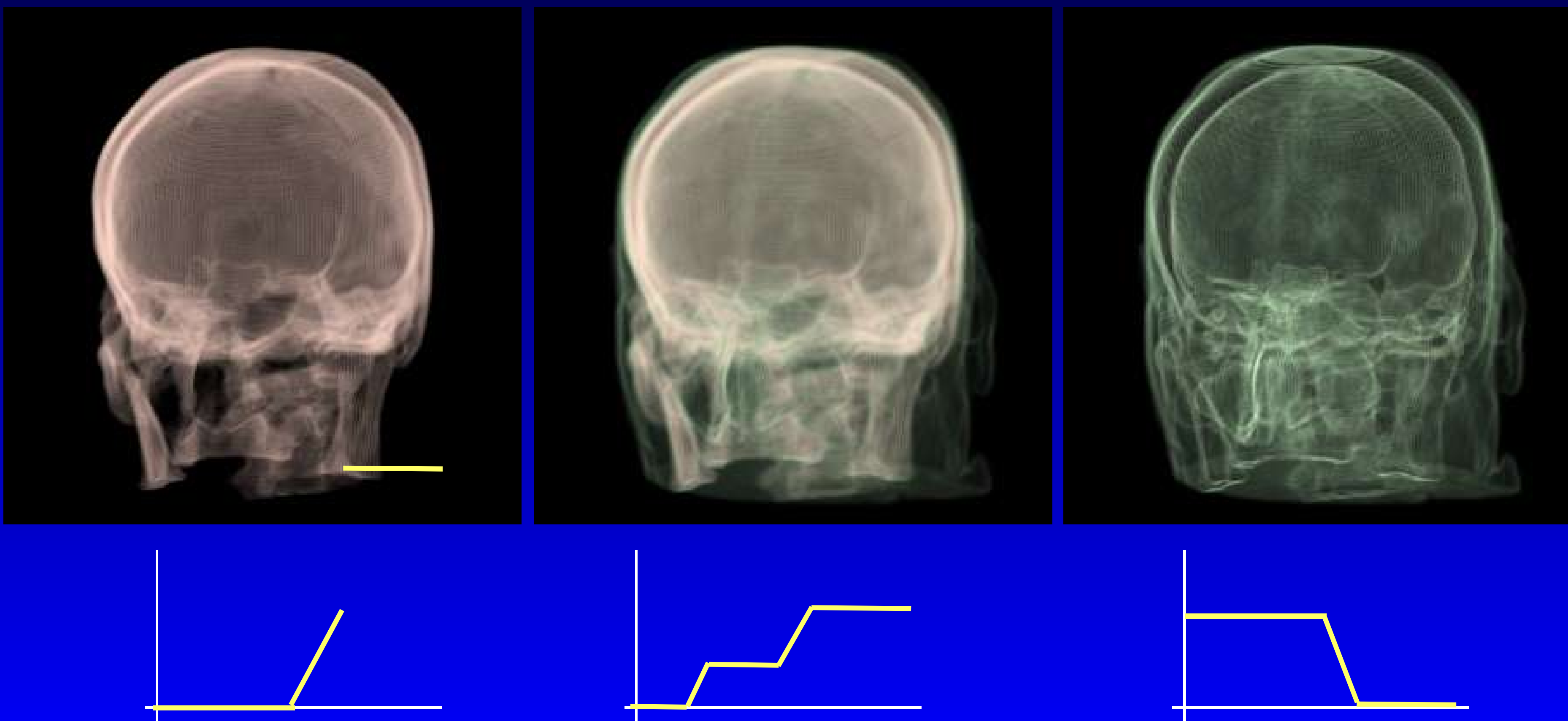
Piecewise Linear Opacity TF

Shaded projection



Piecewise Linear Opacity TF

Unshaded projection with edge enhancement

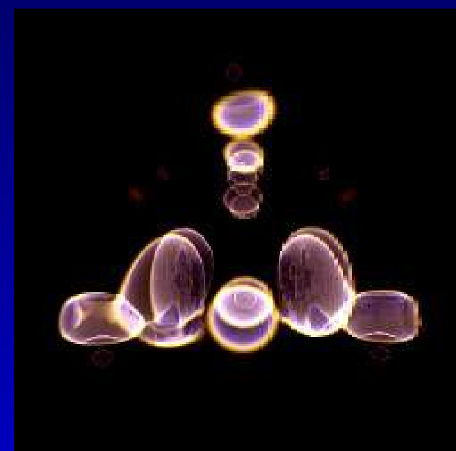


Inverse Design

- Optimization according to a criterion (He 1996):

Image entropy

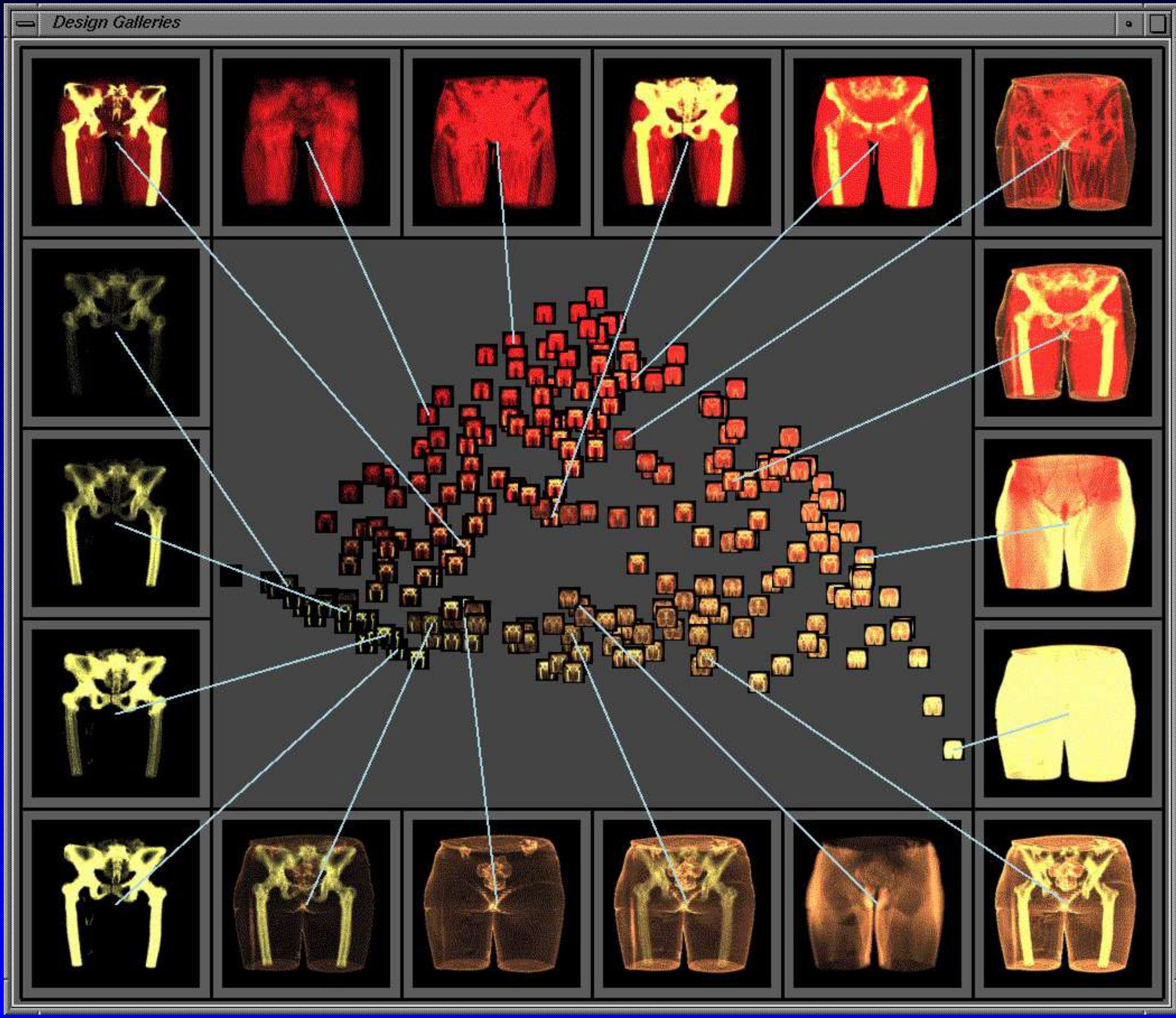
Image variance



Edge content

Combination

Design Gallery™ (Marks 1997)



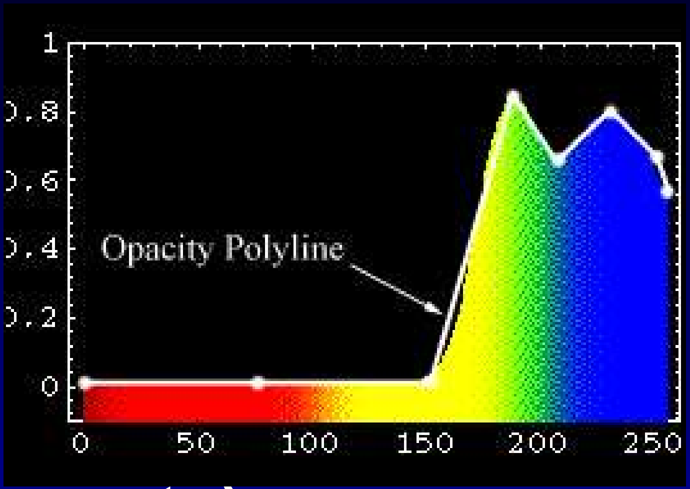
Design Galleries (DG)

- Automatically generated selection of perceptually different images
  - Generated off-line
  - Requires similarity measure (distance between images)
  - No optimality measure required

DG Key Elements

- *Input vector* - a set of parameters, that control output graphics
- *Mapping* from input to output vectors
- *Output vector* - subjectively relevant qualities of output image
- *Distance metrics* between output vectors
- *Dispersion method* - find a well-distributed set of output vectors
- *Arrangement* - result presentation

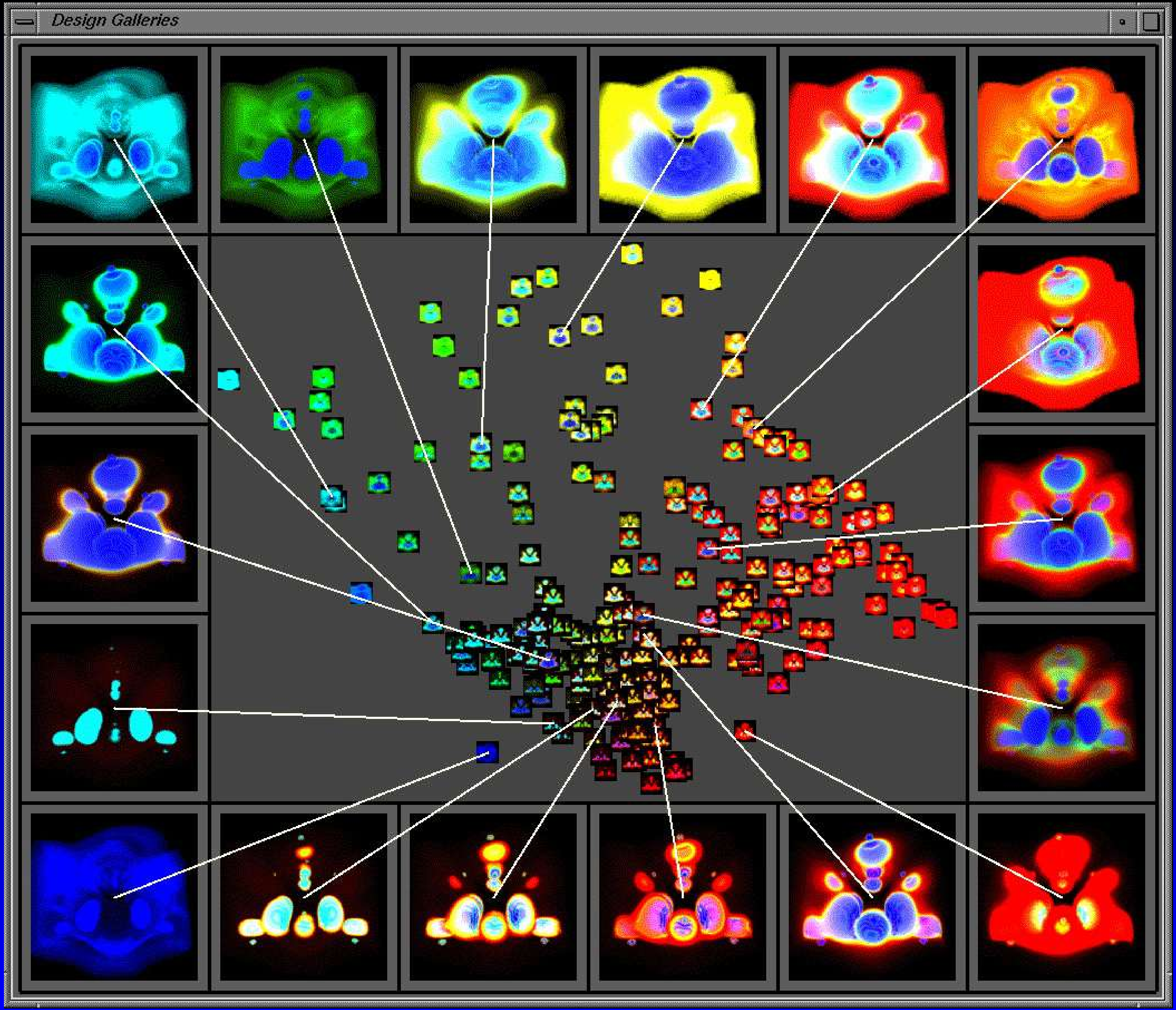
TF generation by means of Design Galleries



- **Input vector**:
  - Opacity TF: 8 control points (16 parameters)
  - Color TF 6 subranges (red, yellow, green, cyan, blue, magenta)
- **Mapping**: A volume rendering technique
- **Output vector**: 8 manualy selected samples (24 values)
- **Distance metrics**: Euclidean

TF generation by means of Design Galleries

- **Dispersion heuristics:**
  - **Repeat (2 000 000 times)**
    - **Select random input vector I**
    - **Perturb the vector I** to **I'**
    - **If I'** ***is better than*** **I**
      - **Replace I by I'**
  - **End**
- **Arrangement**
  - **Embedding in 2D space, with distances kept**
  - **Thumbnail images**

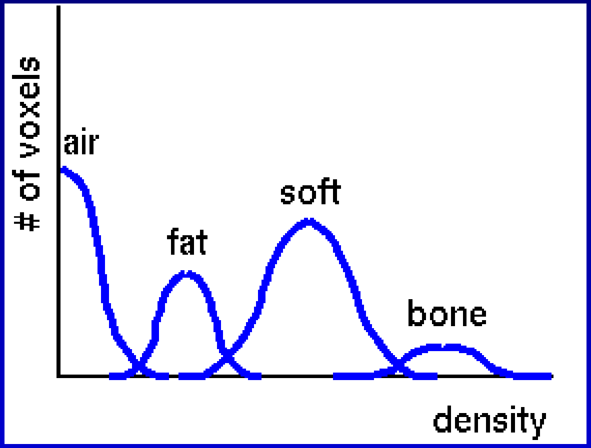

Design Galleries

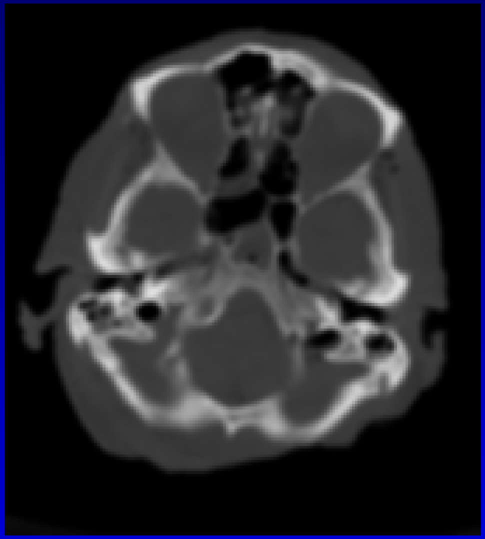
DG - Labor Division

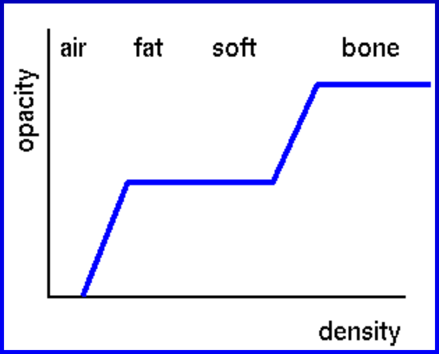
- DG Designer
  - Input and output vectors, metrics, dispersion and arrangement
  - Must understand the visualization technique
- Computer
  - Does the work
- User
  - Uses the results
  - No deeper insight is necessary

Data Suitability for Volume Rendering

- CT data: correspondence between histogram and spatial arrangement







Data Suitability for Volume Rendering

- MR Data: No such correspondence